Mini-Excursion 4:

The Mathematics of Managing Risk

Expected Value

A lot of people approach risk as if it's the enemy, when it's really fortune's accomplice.

Sting

Life is full of risks. Some risks we take because we essentially have no choice—for example, when we fly in an airplane, get in a car, or ride in an elevator to the top of a skyscraper. Some risks we take on willingly and just for thrills—that's why some of us surf big waves, rock climb, or sky-dive. A third category of risks are *calculated risks*, risks that we take because there is a tangible payoff—that's why some of us gamble, invest in the stock market, or try to steal second base.

Implicit in the term *calculated risk* is the idea that there is some type of calculation going on. Often the calculus of risk is informal and fuzzy ("I have a gut feeling my number is coming up," "It's a good time to invest in real estate," etc.), but in many situations both the risk and the associated payoffs can be quantified in precise mathematical terms. In these situations the relationship between the risks we take and the payoffs we expect can be measured using an important mathematical tool called the *expected value*. The notion of expected value gives us the ability to make rational decisions—what risks are worth taking and what risks are just plain foolish.

In this mini-excursion we will briefly explore the concept of the *expected value* (or *expectation*) of a random variable and illustrate the concept with several real-life applications to gambling, investing, and risk-taking in general. The main prerequisites for this mini-excursion are covered in Chapter 15, in particular the concepts covered in Sections 15.4 and 15.5.

Weighted Averages

EXAMPLE 4.1 Computing Class Scores

Imagine that you are a student in Prof. Goodie's Stat 100 class. The grading for the class is based on two midterms, homework, and a final exam. The breakdown for the scoring is given in the first two rows of Table 4-1. Your scores are given in the last row. You needed to average 90% or above to get an A in the course. Did you?

TABLE 4-1

	Midterm 1	Midterm 2	Homework	Final exam
Weight (percentage of grade)	20%	20%	25%	35%
Possible points	100	100	200	200
Your scores	82	88	182	190

One of your friends claims that your A is in the bag, since $(82 + 88 + 182 + 190)/600 = 0.9033\cdots \approx 90.33\%$. A second friend says that you are going to miss an A by just one percentage point, and isn't that too bad! Your second friend's calculation is as follows: You got 82% on the first midterm, 88% on the second midterm, 91% (182/200) on the homework, and 95% (190/200) on the final exam. The average of these four percentages is $(82\% + 88\% + 95\% + 91\%)/4 = 89\%$.

Fortunately, you know more math than either one of your friends. The correct computation, you explain to them patiently, requires that we take into account that (i) the scores are based on different scales (100 points, 200 points), and (ii) the weights of the scores (20%, 20%, 25%, 35%) are not all the same. To take care of (i) we express the numerical scores as percentages (82%, 88%, 91%, 95%), and to take care of (ii) we multiply these percentages by their respective weights ($20\% = 0.20$, $25\% = 0.25$, $35\% = 0.35$). We then add all of these numbers. Your correct average is

$$0.2 \times 82\% + 0.2 \times 88\% + 0.25 \times 91\% + 0.35 \times 95\% = 90\%$$

How sweet it is—you are getting that A after all!

TABLE 4-2 GPA's at Tasmania State University by Class

Class	Freshman	Sophomore	Junior	Senior
Average GPA	2.75	3.08	2.94	3.15

EXAMPLE 4.2 Average GPAs at Tasmania State University

Table 4-2 shows GPAs at Tasmania State University broken down by class. Our task is to compute the overall GPA of *all undergraduates* at TSU.

The information given in Table 4-2 is not enough to compute the overall GPA because the number of students in each class is not the same. After a little digging we find out that the 15,000 undergraduates at Tasmania State are divided by class as follows: 4800 freshmen, 4200 sophomores, 3300 juniors, and 2700 seniors.

To compute the overall school GPA we need to "weigh" the average GPA for each class with the relative size of that class. The relative size of the freshman class is $4800/15{,}000 = 0.32$, the relative size of the sophomore class is $4200/15{,}000 = 0.28$, and so on (0.22 for the junior class and 0.18 for the senior class). When all is said and done, the overall school GPA is

$$0.32 \times 2.75 + 0.28 \times 3.08 + 0.22 \times 2.94 + 0.18 \times 3.15 = 2.9562.$$

Examples 4.1 and 4.2 illustrate how to average numbers that have different relative values using the notion of a *weighted average*. In Example 4.1 the scores (82%, 88%, 91%, and 95%) were multiplied by their corresponding "weights"

(0.20, 0.20, 0.25, and 0.35) and then added to compute the weighted average. In Example 4.2 the class GPAs (2.75, 3.08, 2.94, and 3.15) were multiplied by the corresponding class "weight" (0.32, 0.28, 0.22, 0.18) to compute the weighted average. Note that in both examples the weights add up to 1, which is not surprising since they represent percentages that must add up to 100%.

Weighted Average

Let X be a variable that takes the values $v_1, v_2, \ldots, v_N$, and let $w_1, w_2, \ldots, w_N$ denote the respective weights for these values, with $w_1 + w_2 + \cdots + w_N = 1$. The weighted average for X is given by

$$w_1 \cdot v_1 + w_2 \cdot v_2 + \cdots + w_N \cdot v_N$$

Expected Values

The idea of a weighted average is particularly useful when the weights represent probabilities.

EXAMPLE 4.3 To Guess or Not to Guess (Depends on the Question): Part 1

The SAT is a standardized college entrance exam taken by over a million students each year. In the multiple choice sections of the SAT each question has five possible answers (A, B, C, D, and E). A correct answer is worth 1 point, and, to discourage indiscriminate guessing, an incorrect answer carries a penalty of 1/4 point (i.e. it counts as $-1/4$ points).

Imagine you are taking the SAT and are facing a multiple choice question for which you have no clue as to which of the five choices might be the right answer—they all look equally plausible. You can either play it safe and leave it blank or gamble and take a wild guess. In the latter case, the upside of getting 1 point must be measured against the downside of getting a penalty of 1/4 points. What should you do?

Table 4-3 summarizes the possible options and their respective payoffs (for the purposes of illustration assume that the correct answer is B, but of course, you don't know that when you are taking the test).

TABLE 4-3

Option	Leave Blank	A	B	C	D	E
Payoff	0	−0.25	1	−0.25	−0.25	−0.25

We will now need some basic probability concepts introduced in Chapter 15. When you are randomly guessing the answer to this multiple choice question you are unwittingly conducting a *random experiment* with sample space $S = \{A, B, C, D, E\}$. Since each of the five choices is equally likely to be the correct answer (remember, you are clueless on this one), you assign equal probabilities of $p = 1/5 = 0.2$ to each outcome. This probability assignment, combined with the information in Table 4-3, gives us Table 4-4.

Also note that standardized tests such as the SAT are designed for "key balance" (each of the possible answers occurs with approximately equal frequency).

TABLE 4-4

Outcome	Correct Answer (B)	Incorrect Answer (A, C, D, or E)
Point payoff	1	−0.25
Probability	0.2	0.8

Using Table 4-4 we can compute something analogous to a weighted average for the point payoffs with the *probabilities as weights*. We will call this the *expected payoff*. Here the expected payoff E is

$$E = 0.2 \times 1 + 0.8 \times (-0.25) = 0.2 - 0.2 = 0 \text{ points}$$

The fact that the expected payoff is 0 points implies that this guessing game is a "fair" game—in the long term (if you were to make these kinds of guesses many times) the penalties that you would accrue for wrong guesses are neutralized by the benefits that you would get for your lucky guesses. This knowledge will not impact what happens with an individual question [the possible outcomes are still a +1 (lucky) or a −0.25 (wrong guess)], but it gives you some strategically useful information: on the average, totally random guessing on the multiple choice section of the SAT neither helps nor hurts!

EXAMPLE 4.4 To Guess or Not to Guess (Depends on the Question): Part 2

Example 4.3 illustrated what happens when the multiple choice question has you completely stumped—each of the five possible choices (A, B, C, D, or E) looks equally likely to be the right answer. To put it bluntly, you are clueless! At a slightly better place on the ignorance scale is a question for which you can definitely rule out one or two of the possible answers. Under these circumstances we must do a different calculation for the risk/benefits of guessing.

Let's consider first a multiple choice question for which you can safely rule out one of the five choices. For the purposes of illustration let's assume that the correct answer is B, and that you can definitely rule out choice E. Among the other four possible choices (A, B, C, or D) you have no idea which is most likely to be the correct answer, so you are going to randomly guess. In this scenario we assign the same probability (0.25) to each of the four choices, and the guessing game is described in Table 4-5.

TABLE 4-5

Outcome	Correct Answer (B)	Incorrect Answer (A, C, or D)
Point payoff	1	−0.25
Probability	0.25	0.75

Once again, the *expected payoff* E can be computed as a weighted average:

$$E = 0.25 \times 1 + 0.75 \times (-0.25) = 0.25 - 0.1875 = 0.0625.$$

An expected payoff of 0.0625 points is very small—it takes 16 guesses of this type to generate an expected payoff equivalent to 1 correct answer (1 point). At

$0.0625 = 1/16$.

the same time, the fact that it is a positive expected payoff means that the benefit justifies (barely) the risk.

A much better situation occurs when you can rule out two of the five possible choices (say, D and E). Now the random experiment of guessing the answer is described in Table 4-6.

TABLE 4-6

Outcome	Correct Answer (B)	Incorrect Answer (A or C)
Point payoff	1	−1/4
Probability	1/3	2/3

The switch from decimals to fractions is to avoid having to round-off 1/3 and 2/3 in our calculations.

In this situation the expected payoff E is given by

$$E = (1/3) \times 1 + (2/3) \times (-1/4) = 1/3 - 1/6 = 1/6$$

Examples 4.3 and 4.4 illustrate the mathematical reasoning behind a commonly used piece of advice given to SAT-takers: *guess the answer if you can rule out some of the options, otherwise don't bother*.

For a review of *random variables* see Chapter 16.

The basic idea illustrated in Examples 4.3 and 4.4 is that of the **expected value** (or **expectation**) of a random variable.

Expected Value

Suppose X is a random variable with outcomes $o_1, o_2, o_3, \ldots, o_N$, having probabilities $p_1, p_2, p_3, \ldots, p_N$, respectively. The expected value of X is given by

$$E = p_1 \cdot o_1 + p_2 \cdot o_2 + p_3 \cdot o_3 + \cdots + p_N \cdot o_N$$

Applications of Expected Value

In many real-life situations, we face decisions that can have many different potential consequences—some good, some bad, some neutral. These kinds of decisions are often quite hard to make because there are so many intangibles, but sometimes the decision comes down to a simple question: Is the reward worth the risk? If we can quantify the risks and the rewards, then we can use the concept of *expected value* to help us make the right decision. The classic illustration of this type of situation is provided by "games" where there is money riding on the outcome. This includes not only typical gambling situations (dice games, card games, lotteries, etc.) but also investing in real estate or playing the stock market.

Playing the stock market is a legalized from of gambling where people make investment decisions (buy? sell? when? what?) instead of rolling the dice or spinning a wheel. Some people have a knack for making good investment decisions and in the long run can do very well—others, just the opposite. A lot has to do with what drives the decision-making process—gut feelings, rumors, and can't miss tips from your cousin Vinny at one end of the spectrum, reliable information combined with sound mathematical principles at the other end. Which approach would you rather trust your money to? (If your answer is the former then you might as well skip the next example.)

EXAMPLE 4.5 To Buy or Not to Buy?

Fibber Pharmaceutical is a new start-up in the pharmaceutical business. Its stock is currently selling for $10. Fibber's future hinges on a new experimental vaccine they believe has great promise for the treatment of the avian flu virus. Before the vaccine can be approved by the FDA for use with the general public it must pass a series of clinical trials known as Phase I, Phase II, and Phase III trials. If the vaccine passes all three trials and is approved by the FDA, shares of Fibber are predicted to jump tenfold to $100 a share. At the other end of the spectrum, the vaccine may turn out to be a complete flop and fail Phase I trials. In this case, Fibber's shares will be worthless. In between these two extremes are two other possibilities: the vaccine will pass Phase I trials but fail Phase II trials or pass Phase I and Phase II trials but fail Phase III trials. In the former case shares of Fibber are expected to drop to $5 a share; in the latter case shares of Fibber are expected to go up to $15 a share.

Table 4-7 summarizes the four possible outcomes of the clinical trials. The last row of Table 4-7 gives the probability of each outcome based on previous experience with similar types of vaccines.

TABLE 4-7

Outcome of trials	Fail Phase I	Fail Phase II	Fail Phase III	FDA Approval
Estimated share price	$0	$5	$15	$100
Probability	0.25	0.45	0.20	0.10

Combining the second and third rows of Table 4-7, we can compute the expected value E of a future share of Fibber Pharmaceutical:

$$E = 0.25 \times \$0 + 0.45 \times \$5 + 0.20 \times \$15 + 0.10 \times \$100 = \$15.25$$

To better understand the meaning of the $15.25 expected value of a $10 share of Fibber Pharmaceutical, imagine playing the following game: For a cost of $10 you get to roll a die. This is no ordinary die—this die has only four faces (labeled I, II, III, and IV), and the probabilities of each face coming up are different (0.25, 0.45, 0.20, and 0.10, respectively). If you Roll a I your payoff is $0 (your money is gone); if you roll a II your payoff is $5 (you are still losing $5); if you roll a III your payoff is $15 (you made $5); and if you roll a IV your payoff is $100 (jackpot!). The beauty of this random experiment is that it can be played over and over, hundreds or even thousands of times. If you do this, sometimes you'll roll a I and poof—your money is gone, a few times you'll roll a IV and make out like a bandit, other times you'll roll a III or a II and make or lose a little money. The key observation is that if you play this game long enough the average payoff is going to be $15.25 per roll of the die, giving you an average profit or gain of $5.25 per roll. In purely mathematical terms, this game is a game well worth playing (but since this book does not condone gambling, this is just a theoretical observation).

Returning to our original question, is Fibber Pharmaceutical a good investment or not? At first glance, an expected value of $15.25 per $10 invested looks like a great risk, but we have to balance this with the several years that it might take to collect on the investment (unlike the die game that pays off right away). For example, assuming four years to complete the clinical trials (sometimes it can take even longer), an investment in Fibber shares has a comparable expected payoff as a safe investment at a fixed annual yield of about 11%. This makes for a good, but hardly spectacular, investment.

See Exercise 20.

EXAMPLE 4.6 Raffles and Fundraisers: Part 1

A common event at many fundraisers is to conduct a raffle—another form of legalized gambling. At this particular fundraiser, the raffle tickets are going for \$2.00. In this raffle, they will draw one grand prize winner worth \$500, four second prize winners worth \$50 each, and fifteen third prize winners worth \$20 each. Sounds like a pretty good deal for a \$2.00 investment, but is it? The answer, of course, depends on how many tickets are sold in this raffle.

Suppose that this is a big event, and they sell 1000 tickets. Table 4-8 shows the four possible outcomes for your raffle ticket (first row), their respective net payoffs after subtracting the \$2 cost of the ticket (second row), and their respective probabilities (third row). The last column of the table reflects the fact that if your number is not called, your ticket is worthless and you lost \$2.00.

TABLE 4-8

Outcome	Grand Prize	Second Prize	Third Prize	No Prize
Net gain	\$498	\$48	\$18	−\$2.00
Probability	1/1000	4/1000	15/1000	980/1000

From Table 4-8 we can compute the expected value E of the raffle ticket:

$$\begin{aligned} E &= (1/1000) \times \$498 + (4/1000) \times \$48 + (15/1000) \times \$18 + (980/1000) \times \$(-2) \\ &= \$0.498 + \$0.192 + \$0.27 - \$1.96 = -\$1.00 \end{aligned}$$

The negative expected value is an indication that this game favors the people running the raffle. We would expect this—it is, after all, a fundraiser. But the computation gives us a precise measure of the extent to which the game favors the raffle: on the average we should expect to lose \$1.00 for every \$2.00 raffle ticket purchased. If we purchased, say, 100 raffle tickets we would in all likelihood have a few winning tickets and plenty of losing tickets, but who cares about the details—at the end we would expect a net loss of about \$100.

EXAMPLE 4.7 Raffles and Fundraisers: Part 2

Most people buy raffle tickets to support a good cause and not necessarily as a rational investment, but it is worthwhile asking, What should be the price of a raffle ticket if we want the raffle to be a "fair game"? (Let's assume we are still discussing the raffle in Example 4.6.) In order to answer this question, we set the price of a raffle ticket to be x, set the expected gain to be \$0 (that's what will make it a fair game), and solve the expected value equation for x. Here is how it goes:

$$E = (1/1000) \times \$498 + (4/1000) \times \$48 + (15/1000) \times \$18 + (980/1000) \times \$(-x) = 0$$

$$\$0.498 + \$0.192 + \$0.27 - \$0.98x = 0$$

$$x = \$0.96/0.98 \approx \$0.98$$

EXAMPLE 4.8 Chuck-a-Luck

Chuck-a-luck is an old game, played mostly in carnivals and county fairs. We will discuss it here because it involves a slightly more sophisticated calculation of the probabilities of the various outcomes. To play chuck-a-luck you place a bet, say \$1.00, on one of the numbers 1 through 6. Say that you bet on the number 4. You then roll three dice (presumably honest). If you roll three 4's you win \$3.00; if you roll just two 4's you win \$2.00; if you roll just one 4 you win \$1.00 (and of course, in all of these cases, you get your original \$1.00 back). If you roll no 4's you lose your \$1.00. Sounds like a pretty good game, doesn't it?

To compute the expected payoff for chuck-a-luck we will use some of the ideas introduced in Chapter 15, Section 15.5. When we roll three dice, the sample space consists of $6 \times 6 \times 6 = 216$ outcomes. The different events we will consider are shown in the first row of Table 4-9 (the * indicates any number other than a 4). The second row of Table 4-9 shows the size (number of outcomes) of that event, and the third row shows the respective probabilities.

TABLE 4-9

Event	(4,4,4)	(4, 4, *)	(4, *, 4)	(*, 4, 4)	(4, *, *)	(*, 4, *)	(*, *, 4)	(*, *, *)
Size	1	5	5	5	5×5	5×5	5×5	$5 \times 5 \times 5$
Probability	1/216	5/216	5/216	5/216	25/216	25/216	25/216	125/216

See Exercise 19.

Combining columns in Table 4-9, we can deduce the probability of rolling three 4's, two 4's, one 4, or no 4's when we roll three dice. This leads to Table 4-10. The payoffs are based on a \$1.00 bet.

TABLE 4-10

Roll	Three 4's	Two 4's	One 4	No 4's
Net gain	\$3	\$2	\$1	−\$1
Probability	1/216	15/216	75/216	125/216

The expected value of a \$1.00 bet on chuck-a-luck is given by

$$E = (1/216) \times \$3 + (15/216) \times \$2 + (75/216) \times \$1 + (125/216) \times \$(-1)$$
$$= \$(-17/216) \approx \$-0.08$$

Essentially, this means that in the long run, for every \$1.00 bet on chuck-a-luck, the player will lose on the average about 8 cents, or 8%. This, of course, represents the bad guys' (i.e. the "house") profit, and in gambling is commonly referred to as the *house margin*, or *house advantage*.

The concept of expected value is used by insurance companies to set the price of premiums (the process is called *expectation based pricing*). Our last example is an oversimplification of how the process works, but it illustrates the key ideas behind the setting of life insurance premiums.

EXAMPLE 4.9 Setting Life Insurance Premiums

A life insurance company is offering a $100,000 one-year term life insurance policy to Janice, a 55-year-old nonsmoking female in moderately good health. What should be a reasonable premium for this policy?

For starters, we will let $$P$ denote the *break-even*, or *fair*, premium that the life insurance company should charge Janice if it were not in it to make a profit. This can be done by setting the expected value to be 0. Using mortality tables, the life insurance company can determine that the probability that someone in Janice's demographic group will die within the next year is 1 in 500, or 0.002. The second row of Table 4-11 gives the value of the two possible outcomes to the life insurance company, and the third row gives the respective probabilities.

TABLE 4-11

Outcome	Janice dies	Janice doesn't die
Payoff	$\$(P - 100{,}000)$	$\$P$
Probability	0.002	0.998

See Exercise 14.

Setting the expected payoff equal to 0 gives $P = 200$. This is the premium the insurance company should charge to break even, but of course, insurance companies are in business to make a profit. A standard gross profit margin in the insurance industry is 20%, which in this case would tack on $40 to the premium. We can conclude that the premium for Janice's policy should be about $240.

Exercises

A. Weighted Averages

1. The scoring for Psych 101 is given in the following table. The last row of the table shows Paul's scores. Find Paul's score in the course, expressed as a percent.

	Test 1	**Test 2**	**Test 3**	**Quizzes**	**Paper**	**Final**
Weight (percentage of grade)	15%	15%	15%	10%	25%	20%
Possible points	100	100	100	120	100	180
Paul's scores	77	83	91	90	87	144

2. In his record setting victory in the 1997 Masters, Tiger Woods had the following distribution of scores:

Score	2	3	4	5	6
Percentage of holes	1.4%	36.1%	50%	11.1%	1.4%

What was Tiger Woods' average score per hole during the 1997 Masters?

3. At Thomas Jefferson High School, the student body is divided by age as follows: 7% of the students are 14; 22% of the students are 15; 24% of the students are 16; 23% of the students are 17; 19% of the students are 18; the rest of the students are 19. Find the average age of the students at Thomas Jefferson HS.

4. In 2005 the Middletown Zoo averaged 4000 visitors on sunny days, 3000 visitors on cloudy days, 1500 visitors on rainy days, and only 100 visitors on snowy days. The percentage of days of each type in 2005 is given in the following table. Find the average daily attendance at the Middletown Zoo for 2005.

Weather condition	Sunny	Cloudy	Rainy	Snowy
Percentage of days	47%	27%	19%	7%

B. Expected Values

5. Find the expected value of the random variable with outcomes and associated probability distribution shown in the following table.

Outcome	5	10	15
Probability	1/5	2/5	2/5

6. Find the expected value of the random variable with outcomes and associated probability distribution shown in the following table.

Outcome	10	20	30	40
Probability	0.2	0.3	0.4	?

7. A box contains twenty \$1 bills, ten \$5 bills, five \$10 bills, three \$20 bills, and one \$100 bill. You reach in the box and pull out a bill at random, which you get to keep. Let X represent the value of the bill that you draw.

(a) Find the probability distribution of X.

(b) Find the expected value of X.

(c) How much should you pay for the right to draw from the box if the game is to be a fair game?

8. A basketball player shoots two consecutive free-throws. Each free-throw is worth 1 point and has probability of success $p = 3/4$. Let X denote the number of points scored.

(a) Find the probability distribution of X.

(b) Find the expected value of X.

9. A fair coin is tossed three times. Let X denote the number of *Heads* that come up.

(a) Find the probability distribution of X.

(b) Find the expected value of X.

10. Suppose you roll a pair of honest dice. If you roll a total of 7 you win \$18; if you roll a total of 11 you win \$54; if you roll any other total you lose \$9. Let X denote the payoff in a single play of this game.

(a) Find the expected value of a play of this game.

(b) How much should you pay for the right to roll the dice if the game is to be a fair game?

11. On an American roulette wheel, there are 18 red numbers, 18 black numbers, plus 2 green numbers (0 and 00). If you bet \$$N$ on red you win \$$N$ if a red number comes up (i.e. you get \$$2N$ back—your original bet plus your \$$N$ profit); if a black or green number come up you lose your \$$N$ bet.

(a) Find the expected value of a \$1 bet on red.

(b) Find the expected value of a \$$N$ bet on red.

12. On an American roulette wheel, there are 38 numbers: 00, 0, 1, 2, . . . , 36. If you bet \$$N$ on any one number—say, for example, on 10—you win \$$36N$ if 10 comes up (i.e. you get \$$37N$ back—your original bet plus your \$$36N$ profit); if any other number comes up you lose your \$$N$ bet.

(a) Find the expected value of a \$1 bet on 10 (or any other number).

(b) Find the expected value of a \$$N$ bet on 10.

13. Suppose you roll a single die. If an odd number (1, 3, or 5) comes up you win the amount of your roll (\$1, \$3, or \$5, respectively). If an even number (2, 4, or 6) comes up, you have to pay the amount of your roll (\$2, \$4, or \$6, respectively).

(a) Find the expected value of this game.

(b) Find a way to change the rules of this game so that it is a fair game.

14. This exercise refers to Example 4.9 in this mini-excursion.

(a) Explain how the fair premium of $P = 200$ is obtained.

(b) Find the value of a fair premium for a person with probability of death over the next year estimated to be 3 in 1000.

C. Miscellaneous

15. Joe is buying a new plasma TV at Circuit Town. The salesman offers Joe a three-year extended warranty for \$80. The salesman tells Joe that 24% of these plasma TVs require repairs within the first three years, and the average cost of a repair is \$400. Should Joe buy the extended warranty? Explain your reasoning.

16. Jackie is buying a new MP3 player from Better Buy. The store offers her a two-year extended warranty for \$19. Jackie read in a consumer magazine that for this model MP3, 5% require repairs within the first two years at an average cost of \$50. Should Jackie buy the extended warranty? Explain your reasoning.

17. The service history of the Prego SUV is as follows: 50% will need no repairs during the first year, 35% will have repair costs of $500 during the first year, 12% will have repair costs of $1500 during the first year, and the remaining SUVs (the real lemons) will have repair costs of $4000 during their first year. Determine the price that the insurance company should charge for a one-year extended warranty on a Prego SUV if it wants to make an average profit of $50 per policy.

18. An insurance company plans to sell a $250,000 one-year term life insurance policy to a 60-year-old male. Of 2.5 million men having similar risk factors, the company estimates that 7500 of them will die in the next year. What is the premium that the insurance company should charge if it would like to make a profit of $50 on each policy?

19. This exercise refers the game of chuck-a-luck discussed in Example 4.8. Explain why, when you roll three dice,

(a) the probability of rolling two 4's plus another number (not a 4) is 15/216.

(b) the probability of rolling one 4 plus two other numbers (not 4's) is 75/216.

(c) the probability of rolling no 4's is 125/216.

20. For this exercise you will need to use the general compounding formula introduced in Chapter 10. Explain why the expected value of an investment in Fibber Pharmaceutical is comparable to an investment with an expected annual yield of about 11%. (Assume it takes four years to complete the clinical trials.)

21. In the California Super Lotto game you choose 5 numbers between 1 and 47 plus a Mega number between 1 and 27. Suppose a winning ticket pays $30 million (assume the prize will not be split among several winners). Find the expected value of a $1 lottery ticket. (Round your answer to the nearest penny.) ***Hint***: You may want to review Section 15.3 of Chapter 15.

22. In the California Mega Millions Lottery you choose 5 numbers between 1 and 56 plus a Mega number between 1 and 46. Suppose a winning ticket pays $64 million (assume the prize will not be split among several winners). Find the expected value of a $1 lottery ticket. (Round your answer to the nearest penny.) ***Hint***: You may want to review Section 15.3 of Chapter 15.

References and Further Readings

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