

WEIGHTED AVERAGE

Averaging items with different weights

Example: Calculating grades

Suppose a class has quizzes, homework and three exams. The scores are weighted as follows:

Quiz	HW	Exam1	Exam2	Final
10%	10%	20%	20%	40%

Suppose your averages are as follows:

Quiz	HW	Exam1	Exam2	Final
80%	89%	79%	84%	87%

How is your grade calculated?

Multiply percentages by respective weights (written as decimals)

$$**0.1(80)+0.1(89)+0.2(79)+0.2(84)+0.4(87)**$$

$$**=84.3% final grade**$$

Weighted Average:

Let X be a variable that takes the values v_1, v_2, \dots, v_N , and let w_1, w_2, \dots, w_N denote the respective weights for these values, with $w_1 + w_2 + \dots + w_N = 1$.

The weighted average for X is given by:

$$w_1 \cdot v_1 + w_2 \cdot v_2 + \dots + w_N \cdot v_N$$

Consider weights representing probabilities

Call these problems Expected Value problems

EXPECTED VALUE

Suppose X is a random variable with outcomes o_1, o_2, \dots, o_N , having probabilities p_1, p_2, \dots, p_N , respectively. The expected value of X is given by

$$E = p_1 \cdot o_1 + p_2 \cdot o_2 + \dots + p_N \cdot o_N$$

A positive value for E indicates that on average, we would profit, if it is negative, then on average, we would experience a loss. If the expected value is zero, then we are in a fair experiment.

For example, an American Roulette wheel has 38 equally possible outcomes. A bet placed on a single number pays 35-to-1 (this means that he is paid 35 times his bet, while also his bet is returned, together he gets 36 times his bet).

The expected value of the profit resulting from a \$1 bet on a single number is:

38 possible outcomes

$$(37/38 \times -1) + (1/38 \times 35) = -0.0526$$

Therefore one expects, on average, to lose over 5 cents for every dollar bet

You own a \$5,000 car. The probability that your car will be stolen next year is .02. The probability that your car will be broken into and the radio stolen is .10. The damage of such a break-in and theft is \$200. Cheatem's Insurance Company offers you a policy that would cover both of the above thefts for a cost of only \$150.

(a) What is the expected value of this insurance policy?

(b) What is a fair price for such a policy?