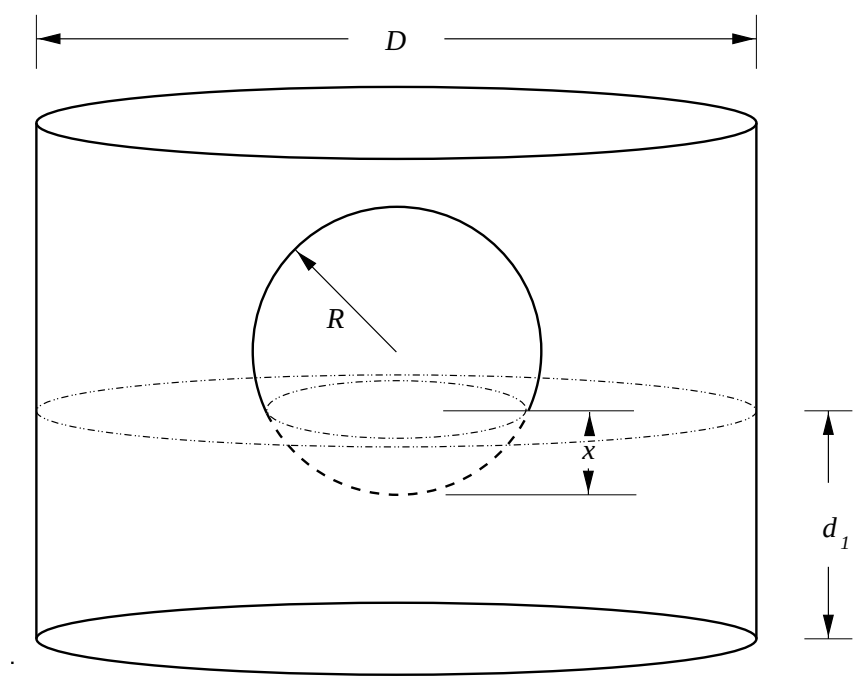
Newton's Method

`Purpose:` Use Newton's Method to (approximately) solve a nonlinear equation for the depth to which a sphere sinks. Compare the calculated and measured depths.

A. Apparatus

1. 2 tanks of different sizes
2. cloth measuring tape
3. water
4. sphere
5. ruler

B. Diagram



C. Nomenclature

1. D = diameter of the tank.
2. C = circumference of the sphere.
3. R = radius of the sphere.
4. d_0 = initial depth of the water in the tank.
5. d_1 = depth of the water after the sphere is placed in the tank.
6. x = depth to which the sphere sinks.
7. x_m = measured depth to which the sphere sinks.
8. x_c = calculated depth to which the sphere sinks.
9. v_s = volume of the sphere below the water surface.
10. $\rho = 1\text{g/ml}$ = the density of water.
11. M = mass of the sphere.

`Note:` All measurements are to be obtained and recorded in S.I. units, using an appropriate number of significant digits.

D. Equations

1. Circumference of the sphere: $C = 2\pi R$.
2. Volume of the sphere below the water level: $v_s = \frac{1}{4}\pi D^2 (d_1 - d_0)$.
3. Volume of the sphere below the water level: $v_s = \frac{1}{3}\pi x^2 (3R - x)$.

E. Basic Procedure

1. Measure D. Calculate R.
2. Place water in tank and measure d_0.
3. Place sphere in tank.
   a. Measure d_1.
   b. Calculate v_s.
4. Measure x_m.
5. Formulate x as the root of a polynomial.
6. Calculate x_c.
7. Repeat E.1–E.6 using the larger tank.
8. Extra Credit: Find the mass of the sphere.

F. Detailed Procedure

1. Use the ruler to measure D. Use the cloth measuring tape to measure the C. Use (D.1) to calculate R. Record D, C and R in table 1.
2. Fill the tank with water to a depth of approximately 10cm. Use the ruler to measure d_0. Record d_0 in table 1.
3. Gently place the sphere into the tank.
   a. There will be a rise in the water level. Use the ruler to measure d_1. Record d_1 in table 1.
   b. Use (D.2) and the measured d_0 and d_1 to calculate v_s. Record v_s in table 1.
4. Use the ruler to measure x_m. Record x_m in table 1.
5. Express (D.3) as a cubic polynomial in x, and write this equation in table 1.
6. Calculate x_c in (F.5) using the following procedure:
   a. Substitute the values for R and v_s obtained in (F.1) and (F.3b).
   b. Use Newton's method to find the root x_c, Record x_c in table 1. Use R as your initial guess.
7. Repeat (F.1)–(F.6) using the larger tank.

8. Extra Credit: Find M.

G. Discussion

I. Two experiments were conducted in which the depth to which a sphere sinks was both measured, x_m, and calculated, x_c. For both cases:

1. Do you expect x_m to equal x_c? Explain.
2. How closely do x_m and x_c agree? For example,

$$\text{if} \quad x_m = x_c \quad \text{then} \quad \frac{x_m - x_c}{x_m} = 0.$$

3. What might account for $(x_m - x_c)/x_m \neq 0$?
   a. Would a different initial guess for the Newton iteration improve the agreement? Why or why not?
   b. How many iterations did you perform? Why did you choose this many?
   c. Would more steps in the Newton iteration improve the agreement? Why or why not?

II. Was the agreement between x_m and x_c better for one of the two tanks? Explain why you might see a difference.

Step	Measured Data	Step	Calculated Data
(1)	$D =$		
(1)	$C =$		
(1)	$R =$		
(2)	$d_0 =$		
(3a)	$d_1 =$	(3b)	$v_s =$
(4)	$x_m =$		
(5)			
		(6)	$x_c =$
		(7)	$M =$

Table 1: Smaller tank

Step	Measured Data	Step	Calculated Data
(1)	$D =$		
(1)	$C =$		
(1)	$R =$		
(2)	$d_0 =$		
(3a)	$d_1 =$	(3b)	$v_s =$
(4)	$x_m =$		
(5)			
		(6)	$x_c =$
		(7)	$M =$

Table 2: Larger tank