Percentages, Interest, Geometric Growth

Percentages- help us compare

Translate to/from decimal: $x\% = \frac{x}{100}$

Example: 50% is the same as 0.5

Example: Compare grade scores:

200 Worksheet points, out of 225 possible,

75 quiz points out of 80.

Which is better, worksheet scores or quiz scores?

200/225=0.8889 or 88.89%

75/80=0.9375 or 93/75%

Markups and Markdowns (percentages)

- **To increase a number** C **by** $x\%$**, multiply** C **by** $1 + x/100$
- **To decrease a number** C **by** $x\%$**, multiply** C **by** $1 - x/100$

A retailer buys an item for 5 dollars.

Retailer marks item up 80%.

Find the customer cost:

$5(1 + .8) = 5(1.8) = 9$

Cost to customer: $9

Example:

You purchase an item that is originally $22. It was marked down 30%, but you bought it on Tuesday for an additional 10% off. What was your price?

$22(1-.3)(1-.1) = 22(.7)(.9) = 13.86$

Simple Interest

Only the original money generates interest

Principal- the original money invested/borrowed

APR- annual interest rate

Example: invest $1000 at 5%APR

Principal is $1000

make 5% of $1000 each year, or $50

Simple Interest Formula: $F = P(1+rt)$

F**- future value of the money**

P**- principal**

r**- rate written as a decimal**

t**- length of time in years**

How much money would you have if you invested $2,000 at a rate of 4% for 25 years using simple interest?

$F = 2000(1 + 0.04 \cdot 25) = 4000$

Compounding Interest

Compound interest- both the prinicpal AND the interest earn interest

Deposit $1000 in retirement account at 6% annual interest. How much money in the account after 25 years?

$$\begin{aligned}
\text{Year } 0 &= 1000 \\
\text{Year } 1 &= (1.06)1000 = 1060 \\
\text{Year } 2 &= (1.06)(1.06)1000 = (1.06)^2(1000) = 1123.60 \\
\text{Year } 3 &= (1.06)(1000(1.06)^2) = (1.06)^3(1000) = 1191.02 \\
\text{Year } 4 &= (1.06)(1000(1.06)^3) = (1.06)^4(1000) = 1262.48
\end{aligned}$$

Recursive formula: multiply by 1.06 each time

$F_N = F_{N-1}(1.06)$

Would have to step through to get to year 25.

Find Explicit formula:

notice that the exponent matches year number

$F = 1000(1.06)^t$

$F = 1000(1.06)^{25} = 4291.87$

Annual Compounding Formula

$F = P(1 + r)^t$

Monthly Compounding:

Same example, but calculate the interest every month.

Annual interest is 6%

Monthly interest is $0.06/12$

$r = 1 + 0.06/12$

$F = 1000(1 + 0.06/12)^{t \cdot 12}$

$F = 1000(1 + 0.06/12)^{25 \cdot 12} = 4464.97$

General Compounding Formula

$F = P(1 + \frac{r}{n})^{nt}$

Continuous Compounding Formula

$F = Pe^{rt}$

Note- need a scientific or better calculator to do these computations. This formula can also be used to model population growth/decay.

Annual Yield- percentage of profit that the investment generates in a one year period

Example: invest $1000 at 6% APR for 1 year, get $1061.68. What is the annual yield?

Net increase: 1061.68-1000 = 61.68

Percent increase: 61.68/1000 = 0.06168

So Annual Yield is 6.168%

Geometric Sequences-

Start with initial term P and multiply by the same constant to get the next number.

$P, cP, c^2P, c^3P, ...$

Example:

5, 10, 20, 40, 80, 160, 320, ...

What is c? We are multiplying by 2 each time

Geometric Sequence Formulas

- $G_N = c\,G_{N-1}; \quad G_0 = P$ **(recursive)**
- $G_N = c^N P$ **(explicit)**

What if we added the terms together?

Geometric Sum Formula

$P + cP + c^2P + \cdots + c^{N-1}P = P\left(\frac{c^N-1}{c-1}\right)$**.**

Example:

Rude Dogg Promotions charges $300 for the first month and then increases their fees by 1.2% each additional month. How much would it cost to hire this company to promote your band for one year?

$300 + 300(1.012) + 300(1.012)^2 + \cdots + 300(1.012)^{11}$

Use the Geometric Sum Formula:

$P =$ **and** $c =$

sum$= P\left(\frac{c^N-1}{c-1}\right)$

Cost $= 300\left(\frac{1.012^{12}-1}{1.012-1}\right) = 3847.37$