Practice Integral Problems (mixed variety) Solutions / hints

1. $\int x^2 e^{2x} dx = x^2 \cdot \frac{1}{2} e^{2x} - \int \frac{1}{2} e^{2x} \cdot 2x \, dx$

$$\begin{bmatrix} u = x^2 & dv = e^{2x} dx \\ du = 2x \, dx & v = \frac{1}{2} e^{2x} \end{bmatrix}$$

$= \frac{1}{2} x^2 e^{2x} - \int x e^{2x} dx$

$$\begin{bmatrix} u = x & dv = e^{2x} dx \\ du = dx & v = \frac{1}{2} e^{2x} \end{bmatrix}$$

so: $\int x^2 e^{2x} dx = \frac{1}{2} x^2 e^{2x} - \left[ \frac{1}{2} x e^{2x} - \int \frac{1}{2} e^{2x} dx \right]$

$= \frac{1}{2} x^2 e^{2x} - \frac{1}{2} x e^{2x} + \frac{1}{4} e^{2x} + C$

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2. $\int_0^5 x^3 \sqrt{x^2 + 2} \, dx$

let $u = x^2 + 2$ and $u - 2 = x^2$

$du = 2x \, dx$    $u(5) = 5^2 + 2 = 27$

$\frac{du}{2x} = dx$    $u(0) = 0^2 + 2 = 2$

$\int_2^{27} \frac{x^3 \sqrt{u}}{2x} \, du = \frac{1}{2} \int_2^{27} x^2 \sqrt{u} \, du = \frac{1}{2} \int_2^{27} (u-2) u^{1/2} \, du = \frac{1}{2} \int_2^{27} u^{3/2} - 2u^{1/2} \, du$

$= \frac{1}{2} \left[ \frac{2}{5} u^{5/2} - 2 \cdot \frac{2}{3} u^{3/2} \right]_2^{27} = 664.8$

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3. answer: $\ln(5x) \cdot (4x^2 + 7x) - 2x^2 + 7x + C$    (IBP)

4. ans: $0.1831$  (u-sub, $u = 2x^3 + 1$)

5. ans: $\frac{1}{2} e^{x^2} + C$  (u-sub, $u = x^2$)

6. ans: $0.13077$  (see #2)

7. ans: $-42.7959$  (see #1)

8. ans: $\ln(3x)(x^2 - x) - \frac{1}{2} x^2 + x + C$

9. ans: $\frac{2}{7} x^2 (x+2)^{3/2} - \frac{16}{35} x (x+2)^{3/2} + \frac{64(x+2)^{3/2}}{105} + C$