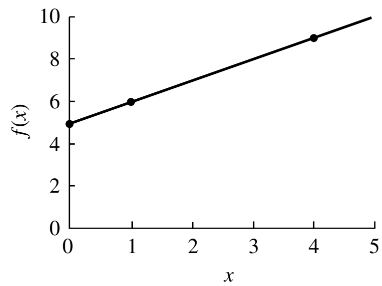
Answers to Odd Exercises

Chapter 1

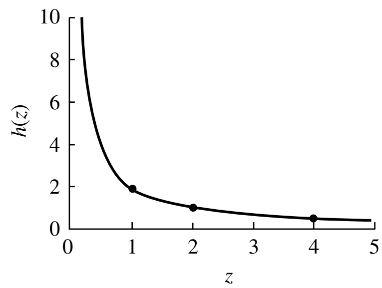
Section 1.2, page 21

1. The variables are the altitude and the wombat density, which we can call a and w, respectively. The parameter is the rainfall, which we can call R.

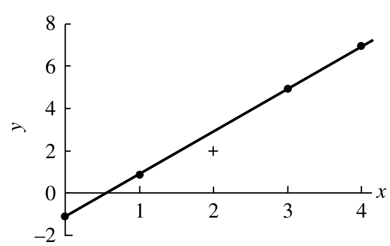
3. $f(0) = 5,\ f(1) = 6,\ f(4) = 9.$



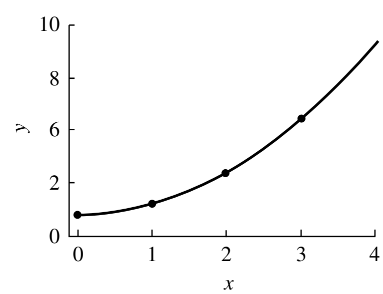
5. $h(1) = \frac{1}{5},\ h(2) = \frac{1}{10},\ h(4) = \frac{1}{20}.$



7. (2, 2).



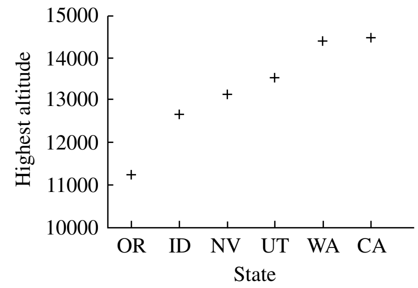
9. (4, 12).



11. $f(a) = a + 5,\ f(a+1) = a + 6,\ f(4a) = 4a + 5.$

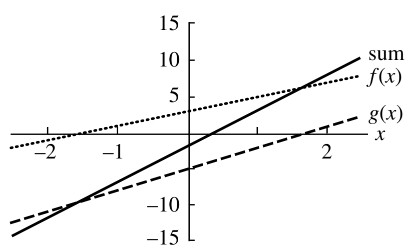
13. $h\left(\frac{c}{5}\right) = \frac{1}{c},\ h\left(\frac{5}{c}\right) = \frac{c}{25},\ h(c+1) = \frac{1}{5c+5}.$

15. I put them in increasing order to look nice.



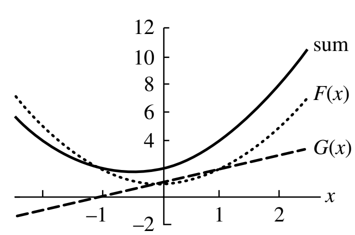
17.

x	$f(x)$	$g(x)$	$(f+g)(x)$
−2	−1	−11	−12
−1	1	−8	−7
0	3	−5	−2
1	5	−2	3
2	7	1	8



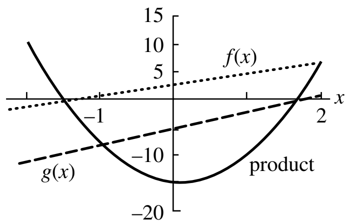
19.

x	$F(x)$	$G(x)$	$(F+G)(x)$
−2	5	−1	4
−1	2	0	2
0	1	1	2
1	2	2	4
2	5	3	8



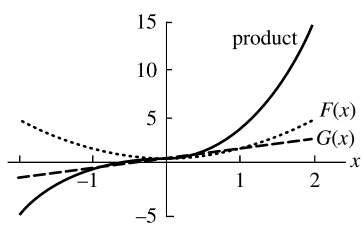
21.

x	$f(x)$	$g(x)$	$(f \cdot g)(x)$
−2	−1	−11	11
−1	1	−8	−8
0	3	−5	−15
1	5	−2	−10
2	7	1	7



23.

x	$F(x)$	$G(x)$	$(F \cdot G)(x)$
−2	5	−1	−5
−1	2	0	0
0	1	1	1
1	2	2	4
2	5	3	15



25. If we write $y = 2x + 3$, we can solve for x with the steps

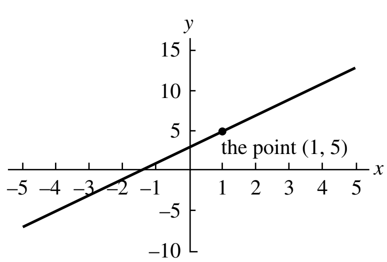
$$y - 3 = 2x \qquad \text{subtract 3 from both sides}$$

$$\frac{y-3}{2} = x \qquad \text{divide both sides by 2.}$$

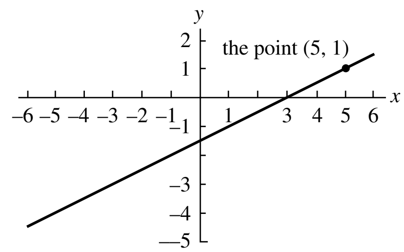
Therefore $f^{-1}(y) = \dfrac{y-3}{2}$. Also, $f(1) = 5$, and $f^{-1}(5) = \dfrac{5-3}{2} = 1$.

27. The function $F(y)$ fails the horizontal line test because, for example, $F(-1) = F(1) = 2$. Therefore it has no inverse.

29. The function

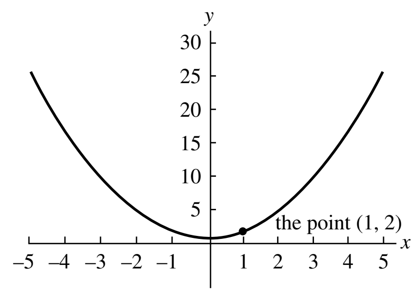


The inverse

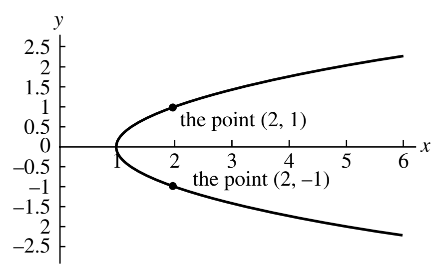


31. This function doesn't have an inverse because it fails the horizontal line test. From the graph, we couldn't tell whether $F^{-1}(2)$ is 1 or −1.

The function



What the inverse would look like



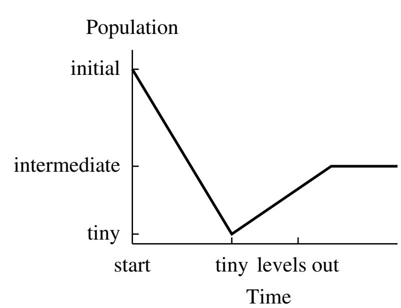
33. $(f \circ g)(x) = f(3x - 5) = 2 \cdot (3x - 5) + 3 = 6x - 7$ and $(g \circ f)(x) = g(2x + 3) = 3 \cdot (2x + 3) - 5 = 6x + 4$. These don't match, so the functions do not commute.

35. $(F \circ G)(x) = F(x + 1) = (x + 1)^2 + 1 = x^2 + 2x + 2$ and $(G \circ F)(x) = G(x^2 + 1) = x^2 + 2$. These do not match, so the functions do not commute.

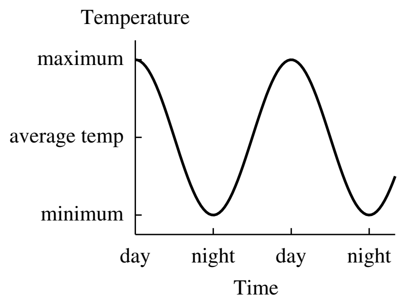
37. The cell volume is generally increasing but decreases during part of its cycle. The cell might get smaller when it gets ready to divide or during the night.

39. The height increases up until about age 30 and then decreases.

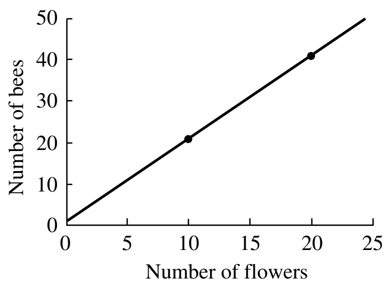
41.



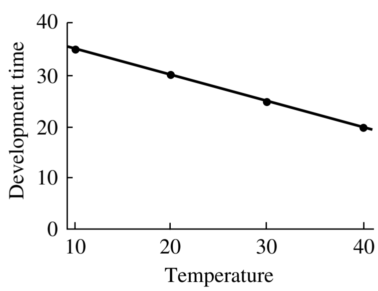
43.



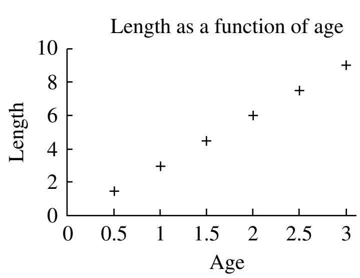
45. When $f = 0$, $b = 1$; when $f = 10$, $b = 21$; when $f = 20$, $b = 41$. Perhaps one bee will check out the plant even if there are no flowers.



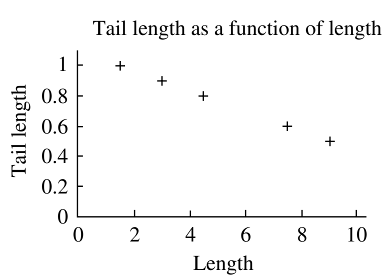
47. When $T = 10$, $A = 35$; when $T = 20$, $A = 30$; when $T = 30$, $A = 25$; when $T = 40$, $A = 20$. The insect develops most quickly (in the shortest time) at the highest temperatures.



49.



51.

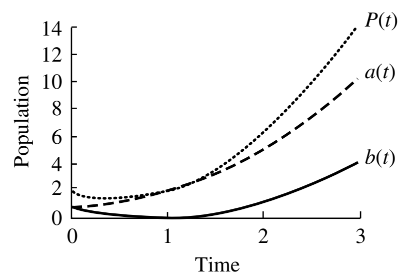


53. $B(M) = B(5W + 2) = 2.5W + 1$. Plugging in $W = 10$ gives 26 bites.

55. $F(V(I)) = F(5I^2) = 37 + 2I^2$. The fever is 39°C if $I = 1$.

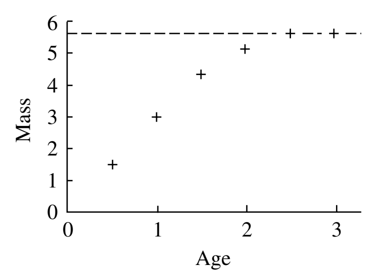
57. The formula is $P(t) = (1 + t^2) + (1 - 2t + t^2) = 2 - 2t + 2t^2$.

t	$a(t)$	$b(t)$	$P(t)$
0.00	1.00	1.00	2.00
0.50	1.25	0.25	1.50
1.00	2.00	0.00	2.00
1.50	3.25	0.25	3.50
2.00	5.00	1.00	6.00
2.50	7.25	2.25	9.50
3.00	10.00	4.00	14.00

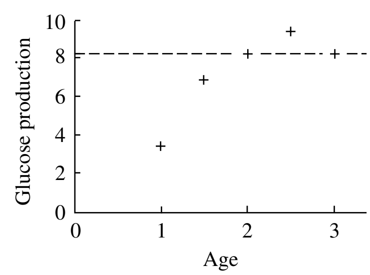


Here a is increasing, b decreases to 0 at time 1.0 and then increases, and the sum P decreases slightly and then increases.

59. Because the mass is the same at ages 2.5 days and 3.0 days, the function relating a and M has no inverse. Knowing the mass does not give us enough information to estimate the age.

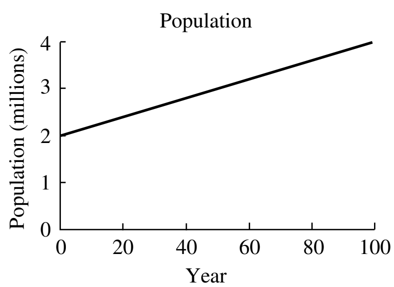


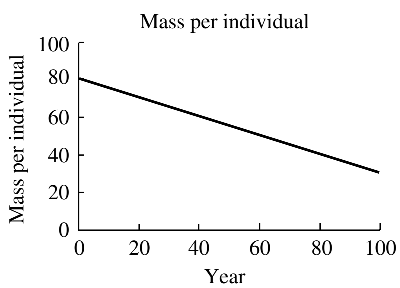
61. Glucose production is 8.2 mg at ages 2.0 days and 3.0 days. We cannot figure out the age from this measurement.

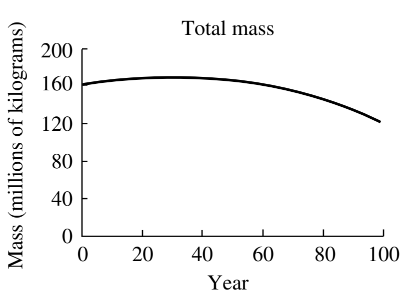


63. Denote the total mass by $T(t)$. Then $T(t) = P(t)W(T) = (2.0 \times 10^6 + 2.0 \times 10^4 t)(80 - 0.5t)$. Measuring population in millions gives

t	$P(t)$	$W(t)$	$T(t)$
0.0	2.0	80.0	160.0
20.0	2.4	70.0	168.0
40.0	2.8	60.0	168.0
60.0	3.2	50.0	160.0
80.0	3.6	40.0	144.0
100.0	4.0	30.0	120.0





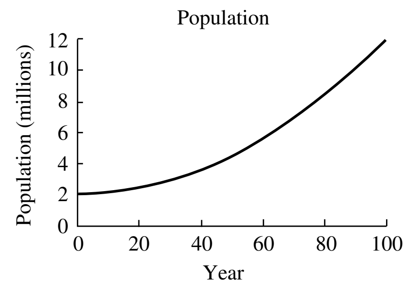


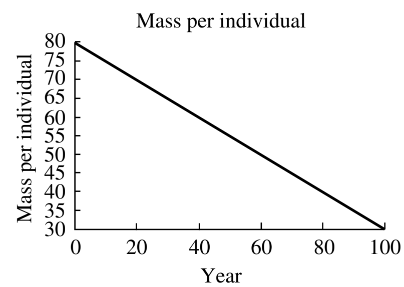
The population increases, the mass per individual decreases, and the total mass increases and then decreases.

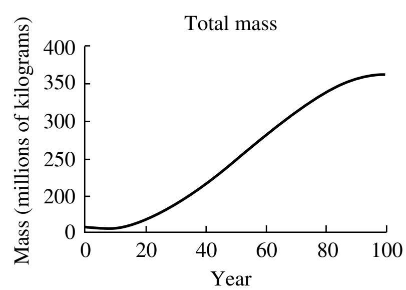
65. We denote the total mass by $T(t)$. Then $T(t) = P(t)W(T) = (2.0 \times 10^6 + 1000t^2)(80 - 0.5t)$. Measuring population in millions gives

t	$P(t)$	$W(t)$	$T(t)$
0.0	2.0	80.0	160.0
20.0	2.4	70.0	168.0
40.0	3.6	60.0	216.0
60.0	5.6	50.0	280.0
80.0	8.4	40.0	336.0
100.0	12.0	30.0	360.0

The population decreases, the mass per individual increases, and the total mass increases.







Section 1.3, page 38

1. $3.4 \text{ lb} \times 16 \dfrac{\text{oz}}{\text{lb}} \times 28.35 \dfrac{\text{g}}{\text{oz}} \approx 1542.24$ g.

3. $60 \text{ yr} \times 365.25 \dfrac{\text{days}}{\text{yr}} \times 24 \dfrac{\text{hr}}{\text{day}} = 525960$ hr.

5. $2.3 \dfrac{\text{g}}{\text{cm}^3} \times \dfrac{1}{28.35} \dfrac{\text{oz}}{\text{g}} \times \dfrac{1}{16} \dfrac{\text{lb}}{\text{oz}} \times 2.54^3 \dfrac{\text{cm}^3}{\text{in.}^3} \times 12^3 \dfrac{\text{in.}^3}{\text{ft}^3} \approx 141.58$ lb/ft^3.

7. 2.3 cm is 0.023 m, so the final height is $1.34 + 0.023 = 1.363$ m.

9. The total weight of apples is $6 \cdot 145 = 870$ g and the total weight of oranges is $7 \cdot 123 = 861$ g, for a grand total of $870 + 861 = 1731$ g.

11. The area of the square is $1.7^2 = 2.89$ cm^2, but the area of the disk is $\pi r^2 = \pi \cdot 1^2$ cm$^2 \approx 3.1415$ cm^2. The disk has a larger area.

13. The volume of the sphere is $\frac{4}{3} \cdot \pi \cdot 100^3 \approx 4.189 \times 10^6$ m^3. The lake has area 3.0×10^6 m^2 and a depth of 0.5 m, giving a volume of merely 1.5×10^6 m^3. The sphere is much larger.

15. Pressure is force per unit area, or

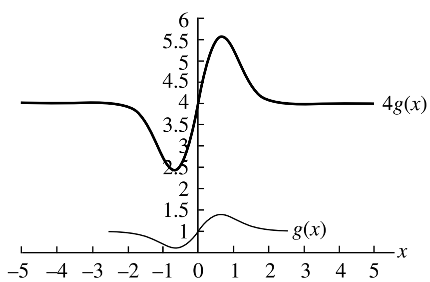
$$\frac{\text{force}}{\text{area}} = \frac{\frac{ML}{T^2}}{L^2} = \frac{M}{LT^2}$$

17. Rate of spread of bacteria on a plate has dimensions of L^2/T, or area per time.

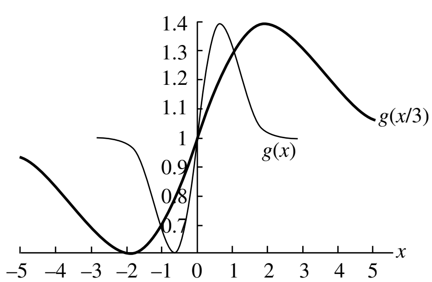
19. This checks, because length $= \frac{\text{length}}{\text{time}} \times$ time.

21. This checks, because $\frac{\text{mass} \times \text{length}}{\text{time}^2} = \text{mass} \times \frac{\text{length}}{\text{time}^2}$.

23. The vertical axis is scaled by a value greater than 1.



25. The horizontal axis is scaled by a value less than 1.



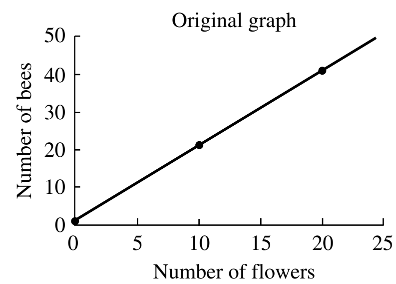
27. The tree with height 23.1 m will have volume of $\pi \cdot 23.1 \cdot 0.5^2 \approx 18.14$ m^3. When the height is 24.1, the volume is 18.93 m^3. The ratio is $\frac{18.93}{18.14} \approx 1.043$.

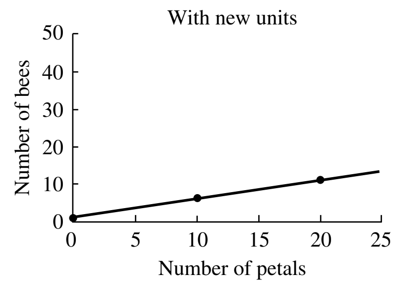
29. The bottom portion of the tree 23.1 m tall will have half the volume found earlier, or 9.07 m^3. The top part is $23.1/2 = 11.55$ m high and is thus a sphere with radius of $11.55/2 \approx 5.78$ m. Substituting into the formula for the volume of a sphere, we find 806.76 m^3. The total volume is 815.83 m^3. The 24.1-m tree has a total volume of $9.46 + 916.13 \approx 925.59$ m^3. The ratio of the volumes is $\frac{925.59}{815.83} \approx 1.134$.

31. The volume is $2.0 \cdot 0.2 \cdot 1.5 = 0.6\,\text{m}^3 = 6.0 \times 10^5$ cm^3. This gives a mass of 6.0×10^5 g = 600 kg.

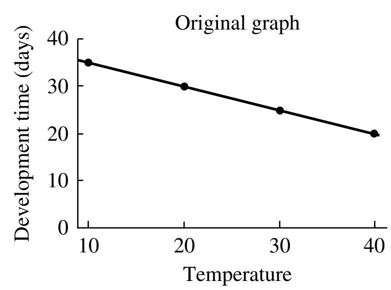
33. $3200 \cdot 0.45 \frac{\text{g}}{\text{individual}} = 1440$ g = 1.44 kg.

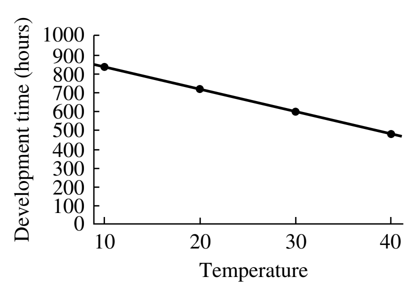
35. Let p be the number of petals. Then $f = p/4$, so $b = \frac{p}{2} + 1$. When $p = 0, b = 1$; when $p = 10, b = 6$; when $p = 20, b = 11$. The number of bees goes up more slowly as a function of the number of petals.





37. Let H be the development time in hours. Then $H = 24A$. Then $H = 24(40 - T/2) = 960 - 12T$. When $T = 10$, $H = 840$; when $T = 20$, $A = 720$; when $T = 30$, $A = 600$; when $T = 40$, $H = 480$.





39.

$$186{,}000 \frac{\text{mile}}{\text{s}} \approx 200{,}000 \frac{\text{mile}}{\text{s}} \approx 200{,}000 \frac{\text{mile}}{\text{s}} \times 60{,}000 \frac{\text{in.}}{\text{mile}}$$
$$= 1.2 \times 10^{10} \frac{\text{in.}}{\text{s}} \approx 1.2 \times 10^{10} \frac{\text{in.}}{\text{s}} \times 2.5 \frac{\text{cm}}{\text{in.}}$$
$$= 3.0 \times 10^{10} \frac{\text{cm}}{\text{s}} = 30 \frac{\text{cm}}{\text{ns}}$$

If a computer is supposed to do an operation in 0.3 ns, it had better not need to move information for more than the distance light can travel in that time, or about 9 cm.

41. The volume is

$$\frac{4}{3}\pi r^3 \approx 3 \cdot 6.5^3 \times 10^9 \text{ km}^3$$
$$\approx 3 \cdot 300 \times 10^9 \text{ km}^3$$
$$\approx 1 \times 10^{12} \text{ km}^3$$

One kilometer is 10^5 cm, so the mass of a cubic kilometer of water is 10^{15} m $= 10^{12}$ kg. Multiplying this by the volume and the density gives a total of 5×10^{24} kg.

43. You'd catch 6 movies per day, or about 2000 per year, for a total of 120,000 in your life.

45. The volume of a sphere of radius r is $4\pi r^3/3 \approx 4r^3$. The radius of the cell is 10^{-3} cm, so the volume is about 4×10^{-9} cm^3. The mass of a cell is therefore around 4×10^{-9} g. I weigh about 60 kg, which is 6×10^4 g. The number of cells is then

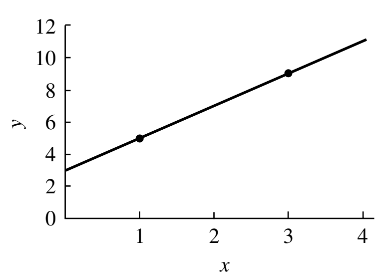
$$\frac{6 \times 10^4 \text{ g}}{4 \times 10^{-9} \text{ g/cell}} = 1.5 \times 10^{13} \text{ cells.}$$

47. The brain is about 2% of my weight and should have about 2% of my cells, or 3×10^{11} cells. The number of neurons is 1×10^{11}, but the total number of cells in the brain is between 1×10^{12} and 5×10^{12}, a bit higher than the previous estimates.

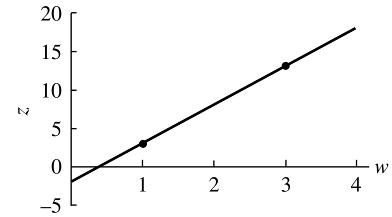
49. The length of the string would be $2\pi r \approx 40840.704$ km. Adding 1 m would make it 40840.705 km. The radius corresponding to this is $r = \frac{40840.705}{2\pi} \approx 6500.0002$ km. The string would be 0.0002 km, or 0.2 m, above the earth. It is amazing that such a relatively tiny change in the length of the string would produce such a big effect.

Section 1.4, page 51

1. The points are (1, 5) and (3, 9). The change in input is 2, the change in output is 4, and the slope is 2. This is not a proportional relation because the ratio of output to input changes from 5 at the first point to 3 at the second point. This relation is increasing because larger values of x lead to larger values of y (and the slope is positive).

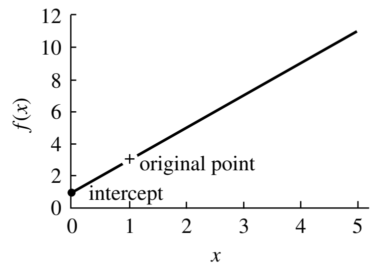


3. The points are (1, 3) and (3, 13). The change in input is 2, the change in output is 10, and the slope is 5. This is not a proportional relation because the ratio of output to input changes from 3 at the first point to 4.333 at the second point. This relation is increasing because larger values of w lead to larger values of z (and the slope is positive).

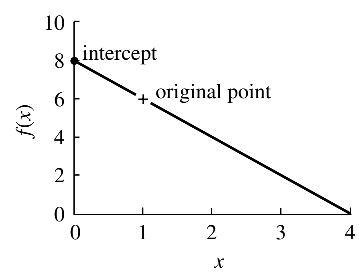


5. The point lies on the line because $f(2) = 2 \cdot 2 + 3 = 7$. The point-slope form is $f(x) = 2(x - 2) + 7$. Multiplying out gives $f(x) = 2x - 4 + 7 = 2x + 3$, as it should.

7. Multiplying out, we find that $f(x) = 2x + 1$. The slope is 2 and the y-intercept is 1. The original point is (1, 2).



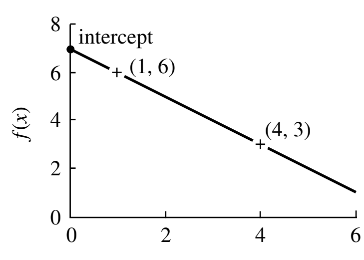
9. In point-slope form, this line has equation $f(x) = -2(x - 1) + 6$. Multiplying out, we find that $f(x) = -2x + 8$. The slope is -2 and the y-intercept is 8.



11. The slope between the two points is

$$\text{slope} = \frac{\text{change in output}}{\text{change in input}} = \frac{3 - 6}{4 - 1} = -1$$

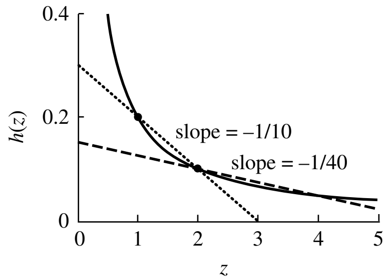
In point-slope form, the line has equation $f(x) = -1 \cdot (x - 1) + 6$. In slope-intercept form, it is $f(x) = -x + 7$. This line has slope -1 and y-intercept 7.



13. This is not linear because the input z appears in the denominator.

15. This is linear because the input q is multiplied only by constants and has constants added to it.

17. $h(1) = \frac{1}{5}, h(2) = \frac{1}{10}, h(4) = \frac{1}{20}$. The slope between $z = 1$ and $z = 2$ is $-1/10$, and that between $z = 2$ and $z = 4$ is $-1/40$.



19. $2x = 7 - 3 = 4$, so $x = 4/2 = 2$. Plugging in, $2 \cdot 2 + 3 = 7$.

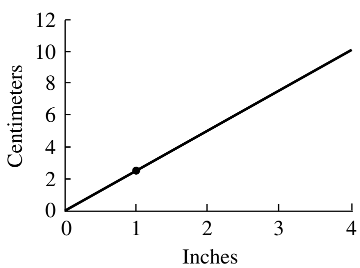
21. $2x - 3x = 7 - 3 = 4$, so $-x = 4$ or $x = -4$. Plugging in, $2 \cdot (-4) + 3 = -5 = 3 \cdot (-4) + 7$.

23. Multiplying out, we get $10x - 4 = 10x + 5$. This has no solution.

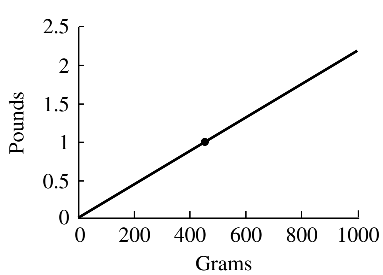
25. $2x = 7 - b$, so $x = \frac{7 - b}{2}$.

27. $(2 - m)x = 7 - b$, so $x = \frac{7 - b}{2 - m}$. There is no solution if $m = 2$. However, if $m = 2$ and $b = 7$, both sides are identical and any value of x works.

29. 1 in. $= 2.54$ cm. The slope is $2.54\ \frac{\text{cm}}{\text{in.}}$.



31. $1\text{ g} = \frac{1}{453.6}\text{ lb} \approx 0.0022\text{ lb}$. The slope is $0.0022\ \frac{\text{lb}}{\text{g}}$.



33. $(f \circ g)(x) = (mx + b) + 1$ and $(g \circ f)(x) = m(x + 1) + b = mx + m + b$. These match only if the intercepts are equal, or $b + 1 = m + b$. This is true for any b as long as $m = 1$. In this case, both f and g have slope 1, meaning that each just adds a constant to its input. The order cannot matter because addition is commutative.

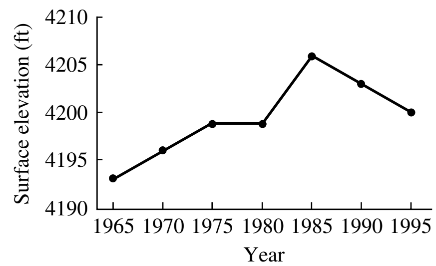
35. The slope is 1.0 cm, and the equation is $V = 1.0A$.

37. The slope is 5.0×10^{-9} g, and the equation is $M = 5.0 \times 10^{-9} b$.

39. The line has slope -0.2 and intercept 10,000. The equation is thus $a = -0.2d + 10{,}000$.

41. $a = -0.2 \cdot 2000 + 10000 = 9600$ ft.

43.



45. The slope is 3 ft every 5 years. It would have been up to 4208 by 1990, 5 ft higher than the actual level.

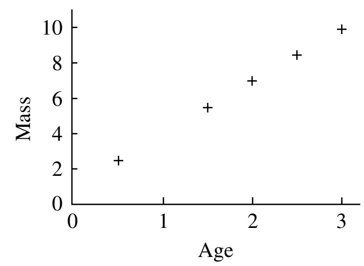
47. Using the first two rows for mass, we find a slope of

$$\text{slope} = \frac{\text{change in mass}}{\text{change in age}} = \frac{4.0 - 2.5}{1.0 - 0.5} = 3.0.$$

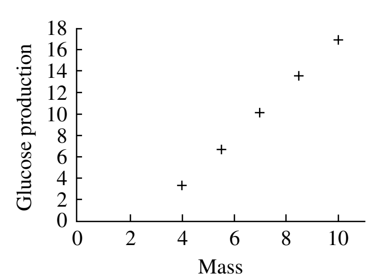
Using (1.0, 4.0) as the base point,

$$M = 3.0(a - 1.0) + 4.0 = 3.0a + 1.0.$$

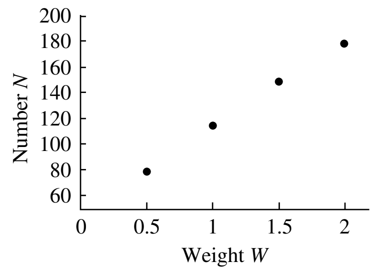
The y-intercept is 1.0, meaning that the mass was 1.0 g at age 0. This might be the mass of a new seedling. Interpolating at $a = 1.75$, $M = 3.0 \cdot 1.75 + 1.0 = 6.25$.



49. The glucose production changes by 3.4 when the mass changes by 1.5, giving a slope of 2.27. The point-slope form of the line, using the last data point, is $G = 2.27(m - 10.0) + 17.0$, which simplifies to $G = 2.27m - 5.7$ in slope-intercept form. The negative intercept must mean that this plant would not start making glucose until the mass got beyond a certain value (around 2). When $m = 20.0$, $G = 39.7$.



51. The point (2.0, 175) lies below the line.



53. $N = 70(0.72 - 0.5) + 80 = 95.4$.

55. The time decreases from 216.8 to 215.9 seconds, giving a change of -0.9 second in 16 years or a slope of $-0.9/16 \approx -0.0563$ second per year. In point-slope form, the men's time m as a function of the year y is

$$m = -0.0563(y - 1972) + 216.8$$

57. Setting equal to 0 and solving for y,

$$\begin{aligned} -0.469(y - 1972) + 241.4 &= 0 \\ 0.469(y - 1972) &= 241.4 \\ (y - 1972) &= \frac{241.4}{0.469} \\ y &= \frac{241.4}{0.469} + 1972 = 2486. \end{aligned}$$

It seems likely that this will never happen, so we can assume that women will not improve this quickly forever.

Section 1.5, page 64

1. The updating function is $f(p_t) = p_t - 2$, and $f(5) = 3$, $f(10) = 8$, $f(15) = 13$. This is a linear function.

3. The updating function is $f(x_t) = x_t^2 + 2$, and $f(0) = 2$, $f(2) = 6$, $f(4) = 18$. This is not a linear function because the input x_t is squared.

5. Denote the updating function by $f(v) = 1.5v$. Then $(f \circ f)(v) = f(1.5v) = 1.5(1.5v) = 2.25v$, so $v_{t+2} = 2.25v_t$. Applying f to the initial condition twice gives $f(1220) = 1830$ and $f(1830) = 2745$, which is equal to $2.25 \cdot 1220$.

7. Denote the updating function by $h(n) = 0.5n$; then $(h \circ h)(n) = h(0.5n) = 0.5(0.5n) = 0.25n$ and $n_{t+2} = 0.25n_t$. Applying the updating function to the initial condition twice gives $h(1200) = 600$ and $h(600) = 300$, matching $(h \circ h)(1200) = 0.25 \cdot 1200 = 300$.

9. Solving for v_t gives $v_t = \frac{v_{t+1}}{1.5}$. Then $v_0 = \frac{1220}{1.5} = 813.3$.

11. Solving for n_t gives $n_t = 2n_{t+1}$. Then $n_0 = 2 \cdot 1200 = 2400$.

13.
$$(f \circ f)(x) = f(f(x)) = f\left(\frac{x}{1+x}\right) = \frac{\frac{x}{1+x}}{1 + \frac{x}{1+x}}$$

$$= \frac{\frac{x}{1+x}}{\frac{1+2x}{1+x}} = \frac{x}{1+2x}.$$

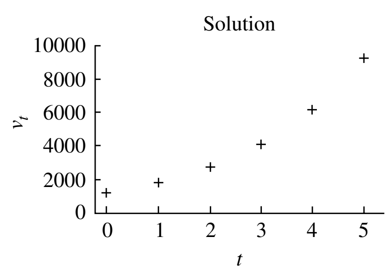
To find the inverse, set $y = f(x)$ and solve

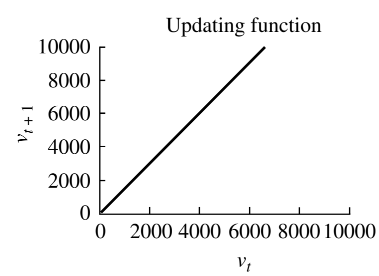
$$\begin{aligned} y &= \frac{x}{1+x} \\ (1+x)y &= x \\ y + xy &= x \\ y &= x - xy \\ \frac{y}{1-y} &= x. \end{aligned}$$

Therefore, $f^{-1}(y) = \frac{y}{1-y}$.

15. $v_t = 1.5^t \cdot 1220\ \mu\text{m}^3$.

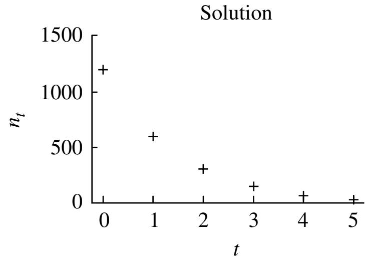
$$\begin{aligned} v_1 &= 1.5 \cdot 1220 = 1830 \\ v_2 &= 1.5 \cdot 1830 = 2745 \\ v_3 &= 1.5 \cdot 2745 = 4117.5 \\ v_4 &= 1.5 \cdot 4117.5 = 6176.25 \\ v_5 &= 1.5 \cdot 6176.25 = 9264.375. \end{aligned}$$

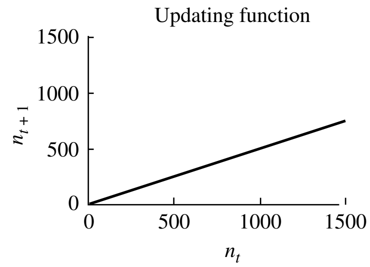




17. $n_t = 0.5^t \cdot 1200$.

$$\begin{aligned} n_1 &= 0.5 \cdot 1200 = 600 \\ n_2 &= 0.5 \cdot 600 = 300 \\ n_3 &= 0.5 \cdot 300 = 150 \\ n_4 &= 0.5 \cdot 150 = 75 \\ n_5 &= 0.5 \cdot 75 = 37.5. \end{aligned}$$





19. Plugging $t = 20$ into the solution $v_t = 1.5^t \cdot 1220\ \mu\text{m}^3$, we get $v_{20} = 4.05 \times 10^6\ \mu\text{m}^3$. This might be reasonable.

21. Plugging $t = 20$ into the solution $n_t = 0.5^t \cdot 1200$, we get $n_{20} = 0.0011$. This is an unreasonably small population.

23. $x_1 = 1/2$, $x_2 = 1/3$, $x_3 = 1/4$, $x_4 = 1/5$. It looks like $x_t = \dfrac{1}{1+t}$.

25. $x_1 = 3$, $x_2 = 1$, $x_3 = 3$, $x_4 = 1$. It seems to be jumping back and forth between 1 and 3. If I start at $x_0 = 0$, the results jump back and forth between 0 and 4.

27. These do not commute. If you started with 100, doubled (giving 200), and then removed 10, you would end up with 190. If you started with 100, removed 10 (leaving 90), and then doubled, you'd have only 180. In general, if we call the starting population P_t, if we double first and then remove 10, we end up with $P_{t+1} = 2P_t - 10$. If we first remove 10 and then double, we end up with $P_{t+1} = 2(P_t - 10) = 2P_t - 20$, which never matches the result in the other order.

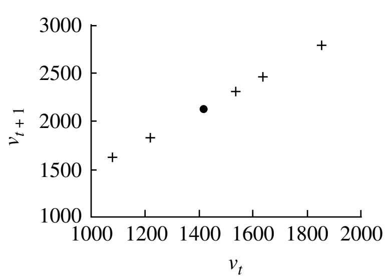
29. These do commute. Either way, it ends up 1.0 cm taller.

31. $h_{20} = 10.0 + 20 = 30.0$ m, a reasonable height for a tree.

33. 1.05×10^6 million bacteria. These will weigh about 10^{-6} g, which sounds reasonable.

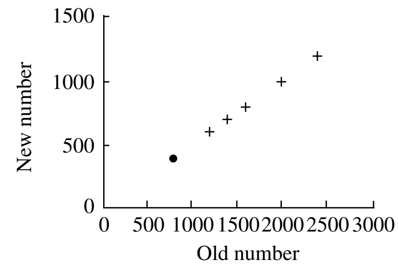
35. The solution is $x_1 = 50$, $x_2 = 130$, $x_3 = 290$. Adding 30, we see that $x_0 + 30 = 40$, $x_1 + 30 = 80 = 40 \cdot 2$, $x_2 + 30 = 160 = 40 \cdot 2^2$, and $x_3 + 30 = 320 = 40 \cdot 2^3$. It looks like $x_t + 30 = 40 \cdot 2^t$, so $x_t = 40 \cdot 2^t - 30$.

37.



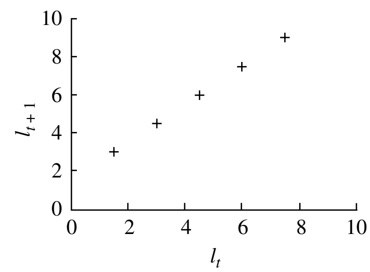
The discrete-time dynamical system is $v_{t+1} = 1.5 v_t$ and the missing value is 2130.

39.

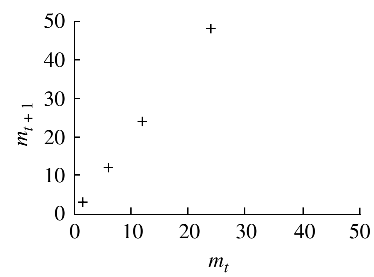


The discrete-time dynamical system is $n_{t+1} = 0.5 n_t$ and the missing value is 4.0×10^2.

41. The length increases by 1.5 cm each half-day, so $l_{t+1} = l_t + 1.5$ cm.

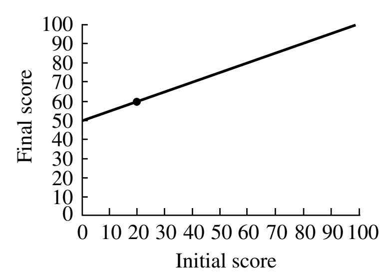


43. The mass doubles each half-day, so $m_{t+1} = 2m_t$.



45. The argument is the initial score. The value is the final score.

47.



49. Let v_{t+1} and v_t be the total volume before and after the experiment. Then

$$v_t = 10^4 b_t \quad \text{and} \quad v_{t+1} = 10^4 b_{t+1}.$$

The original discrete-time dynamical system is

$$b_{t+1} = 2.0 b_t.$$

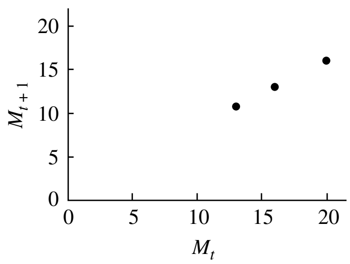
Therefore,

$$\begin{aligned} v_{t+1} &= 10^4 b_{t+1} \\ &= 10^4(2.0b_t) \\ &= 2.0 \cdot 10^4 b_t \\ &= 2.0v_t. \end{aligned}$$

51. $V_t = \pi h_t 0.5^2$, and $V_{t+1} = \pi h_{t+1} 0.5^2$. Therefore,

$V_{t+1} = \pi(h_t + 1)0.5^2 = \pi h_t \cdot 0.5^2 + \pi \cdot 0.5^2 = V_t + \pi \cdot 0.5^2$.

53. The points for the first patient are (20.0, 16.0), (16.0, 13.0), and (13.0, 10.75).



Let the level be M_t at the beginning of the day. The two points are (20.0, 16.0) and (16.0, 13.0). The slope is

$$\text{slope} = \frac{\text{change in output}}{\text{change in input}} = \frac{13.0 - 16.0}{16.0 - 20.0} = 0.75.$$

In point-slope form, the discrete-time dynamical system is

$$M_{t+1} = 0.75\,(M_t - 20.0) + 16.0 = 0.75M_t + 1.0.$$

55. The solution for the first is $b_t = 2.0^t \cdot 1.0 \times 10^6$, and the solution for the second is $b_t = 2.0^t \cdot 3.0 \times 10^5$. The difference is $2.0^t \cdot 0.7 \times 10^6$, but the ratio is always approximately 3.33. Both populations are growing at the same rate, but the first has a head start. It is always 3.33 times bigger, which becomes a larger difference as the populations become larger.

57. a. $b_1 = 2b_0 - 1.0 \times 10^6 = 2 \cdot 3.0 \times 10^6 - 1.0 \times 10^6 = 5.0 \times 10^6$, $b_2 = 9.0 \times 10^6$, $b_3 = 17.0 \times 10^6$.

b. There were three harvests of 1.0×10^6 bacteria, for a total of 3.0×10^6 bacteria.

c. $b_{t+1} = 2.0b_t - 1.0 \times 10^6$.

d. The population would have doubled three times, so there would be 24.0×10^6 bacteria. You could harvest 7.0×10^6 bacteria and still have a population of 17.0×10^6, which means harvesting 4.0×10^6 more than in part **b.** The bacteria removed early never had a chance to reproduce.

59. a. If the old fraction is f_t, then $f_{t+1} = f_t + 0.1f_t$, or $f_{t+1} = 1.1f_t$.

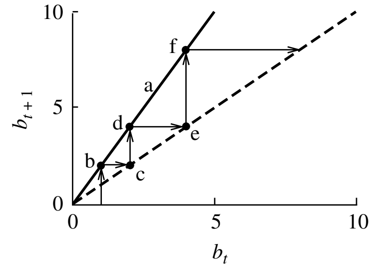
b. $f_t = 0.001 \cdot 1.1^t$.

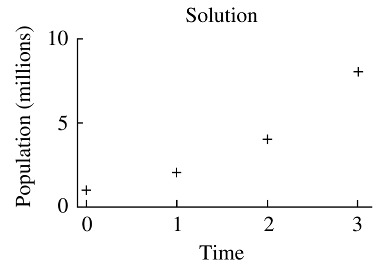
c. The fraction gets larger and larger and will eventually exceed 1. The discrete-time dynamical system doesn't make sense when $f_{t+1} > 1$ because a fraction can't be bigger than 1.

61. We have that $b_{t+1} = 2b_t$ and $m_{t+1} = 3m_t$. Then the total mass $M_{t+1} = m_{t+1}b_{t+1} = 3m_t \cdot 2b_t = 6m_tb_t = 6M_t$.

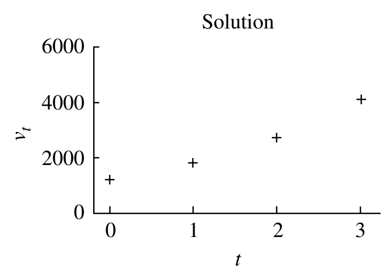
Section 1.6, page 76

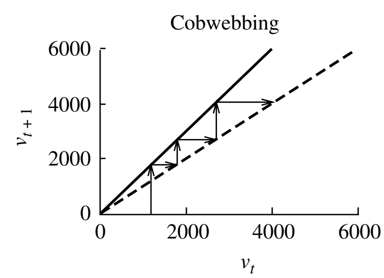
1.



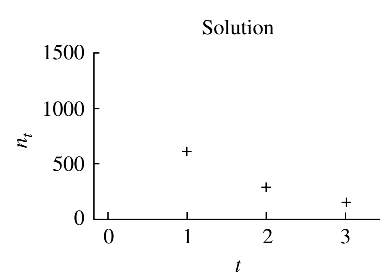


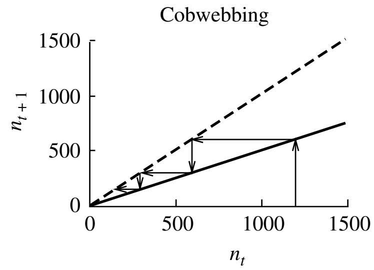
3. The solution was $v_t = 1.5^t \cdot 1220\ \mu\text{m}^3$, consistent with a cobweb diagram that predicts a solution that increases faster and faster.





5. The solution was $n_t = 0.5^t \cdot 1200$, consistent with a cobweb diagram that decays toward 0.





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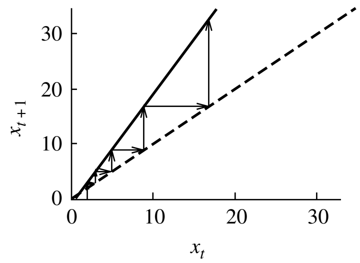

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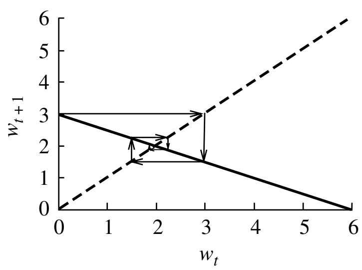
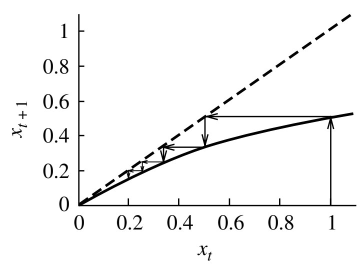
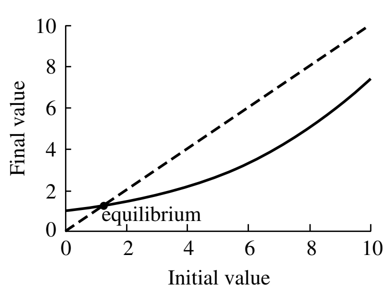


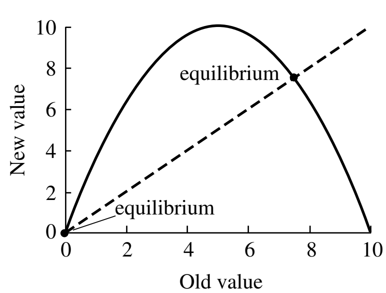
11.



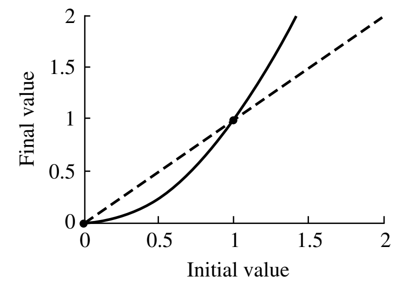
13. The equilibrium seems to be at about 1.3.



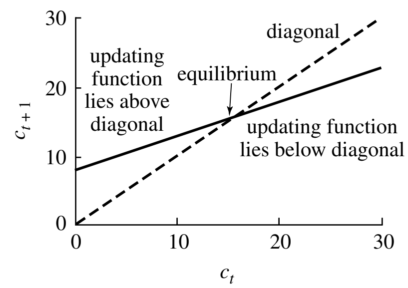
15. The equilibria seem to be at about 0.0 and 7.5.



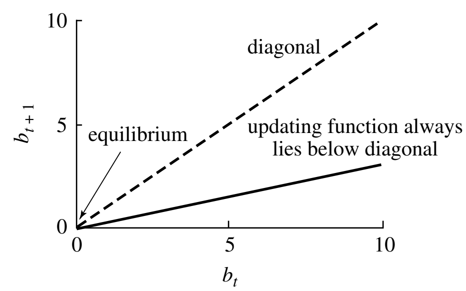
17. $f(x) = x$ when $x^2 = x$, or $x^2 - x = 0$, or $x(x-1) = 0$, which has solutions at $x = 0$ and $x = 1$.



19. The equilibrium is where $c^* = 0.5c^* + 8.0$, or $c^* = 16.0$.



21. The equilibrium is $b^* = 0$.



23. $v^* = 1.5v^*$ if $v^* = 0$.

25. $x^* = 2x^* - 1$ has solution $x^* = 1$.

27. $w^* = -0.5w^* + 3$ has solution $w^* = 2$.

29. $x^* = \dfrac{x^*}{1 + x^*}$ has solution $x^* = 0$.

31.

$$
\begin{aligned}
w^* &= aw^* + 3 \\
w^* - aw^* &= 3 \\
w^* &= \frac{3}{1-a}.
\end{aligned}
$$

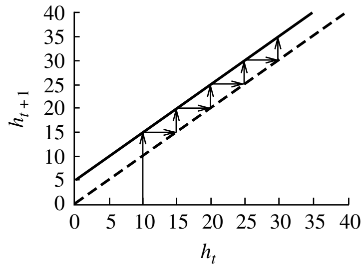
This solution does not exist if $a = 1$, and it is negative if $a > 1$.

33.

$$
\begin{aligned}
x^* &= \frac{ax^*}{1+x^*} \\
x^*(1+x^*) &= ax^* \\
x^*(1+x^*) - ax^* &= 0 \\
x^*(1+x^*-a) &= 0.
\end{aligned}
$$

There are two solutions, $x^* = 0$ and $x* = a - 1$. The second is negative if $a < 1$. The two solutions are equal (leaving only one) when $a = 1$.

35.



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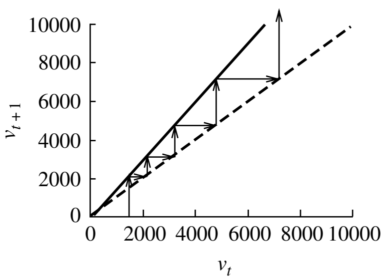

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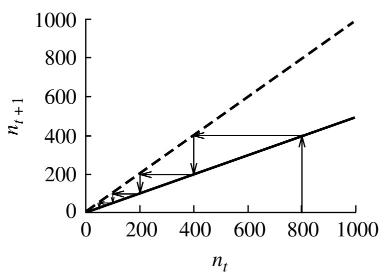
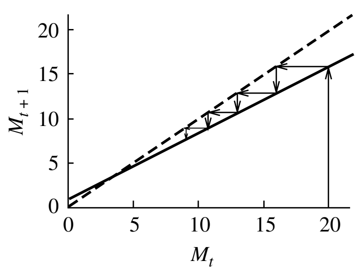


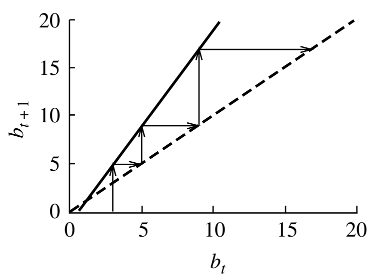
41.



The equilibrium is

$$\begin{aligned} M^* &= 0.75M^* + 1 \\ M^* - 0.75M^* &= 1 \\ 0.25M^* &= 1 \\ M^* &= 4. \end{aligned}$$

43.



The equilibrium is

$$\begin{aligned} b^* &= 2.0b^* - 1.0 \times 10^6 \\ b^* - 2.0b^* &= -1.0 \times 10^6 \\ -b^* &= -1.0 \times 10^6 \\ b^* &= 1.0 \times 10^6. \end{aligned}$$

The population grows, as we found in Section 1.5, Exercise 57, and seems to be moving away from the equilibrium.

45. The equilibrium is

$$\begin{aligned} b^* &= 2.0b^* - h \\ b^* - 2.0b^* &= -h \\ -b^* &= -h \\ b^* &= h. \end{aligned}$$

It is strange that the equilibrium gets larger as the harvest gets larger. However, the cobwebbing diagram indicates that only populations above the equilibrium will grow, and those below it will shrink. The equilibrium in this case is the minimum population required for the population to survive.

Section 1.7, page 89

1. Law 6: $43.2^0 = 1$.

3. Law 3: $43.2^{-1} = 1/43.2 \approx 0.023$.

5. Law 4: $43.2^{7.2}/43.2^{6.2} = 43.2^{7.2-6.2} = 43.2^1 = 43.2$.

7. Law 2: $(3^4)^{0.5} = 3^{4\cdot 0.5} = 3^2 = 9$.

9. To use law 1, we must first multiply out the exponents, finding $2^{2^3} \cdot 2^{2^2} = 2^8 \cdot 2^4 = 2^{12} = 4096$.

11. Law 6: $\ln(1) = 0$.

13. Law 5: $\log_{43.2} 43.2 = 1$.

15. Law 1: $\log_{10}(5) + \log_{10}(20) = \log_{10}(5 \cdot 20) = \log_{10}(100) = \log_{10}(10^2) = 2$.

17. Law 4: $\log_{10}(500) - \log_{10}(50) = \log_{10}(500/50) = \log_{10}(10) = 1$.

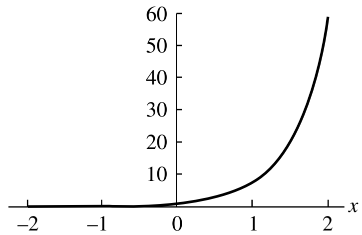
19. Law 2: $\log_{43.2}(43.2^7) = 7$.

21. Law 3: $\log_7\left(\frac{1}{43.2}\right) = -\log_7 43.2 \approx -1.935$.

23. $e^{3x} = \frac{21}{7} = 3$. Taking logs of both sides, $3x = \ln(3) \approx 1.099$, and $x \approx 0.366$. Checking, $7e^{3\cdot 0.366} \approx 21.0$.

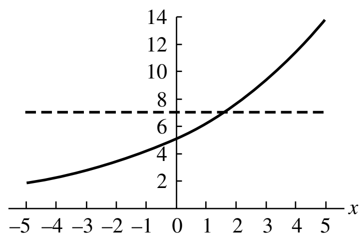
25. Taking logs of both sides gives $\ln(4) - 2x + 1 = \ln(7) + 3x$. Moving the x's to one side gives $5x = \ln(4) + 1 - \ln(7) = 1 + \ln\left(\frac{4}{7}\right) \approx 0.440$, and $x = 0.088$. Checking, $4e^{-2\cdot 0.088+1} = 7e^{3\cdot 0.088} \approx 9.11$.

27. $e^{2x} = 7$ at $2x = \ln(7.0) \approx 1.946$ or $x \approx 0.973$. This function is increasing, and it doubles after a "time" of $x = \frac{\ln(2)}{2} \approx 0.346$.



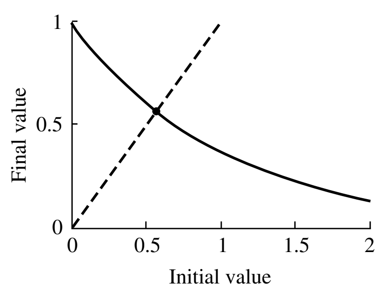
Therefore, $e^{2x} = 14.0$ when $x \approx 0.973 + 0.346 = 1.319$, and $e^{2x} = 3.5$ when $x \approx 0.973 - 0.346 = 0.627$.

29. $5e^{0.2x} = 7$ when $e^{0.2x} = 1.4$, or $0.2x = \ln(1.4) \approx 0.336$ or $x \approx 1.68$. This function is increasing, and it doubles after a "time" of $x = \frac{\ln(2)}{0.2} \approx 3.46$.

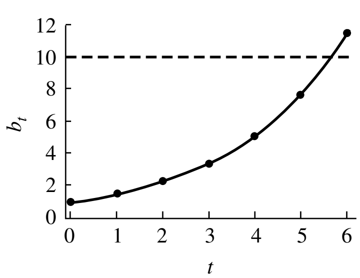


Therefore, $5e^{0.2x} = 14.0$ when $x \approx 1.68 + 3.46 = 5.14$, and $5e^{0.2x} = 3.5$ when $x \approx 1.68 - 3.46 = -1.78$.

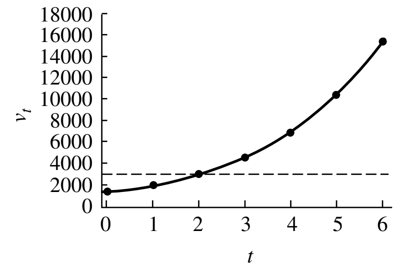
31.



33. The solution is $b_t = 1.5^t \cdot 1.0 \times 10^6$. In exponential notation, $b_t = 1.0 \times 10^6 e^{\ln(1.5)t} \approx 1.0 \times 10^6 e^{0.405t}$. The population reaches 1.0×10^7 when $e^{0.405t} = 10$, or $0.405t = \ln(10) \approx 2.302$, or $t \approx \frac{2.302}{0.405} \approx 5.865$.



35. The solution is $v_t = 1.5^t \cdot 1350$. In exponential notation, $v_t = 1350e^{\ln(1.5)t} \approx 1350e^{0.405t}$. The volume reaches 3250 when $e^{0.405t} = \frac{3250}{1350} = 2.407$, or $0.405t = \ln(2.407) \approx 0.878$, or $t \approx \frac{0.878}{0.405} \approx 2.17$.



37. $t_d \approx 0.6931/1.0 = 0.6931$ days. It will take twice this long to quadruple, or 1.386 days.

39. $t_d \approx 0.6931/0.1 = 6.931$ hr. It will quadruple in 13.86 hr.

41. Converting to base e, we have that $S(t) = 2.34e^{\ln(10)\cdot 0.5t} \approx 2.34e^{1.151t}$. The doubling time is $t_d \approx \frac{0.6931}{1.151} = 0.602$ days. It will have increased by a factor of 10 when $\alpha t = 1$, which occurs when $t = 2$.

43. $Q(50000) = 6.0 \times 10^{10} e^{-0.000122 \cdot 50000} \approx 1.34 \times 10^8$. This is $\frac{1.34 \times 10^8}{6.0 \times 10^{10}} \approx 0.00223$ of the original amount.

45. $t_h \approx \frac{0.693}{0.000122} \approx 5680$ years.

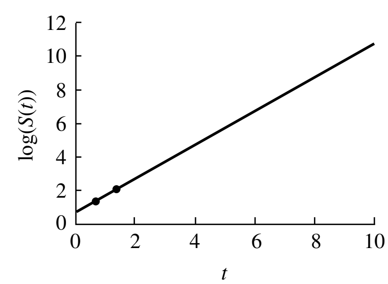
47. It will have doubled twice and will be 2000.

49. If the doubling time is 24 years, the parameter α is $\frac{0.693}{24} \approx 0.0289$. Therefore, $P(t) \approx 500e^{0.0289t}$.

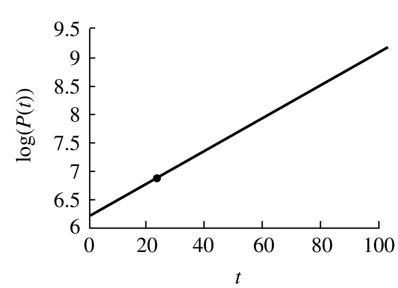
51. It will have halved three times, which takes 129 years.

53. If the half-life is 43 years, the parameter α is $-\frac{0.693}{43} \approx -0.0161$, and the equation is $P(t) \approx 1600e^{-0.0161t}$.

55. We plot the line $\ln(S(t)) = \ln(2.0e^t) \approx 0.693 + t$.

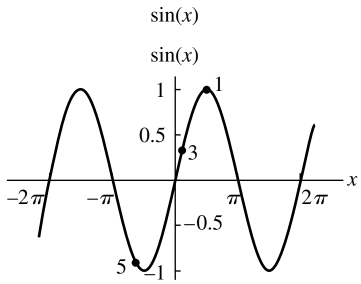


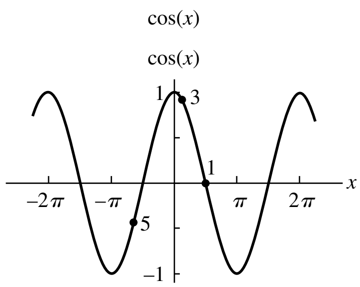
57. $\ln(P(t)) = \ln(500e^{0.0289t}) \approx 6.21 + 0.0289t$.

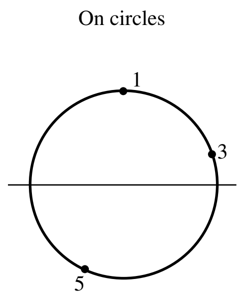


Section 1.8, page 98

1. $\sin(\pi/2) = 1$, $\cos(\pi/2) = 0$.







3. $\sin(\pi/9) \approx 0.342$, $\cos(\pi/9) \approx 0.940$.

5. $\sin(-2.0) \approx -0.909$, $\cos(-2.0) \approx -0.416$.

7. $\pi/6$ rad.

9. 0.017 rad.

11. 114.6°.

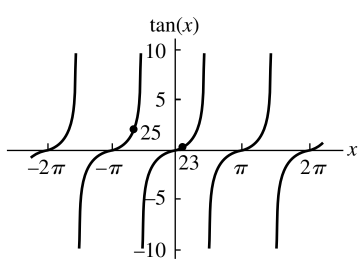
13. $-36^\circ = 324^\circ$.

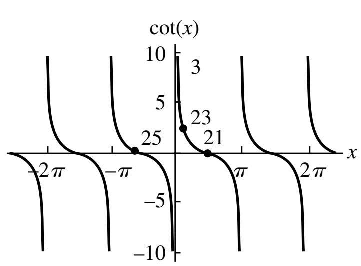
15. 200 grads, after multiplying by $\frac{400 \text{ grads}}{360^\circ}$.

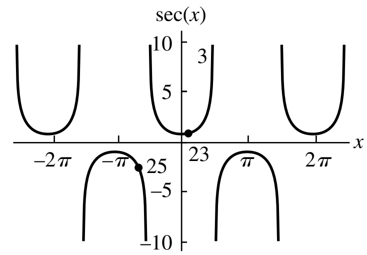
17. 50.0 grads, after multiplying by $\frac{400 \text{ grads}}{2\pi}$.

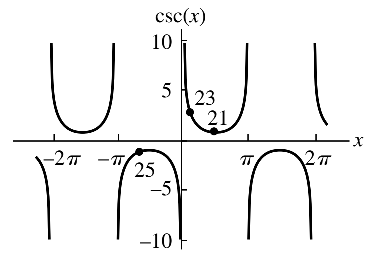
19. 135°, after multiplying by $\frac{360^\circ}{400 \text{ grads}}$.

21. No answer, 0, no answer, 1.









23. $\tan(\pi/9) \approx 0.36397$, $\cot(\pi/9) \approx 2.74748$, $\sec(\pi/9) \approx 1.06418$, $\csc(\pi/9) \approx 2.92380$.

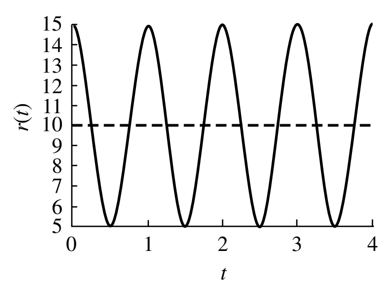
25. $\tan(-2.0) \approx 2.18504$, $\cot(-2.0) \approx 0.45766$, $\sec(-2.0) \approx -2.40300$, $\csc(-2.0) \approx -1.09975$.

27. $\cos(0) = \sqrt{1}$, $\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} = \sqrt{\frac{1}{2}}$, $\cos\left(\frac{\pi}{2}\right) = 0 = \sqrt{\frac{0}{2}}$.

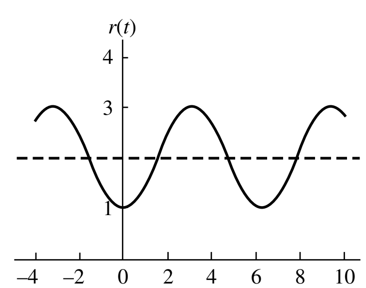
29. $\cos(0 - \pi) = \cos(-\pi) = -1 = -\cos(0)$, $\cos(\pi/4 - \pi) = \cos(-3\pi/4) = -\sqrt{2}/2 = -\cos(\pi/4)$, $\cos(\pi/2 - \pi) = \cos(-\pi/2) = 0 = -\cos(\pi/2)$, $\cos(\pi - \pi) = \cos(0) = 1 = -\cos(\pi)$.

31. $\cos(2 \cdot 0) = \cos^2(0) - \sin^2(0) = 1 - 0 = 1 = \cos(0)$, $\cos(2 \cdot \pi/4) = \cos^2(\pi/4) - \sin^2(\pi/4) = 1/2 - 1/2 = 0 = \cos(\pi/2)$, $\cos(2 \cdot \pi/2) = \cos^2(\pi/2) - \sin^2(\pi/2) = 0 - 1 = -1 = \cos(\pi)$, $\cos(2 \cdot \pi) = \cos^2(\pi) - \sin^2(\pi) = 1 - 0 = 1 = \cos(2\pi)$.

33. Multiplying the factor of 5.0 through gives $r(t) = 10.0 + 5.0\cos(2\pi t)$ with average 10.0, amplitude 5.0, period 1.0, and phase 0.



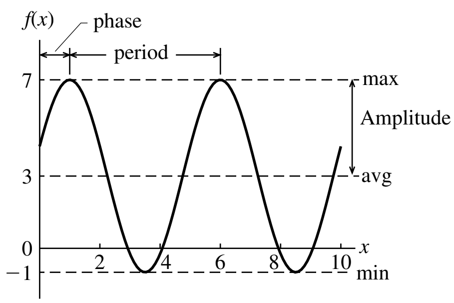
35. Use the fact that $\cos(t - \pi) = -\cos(t)$. Then $f(t) = 2.0 + 1.0\cos(t - \pi)$ with average 2.0, amplitude 1.0, period 2π, and phase π.



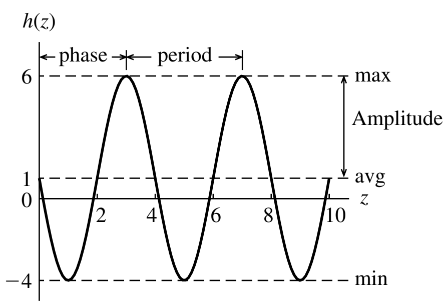
37. Average is 6.0, minimum is 4.0, maximum is 8.0, amplitude is 2.0, period is 5.0, and phase is 2.0.

39. Average is 4.0, minimum is -1.0, maximum is 9.0, amplitude is 5.0, period is 2.0, and phase is 0.0.

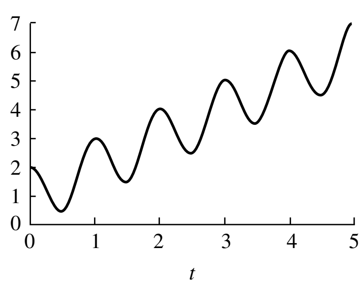
41. The average is 3.0, the amplitude is 4.0, the maximum is 7.0, the minimum is -1.0, the period is 5.0, and the phase is 1.0.



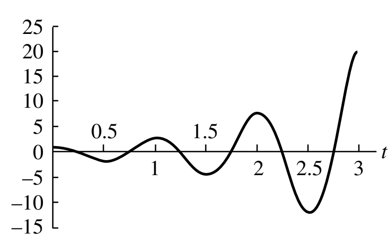
43. The average is 1.0, the amplitude is 5.0, the maximum is 6.0, the minimum is -5.0, the period is 4.0, and the phase is 3.0.



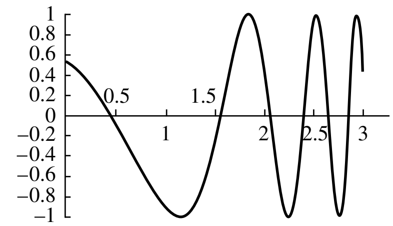
45. This function increases overall but wiggles around quite a bit. It might describe the size of an organism that grows on average, but grows quickly during the day and shrinks down a bit at night.



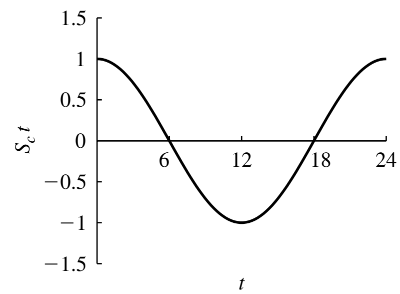
47. This function wiggles up and down with increasing amplitude. Perhaps it describes the insane temperature oscillations that will precede the next ice age.



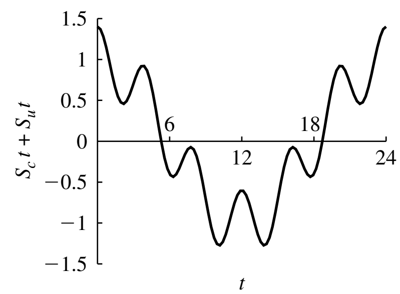
49. This function wiggles up and down with increasing period but with constant amplitude. Perhaps it describes the insane temperature oscillations that will precede the next ice age.



51. Let $S_c(t)$ represent the circadian rhythm. Then $S_c(t) = \cos\left(\frac{2\pi t}{24}\right)$.



53. To plot, compute the value of the combined function $S_c(t) + S_u(t)$ every hour, and smoothly connect the dots.



Section 1.9, page 109

1. 2/3 of the water is at 100°C, and 1/3 is at 30°C. The final temperature is then the weighted average $T = \frac{1}{3} \cdot 30°\text{C} + \frac{2}{3} \cdot 100°\text{C} \approx 76.7°\text{C}$.

3. A fraction $\frac{20}{52}$ got 50, $\frac{18}{52}$ got 75, and $\frac{14}{52}$ got 100, for an average of $\frac{20}{52} \cdot 50 + \frac{18}{52} \cdot 75 + \frac{14}{52} \cdot 100 \approx 72.1$.

5. 1/3 of the water is at T_1, and 2/3 is at T_2. The final temperature is then the weighted average $T = \frac{1}{3} \cdot T_1 + \frac{2}{3} \cdot T_2$. If $T_1 = 30$ and $T_2 = 100$, we get $T = \frac{1}{3} \cdot 30 + \frac{2}{3} \cdot 100 = 76.7$ as before.

7. A fraction $V_1/(V_1 + V_2)$ is at T_1, and a fraction $V_2/(V_1 + V_2)$ is at T_2. The final temperature is the weighted average

$$T = T_1 \frac{V_1}{V_1 + V_2} + T_2 \frac{V_2}{V_1 + V_2}.$$

9. The 100°C water cools to 50°C, so 2/3 of the water is at 50°C, and 1/3 is at 15°C. The final temperature is then the weighted average $T = \frac{1}{3} \cdot 15°C + \frac{2}{3} \cdot 50°C \approx 38.3°C$. This is indeed half the value in Exercise 1.

11. After the deduction, a fraction $\frac{20}{52}$ got 40, $\frac{18}{52}$ got 65, and $\frac{14}{52}$ got 90, for an average of $\frac{20}{52} \cdot 40 + \frac{18}{52} \cdot 65 + \frac{14}{52} \cdot 90 = 62.1$. This is indeed 10 less than the average before the deduction.

13. a. amount = volume times concentration, or $V \cdot c_0 = 2.0 \cdot 1.0 = 2.0$ mmol.

b. 0.5 L at 1.0 mmol/L = 0.5 mmol.

c. 1.5 L at 1.0 mmol/L = 1.5 mmol.

d. 0.5 L at 5.0 mmol/L = 2.5 mmol.

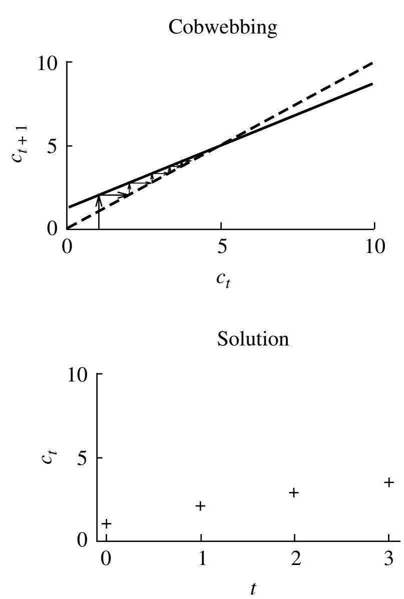
e. 1.5 mmol + 2.5 mmol = 4.0 mmol.

f. 4.0 mmol/2.0 L = 2.0 mmol/L.

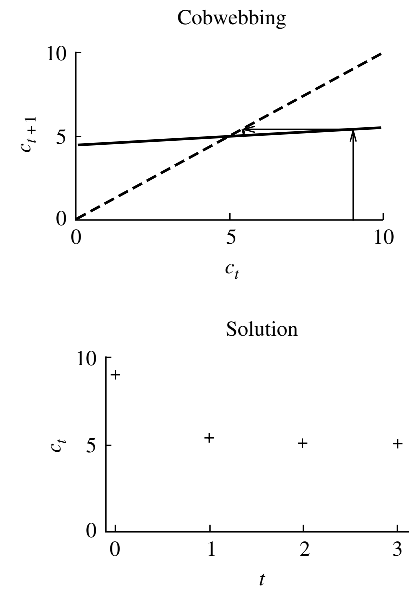
g. $q = 0.5/2.0 = 0.25$. Then $c_{t+1} = (1 - q)c_t + q\gamma = 0.75 \cdot c_t + 0.25 \cdot 5.0$. When $c_0 = 1.0$, $c_1 = 0.75 \cdot 1.0 + 0.25 \cdot 5.0 = 2.0$ mmol/L.

15. Start with 9.0 mmol, breathe out 8.1 mmol, leaving 0.9 mmol, breathe in 4.5 mmol, ending with 5.4 mmol and a concentration of 5.4 mmol/L. This checks with the discrete-time dynamical system. In this case, $q = 0.9$ and $\gamma = 5.0$, so $c_{t+1} = 0.1c_t + 0.9 \cdot 5.0$. Substituting $c_0 = 9.0$, we find $c_1 = 5.4$.

17. The discrete-time dynamical system has $q = 0.5/2.0 = 0.25$ and $\gamma = 5.0$ and thus has formula $c_{t+1} = (1 - 0.25)c_t + 0.25 \cdot 5.0 = 0.75c_t + 1.25$. We want to start from 1.0 mmol/L.



19. The discrete-time dynamical system is $c_{t+1} = 0.1c_t + 4.5$, starting from $c_0 = 9.0$.



21. The discrete-time dynamical system is $c_{t+1} = 0.75 \cdot c_t + 1.25$. Solving for the equilibrium, we find $c^* = 0.75 \cdot c^* + 1.25$, or $0.25c^* = 1.25$, or $c^* = 5.0$. This matches the value of γ.

23. The discrete-time dynamical system is $c_{t+1} = 0.1 \cdot c_t + 4.5$. Solving for the equilibrium, we find $c^* = 0.1 \cdot c^* + 4.5$, or $0.9c^* = 4.5$, or $c^* = 5.0$. This matches the value of γ.

25. Using the equation $c^* = \frac{q\gamma}{1 - (1 - q)(1 - \alpha)}$ for the equilibrium, we find that $c^* = \frac{0.4 \cdot 0.21}{1 - 0.6 \cdot 0.9} \approx 0.183$. The concentration is higher because more of the air in the lung at any one time comes from outside.

27. The concentration after absorption is $c_t - 0.02$. Using the weighted-average idea,

$$\begin{aligned} c_{t+1} &= (1 - q)\,(c_t - 0.02) + q\gamma = 0.8\,(c_t - 0.02) + 0.042 \\ &= 0.8c_t + 0.026. \end{aligned}$$

The equilibrium solves

$$\begin{aligned} c^* &= 0.8c^* + 0.026 \\ 0.2c^* &= 0.026 \\ c^* &= 0.13. \end{aligned}$$

This function does not really make sense if $c_t < 0.02$ because there not would be enough there to absorb.

29. The concentration after absorption is $c_t - 0.2(c_t - 0.05) = 0.8c_t + 0.01$. Then

$$\begin{aligned} c_{t+1} &= (1 - q)(0.8c_t + 0.01) + q\gamma = 0.8(0.8c_t + 0.01) \\ &\quad + 0.042 = 0.64c_t + 0.05. \end{aligned}$$

The equilibrium is then

$$\begin{aligned} c^* &= 0.64c^* + 0.05 \\ 0.36c^* &= 0.05 \\ c^* &= 0.0139. \end{aligned}$$

31. The concentration after absorption is $c_t - A$. Then

$$c_{t+1} = (1-q)(c_t - A) + q\gamma = 0.8(c_t - A) + 0.042.$$

The equilibrium solves

$$\begin{aligned} c^* &= 0.8c^* + 0.042 - 0.8A \\ 0.2c^* &= 0.042 - 0.8A \\ c^* &= 0.21 - 4A. \end{aligned}$$

Then $c^* = 0.15$ if $0.21 - 4A = 0.15$, or $A = 0.015$. The lung must reduce the oxygen concentration by 1.5%. In Example 1.9.9, we found that the lung absorbs 10% of the equilibrium concentration of 0.15, which is equivalent to $A = 0.15$.

33. The concentration before exchanging air is $c_t + 0.001$, so the discrete-time dynamical system is the weighted average

$$\begin{aligned} c_{t+1} &= (1-q)(c_t + 0.001) + q\gamma \\ c_{t+1} &= 0.8(c_t + 0.001) + 0.2 \cdot 0.0004 = 0.8c_t + 0.00088. \end{aligned}$$

The equilibrium is

$$\begin{aligned} c^* &= 0.8c^* + 0.00088 \\ 0.2c^* &= 0.00088 \\ c^* &= 0.0044. \end{aligned}$$

This is about 11 times higher than the external concentration.

35. a. Population after reproduction is $0.6 \cdot 3.0 \times 10^6 = 1.8 \times 10^6$.

b. Population after supplementation is $1.8 \times 10^6 + 1.0 \times 10^6 = 2.8 \times 10^6$.

c. $b_{t+1} = 0.6b_t + 1.0 \times 10^6$.

37. The discrete-time dynamical system is $b_{t+1} = 0.6b_t + 1.0 \times 10^6$. The equilibrium satisfies $b^* = 0.6b^* + 1.0 \times 10^6$, or $0.4b^* = 1.0 \times 10^6$, or $b^* = 2.5 \times 10^6$.

39. The discrete-time dynamical system is $b_{t+1} = 0.5b_t + S$. The equilibrium satisfies $b^* = 0.5b^* + S$, or $0.5b^* = S$, or $b^* = 2S$. The equilibrium becomes larger when S is large. This makes sense because the population will be larger when more bacteria are added.

41. Let s_t be the concentration of salt before inflow and loss through outflow. There is then $3.3 \times 10^7 s_t$ salt in the lake, which receives 3.0×10^3 of salt from inflow, for a total of $3.3 \times 10^7 s_t + 3.0 \times 10^3$ of salt in 3.6×10^7 cubic meters of water. The concentration is

$$s_{t+1} = \frac{3.3 \times 10^7 s_t + 3.0 \times 10^3}{3.6 \times 10^7} \approx 0.917s_t + 8.33 \times 10^{-5}.$$

Because water that flows out is well mixed, this gives the discrete-time dynamical system.

43. As before, there are $3.3 \times 10^7 s_t + 3.0 \times 10^3$ cubic meters of salt in 3.6×10^7 cubic meters of water. After evaporation, there are 3.3×10^7 cubic meters of water, in which the concentration is

$$s_{t+1} = \frac{3.3 \times 10^7 s_t + 3.0 \times 10^3}{3.3 \times 10^7} \approx s_t + 1.0 \times 10^{-4}.$$

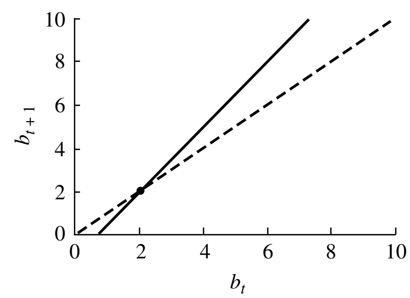
45. The discrete-time dynamical system is $s_{t+1} = 0.917s_t + 8.33 \times 10^{-5}$. The equilibrium solves

$$\begin{aligned} s^* &= 0.917s^* + 8.33 \times 10^{-5} \\ 0.083s^* &= 8.33 \times 10^{-5} \\ s^* &= 0.001. \end{aligned}$$

The water ends up like the water that flows in.

47. The discrete-time dynamical system is $s_{t+1} = s_t + 1.0 \times 10^{-4}$. The equilibrium equation is $s^* = s^* + 1.0 \times 10^{-4}$, which has no solution. This lake has no equilibrium and will get saltier and saltier.

49. The discrete-time dynamical system is $b_{t+1} = 1.5b_t - 1.0 \times 10^6$, and the equilibrium is $b^* = 2.0 \times 10^6$.

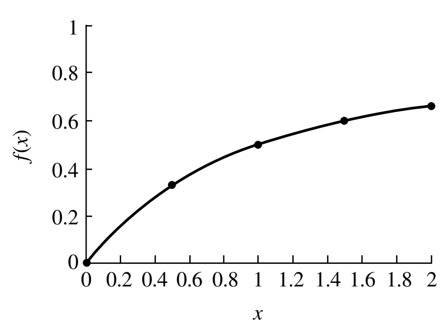


Section 1.10, page 118

1. There are now 400 red birds and 800 blue birds, or 1/3 red birds and 2/3 blue birds. These fractions add to 1.

3. There are now $200r$ red birds and 800 blue birds. The fraction of red birds is $200r$ out of $200r + 800$, or a fraction $\frac{r}{r+4}$. There are 800 blue birds out of $200r + 800$, or a fraction $\frac{4}{r+4}$. These fractions add to 1 no matter what r is.

5.



7.

$$p_t = \frac{m_t}{m_t + b_t} = \frac{1.2 \times 10^5}{1.2 \times 10^5 + 3.5 \times 10^6} \approx 0.033$$

$$\begin{aligned} m_{t+1} &= 1.2m_t = 1.44 \times 10^5 \\ b_{t+1} &= 2.0b_t = 7.0 \times 10^6 \\ p_{t+1} &= \frac{m_{t+1}}{m_{t+1} + b_{t+1}} = \frac{1.44 \times 10^5}{1.44 \times 10^5 + 7.0 \times 10^6} \approx 0.020. \end{aligned}$$

9. $p_t \approx 0.033$, $m_{t+1} = 0.36 \times 10^5$, $b_{t+1} = 1.75 \times 10^6$, $p_{t+1} \approx 0.020$.

11. The equation for the equilibria is $p^* = \dfrac{p^*}{p^* + 2.0(1 - p^*)}$. Then

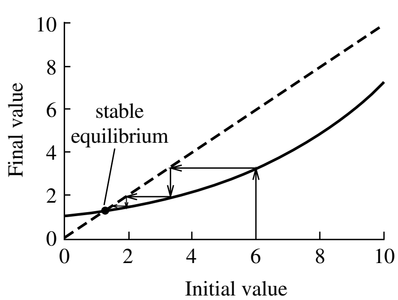
$p^*(p^* + 2.0(1 - p^*)) = p^*$	multiply both sides by the denominator
$p^*(p^* + 2.0(1 - p^*)) - p^* = 0$	subtract p^* from both sides
$p^*(p^* + 2.0(1 - p^*) - 1) = 0$	factor out p^*
$p^*(1.0 - p^*) = 0$	simplify
$p^* = 0$	
or $p^* = 1.0$	solve each piece.

13. The equation for equilibria is $x^* = \dfrac{x^*}{1 + ax^*}$. Solving,

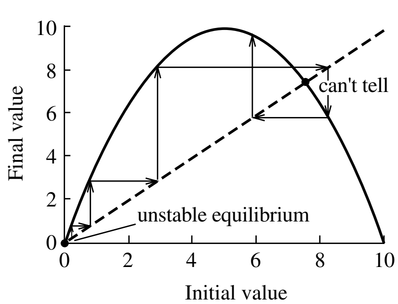
$x^*(1 + ax^*) = x^*$	multiply both sides by the denominator
$x^*(1 + ax^*) - x^* = 0$	subtract x^* from both sides
$x^*(1 + ax^* - 1) = 0$	factor out x^*
$x^*(ax^*) = 0$	simplify.

The only equilibrium is at $x^* = 0$, as long as $a \neq 0$. If $a = 0$, the system is $x_{t+1} = x_t$, which has every value of x^* as an equilibrium.

15. The equilibrium seems to be at about 1.3.



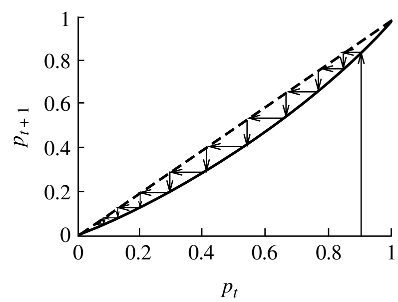
17. The equilibria seem to be at about 0.0 and 7.5.



19. The discrete-time dynamical system is

$$p_{t+1} = \frac{1.2p_t}{1.2p_t + 2.0(1 - p_t)}.$$

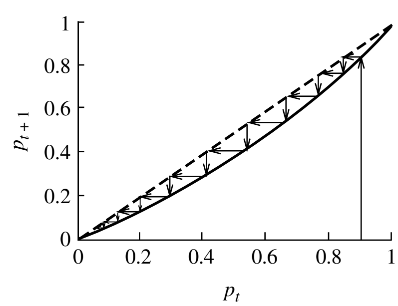
The equilibrium at $p^* = 0$ is stable, and the one at $p^* = 1$ is unstable. This makes sense because the wild type are reproducing faster than the mutants ($r > s$) and should dominate the population.



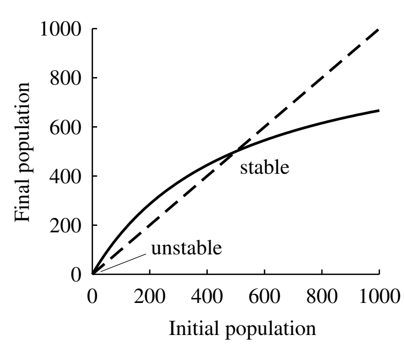
21. The discrete-time dynamical system is

$$p_{t+1} = \frac{0.3p_t}{0.3p_t + 0.5(1 - p_t)}.$$

The equilibrium at $p^* = 0$ is stable, and the one at $p^* = 1$ is unstable. The picture looks the same as in **a** because the ratio of r to s is the same. But in this case, both populations are decreasing.



23.



25. a. 2.0×10^5 mutate and 1.0×10^4 revert.

b. There are $1.0 \times 10^6 - 2.0 \times 10^5 + 1.0 \times 10^4 = 8.1 \times 10^5$ wild type and $1.0 \times 10^5 - 1.0 \times 10^4 + 2.0 \times 10^5 = 2.9 \times 10^5$ mutants.

c. The total number before and after is 1.1×10^6. It does not change because the bacteria are not reproducing or dying, just changing their type.

d. The fraction before is $1.0 \times 10^5/1.1 \times 10^6 = 0.091$. The fraction after is $2.9 \times 10^5/1.1 \times 10^6 \approx 0.264$.

27. a. $0.2b_t$ mutate and $0.1m_t$ revert.

b. $b_{t+1} = b_t - 0.2b_t + 0.1m_t = 0.8b_t + 0.1m_t$. $m_{t+1} = m_t - 0.1m_t + 0.2b_t = 0.9m_t + 0.2b_t$.

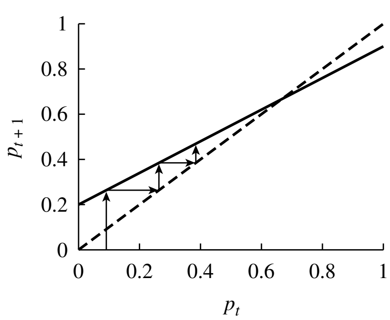
c. The total number before is $b_t + m_t$. The total number after is

$$\begin{aligned} b_{t+1} + m_{t+1} &= (0.8b_t + 0.1m_t) + (0.9m_t + 0.2b_t) \\ &= 0.8b_t + 0.2b_t + 0.1m_t + 0.9m_t \\ &= b_t + m_t. \end{aligned}$$

d. Divide the discrete-time dynamical system for m_{t+1} by $b_{t+1} + m_{t+1}$,

$$\begin{aligned} p_{t+1} &= \frac{m_{t+1}}{b_{t+1} + m_{t+1}} = \frac{0.9m_t + 0.2b_t}{b_{t+1} + m_{t+1}} = \frac{0.9m_t + 0.2b_t}{b_t + m_t} \\ &= \frac{0.9m_t}{b_t + m_t} + \frac{0.2b_t}{b_t + m_t} = 0.9p_t + 0.2(1 - p_t) \\ &= 0.2 + 0.7p_t. \end{aligned}$$

e. The equilibrium solves $p^* = 0.2 + 0.7p^*$, or $p^* \approx 0.667$.



f. The equilibrium seems to be stable. The fraction of mutants will increase until it reaches 66.7%.

29. a. 1.0×10^5 mutate.

b. There are $1.0 \times 10^6 - 1.0 \times 10^5 = 9.0 \times 10^5$ wild type and $1.0 \times 10^5 + 1.0 \times 10^5 = 2.0 \times 10^5$ mutants.

c. There are 1.8×10^6 wild type and 3.0×10^5 mutants.

d. The total number after is 2.1×10^6.

e. The fraction before is $1.0 \times 10^5/1.1 \times 10^6 \approx 0.091$. The fraction after is $3.0 \times 10^5/2.1 \times 10^6 \approx 0.143$.

31. a. $0.1b_t$ mutate.

b. There are $0.9b_t$ wild type and $m_t + 0.1b_t$ mutants after mutation.

c. There are $b_{t+1} = 1.8b_t$ wild type and $m_{t+1} = 1.5m_t + 0.15b_t$ mutants after reproduction.

d. The total number after is $1.95b_t + 1.5m_t$.

e. Divide the discrete-time dynamical system for m_{t+1} by $b_{t+1} + m_{t+1}$,

$$\begin{aligned} p_{t+1} &= \frac{m_{t+1}}{b_{t+1} + m_{t+1}} \\ &= \frac{1.5m_t + 0.15b_t}{1.95b_t + 1.5m_t} \\ &= \frac{\frac{1.5m_t}{m_t + b_t} + \frac{0.15b_t}{m_t + b_t}}{\frac{1.95b_t}{m_t + b_t} + \frac{1.5m_t}{m_t + b_t}} \\ &= \frac{1.5p_t + 0.15(1 - p_t)}{1.5p_t + 1.95(1 - p_t)}. \end{aligned}$$

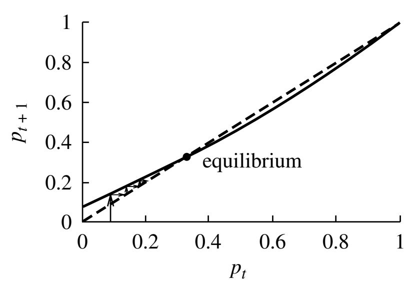
f. The equilibrium solves

$$p^* = \frac{1.5p^* + 0.15(1 - p^*)}{1.5p^* + 1.95(1 - p^*)}.$$

Following the algebra gives

$$\begin{aligned} &p^*(1.5p^* + 1.95(1 - p^*)) = 1.5p^* + 0.15(1 - p^*) \\ &\quad p^*(1.5p^* + 1.95(1 - p^*)) \\ &\quad -1.5p^* + 0.15(1 - p^*) = 0 \\ &0.15 - 0.6p^* + 0.45(p^*)^2 = 0 \\ &0.45(p^* - 1/3)(p^* - 1.0) = 0. \end{aligned}$$

Therefore, $p^* = 1/3$ or $p^* = 1.0$.



g. The equilibrium at $1/3$ seems to be stable. The fraction of mutants will end up at about 33.3%.

33. $x_1 = 100 - 20 + 30 = 110$. $y_1 = 100 - 30 + 20 = 90$. $x_2 = 115$ and $y_2 = 85$.

35. $x_{t+1} = x_t - 0.2x_t + 0.3y_t = 0.8x_t + 0.3y_t$. $y_{t+1} = y_t - 0.3y_t + 0.2x_t = 0.7y_t + 0.2x_t$.

37. a. Consider the first island. After migration, there are 80 butterflies that started on the first island and 30 that started on the second. The 80 reproduce, making a total of 190. On the second island, there are 20 from the first and 70 from the second after migration. The 20 reproduce, making a total of 110. Following the same reasoning, $x_2 = 337$ and $y_2 = 153$.

b. $x_{t+1} = 2(x_t - 0.2x_t) + 0.3y_t = 1.6x_t + 0.3y_t$. $y_{t+1} = y_t - 0.3y_t + 2 \cdot 0.2x_t = 0.7y_t + 0.4x_t$.

c. $x_{t+1} + y_{t+1} = 2x_t + y_t$. Then

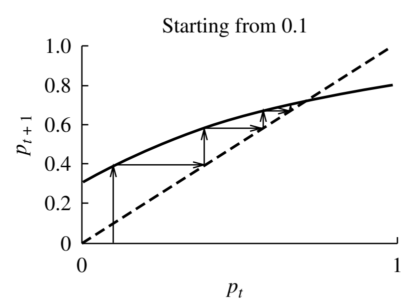
$$
\begin{aligned}
p_{t+1} = \frac{x_{t+1}}{x_{t+1} + y_{t+1}} &= \frac{1.6x_t + 0.3y_t}{x_{t+1} + y_{t+1}} \\
&= \frac{1.6x_t + 0.3y_t}{2x_t + y_t} \\
&= \frac{\frac{1.6x_t}{x_t + y_t} + \frac{0.3y_t}{x_t + y_t}}{\frac{2x_t}{x_t + y_t} + \frac{y_t}{x_t + y_t}} \\
&= \frac{1.6p_t + 0.3(1 - p_t)}{2p_t + 1 - p_t}.
\end{aligned}
$$

d. We must solve the equation

$$p^* = \frac{1.6p^* + 0.3(1 - p^*)}{2p^* + 1 - p^*} = \frac{0.3 + 1.3p^*}{1 + p^*}.$$

Multiplying out and using the quadratic formula, we find $p^* \approx 0.718$. This is larger than the result in Exercise 1.10.36 because the butterflies from the first island reproduce.

e.



39. The discrete-time dynamical system is

$$M_{t+1} = M_t - \frac{0.5}{1.0 + 0.1M_t} M_t + 1.0.$$

To find the equilibrium,

$$
\begin{aligned}
M^* &= M^* - \frac{0.5}{1.0 + 0.1M^*} M^* + 1.0 \\
\frac{0.5}{1.0 + 0.1M^*} M^* &= 1.0 \\
0.5M^* &= 1.0 + 0.1M^* \\
0.4M^* &= 1.0.
\end{aligned}
$$

The equilibrium is therefore 2.5. It is larger because the fraction absorbed is always less than 0.5.

41. The discrete-time dynamical system is

$$M_{t+1} = M_t - \frac{\beta}{1.0 + 0.1M_t} M_t + 1.0.$$

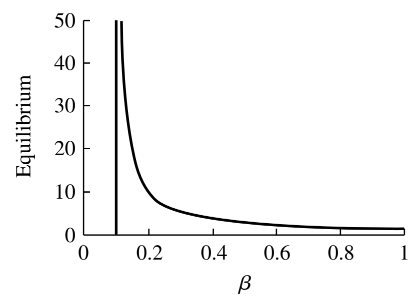
To find the equilibrium,

$$
\begin{aligned}
M^* &= M^* - \frac{\beta}{1.0 + 0.1M^*} M^* + 1.0 \\
\frac{\beta}{1.0 + 0.1M^*} M^* &= 1.0 \\
\beta M^* &= 1.0 + 0.1M^* \\
(\beta - 0.1)M^* &= 1.0.
\end{aligned}
$$

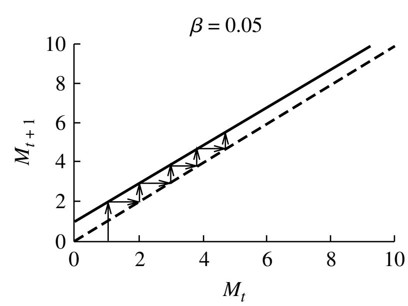
The equilibrium is therefore

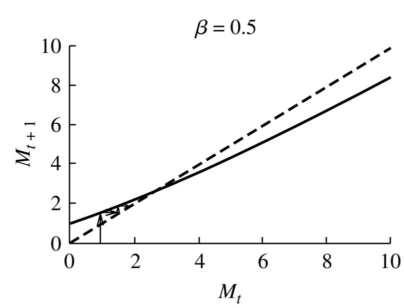
$$M^* = \frac{1.0}{1.0\beta - 0.1}.$$

This becomes smaller as β becomes larger. Large values of β indicate that the fraction absorbed is large, which makes sense. The fact that the equilibrium is negative when $\beta \le 0.1$ indicates that there is no equilibrium. The body cannot use up the dose of 1.0 each day, and the level just keeps building up.

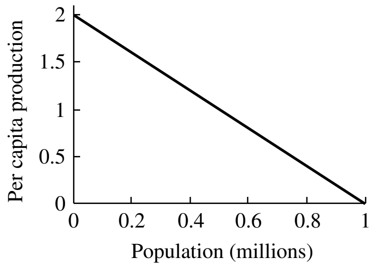


The second diagram looks a lot like the tree growth model. Once the concentration becomes large, it simply increases by 1.0 per day.



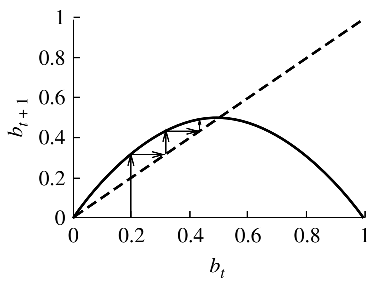


43. a.

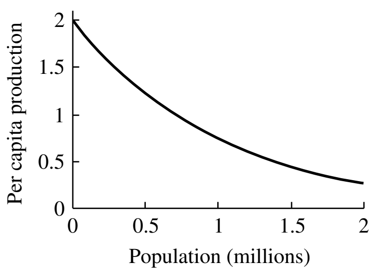


b. $b_{t+1} = 2.0 b_t \left(1 - \frac{b_t}{1.0 \times 10^6}\right)$.

c. The equilibria are $b^* = 0$ and $b^* = 5.0 \times 10^5$.

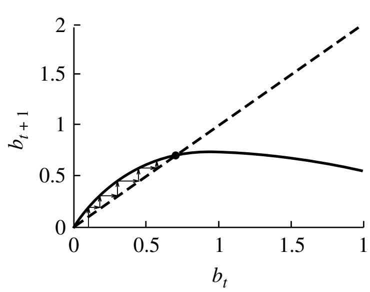


45. a.



b. $b_{t+1} = 2.0 b_t e^{\frac{-b_t}{1.0 \times 10^6}}$.

c. The equilibria are $b^* = 0$ and $b^* = \ln(2) 1.0 \times 10^6 \approx 6.93 \times 10^5$.



Section 1.11, page 127

1. $\hat{V}_t = 0.5 \cdot 30.0 = 15.0 < V_c$. The heart will beat, and $V_{t+1} = 25.0$ mV.

3. $\hat{V}_t = 0.7 \cdot 30.0 = 21.0 > V_c$. The heart will not beat, and $V_{t+1} = 21.0$ mV.

5. $V^* = u/(1-c) = 20.0$. Because $cV^* = 10.0 < V_c = 20.0$, the inequality in Equation 1.11.2 is satisfied and the equilibrium makes sense. This heart will beat every time.

7. $V^* = 33.3$. Because $cV^* = 23.1 > V_c = 20.0$, the inequality in Equation 1.11.2 is not satisfied. The equilibrium does not make sense. Is this a case of 2 : 1 AV block? We find that $\bar{V} = 19.61$ from Equation 1.11.5. However, $c\bar{V} < V_c$, so that the second inequality in Equation 1.11.4 is not satisfied. This is not a case of 2 : 1 AV block. In fact, this turns out to be an example of the Wenckebach phenomenon where the heart skips every fourth beat.

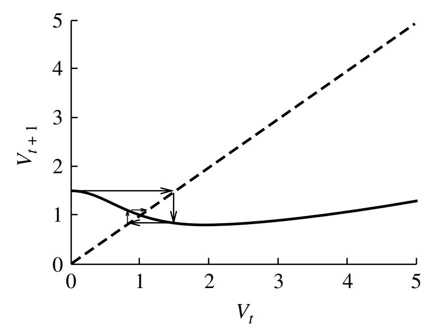
9. $c = e^{-\alpha\tau} = 0.3678$. The heart beats every time.

11. $c = e^{-\alpha\tau} = 0.3678$. The heart beats every time, twice as fast as in part **a**, but with recovery also twice as fast.

13. The dynamical system is

$$V_{t+1} = cV_t + \frac{2(1-c)}{1+V_t^2}.$$

Substituting $V_t = 1$ gives $V_{t+1} = 1$, so this is an equilibrium.



From the cobweb, it seems to be stable.

15. I got the following strange results. These are jumping all over the place.

Time	Solution 1	Solution 2	Solution 3
0	500	600.0	800.0
1	750.0	900.0	1200.0
2	1125.0	1350.0	800.0
3	687.50	1025.0	1200.0
4	1031.25	537.5	800.0
5	546.88	806.25	1200.0
6	820.31	1209.37	800.0
7	1230.47	814.062	1200.0
8	845.70	1221.10	800.0
9	1268.55	831.64	1200.0
10	902.83	1247.46	800.0

17. I got the following strange results. The first jumps around like the earlier ones, but the second seems to be shooting off to infinity.

Time	Solution 1	Solution 2	Solution 3
0	500.0	600.0	800.0
1	825.0	990.00	1320.0
2	1361.25	1633.50	1178.0
3	1246.06	1695.27	943.70
4	1056.00	1797.20	1557.10
5	742.40	1965.39	1569.22
6	1224.97	2242.89	1589.22
7	1021.20	2700.76	1622.21
8	684.98	3456.26	1676.65
9	1130.21	4702.83	1766.47
10	864.85	6759.67	1914.67

Supplementary Problems, page 128

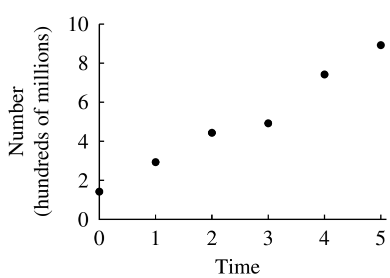
1. **a.** 3.0×10^{10} bacteria. **b.** $2.0 \times 10^{10} \le \text{number} \le 5.0 \times 10^{10}$.

3. a. $f^{-1}(y) = -\ln(y)/2$, $g^{-1}(y) = \sqrt[3]{y-1}$. $f(x) = 2$ when $x = f^{-1}(2) = -\frac{\ln(2)}{2} \approx -0.345$. Similarly, we find $g(1) = 2$.

b. $(f \circ g)(x) = e^{-2(x^3+1)}$, $(g \circ f)(x) = e^{-6x} + 1$, $(f \circ g)(2) = e^{-18} = 1.5 \times 10^{-8}$, $(g \circ f)(2) = 1 + e^{-12} = 1.00006$.

c. $(g \circ f)^{-1}(y) = -\ln(y-1)/6$, domain is $y > 1$.

5. a.



b. Line has slope of $1.5 \frac{\text{million bacteria}}{\text{hour}}$ and equation $b(t) = 1.5 + 1.5t$. At $t = 3$, the point 5.0 lies below the line.

c. Substituting $t = 3$ into the equation, we get 6.0 million.

d. Substituting $t = 7$ into the equation, we get 12.0 million.

7. a. $\pi/3$ radians. $\sin(\theta) = \sqrt{3}/2$, $\cos(\theta) = 1/2$.

b. $-\pi/3$ radians. $\sin(\theta) = -\sqrt{3}/2$, $\cos(\theta) = 1/2$.

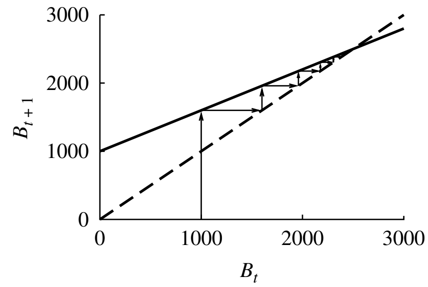
c. 1.919 radians. $\sin(\theta) \approx 0.9397$, $\cos(\theta) \approx -0.3420$.

d. -3.316 radians. $\sin(\theta) \approx 0.1736$, $\cos(\theta) \approx -0.9848$.

e. 20.25 radians. $\sin(\theta) \approx 0.9848$, $\cos(\theta) \approx 0.1736$.

9. a. Let B_t be the number of butterflies in the late summer. There are then $1.2B_t$ eggs, leading to $0.6B_t$ new butterflies from reproduction plus 1000 from immigration. The discrete-time dynamical system is $B_{t+1} = 0.6B_t + 1000$.

b.

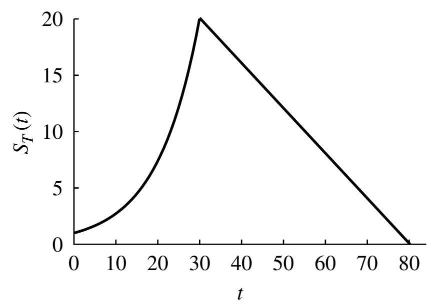


c. Equilibrium has 2500 butterflies.

11. a. The size at $t = 30$ is $S(30) = 1.0e^{0.1 \cdot 30} \approx 20.08$. The size after treatment, which we can denote $S_T(t)$, is a line through the point (30, 20.08) with slope -0.4, so

$$S_T(t) = -0.4(t - 30) + 20.08.$$

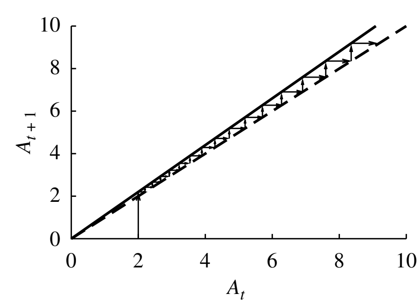
b.



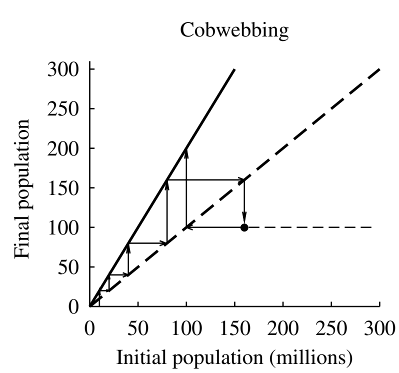
c. We solve $S_T(t) = 0$, or

$$\begin{aligned} -0.4(t-30) + 20.08 &= 0 \\ 0.4(t-30) &= 20.08 \\ t - 30 &= \frac{20.08}{0.4} \\ t &= 30 + \frac{20.08}{0.4} \approx 80.2. \end{aligned}$$

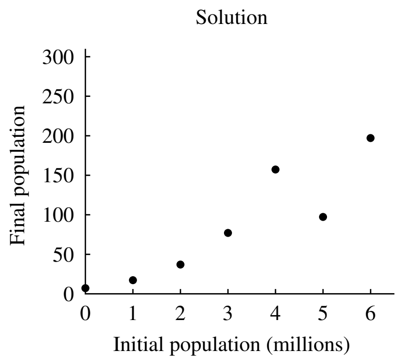
13. **a.** 2.66cm^2. **b.** The discrete-time dynamical system is $A_{t+1} = 1.1A_t$. **c.** 1.82cm^2. **d.** 0.55. **e.** When $2.0 \times 1.1^t = 10$ or in 16.9 hours.



15. a.



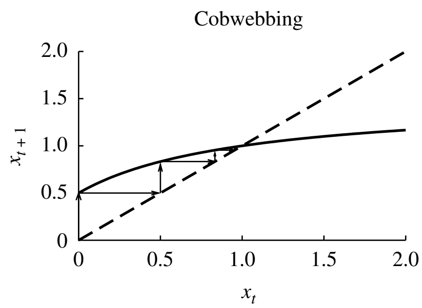
b.

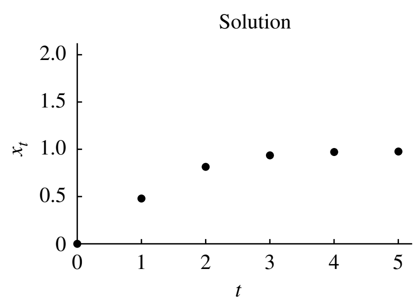


c. The only equilibrium is at 0.

17. a. $x_{t+1} = 0.5 + x_t/(1 + x_t)$.

b.





c. The equilibrium is at 1.0.

19. a. She has \$1120, and the casino has \$10,880.

b. Let g represent the money the gambler has and c the amount the casino has. Then

$$g_{t+1} = g_t - 0.1g_t + 0.02c_t = 0.9g_t + 0.02c_t$$
$$c_{t+1} = c_t + 0.1g_t - 0.02c_t = 0.1g_t + 0.98c_t.$$

c.

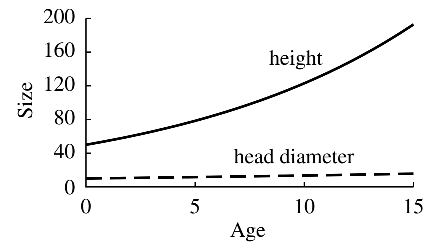
$$p_{t+1} = \frac{g_{t+1}}{g_{t+1} + c_{t+1}} = \frac{0.9g_t + 0.02c_t}{g_t + c_t}$$
$$= \frac{0.9p_t + 0.02(1 - p_t)}{p_t + (1 - p_t)} = 0.88p_t + 0.02.$$

d. The equilibrium is at $p = \frac{1}{6}$.

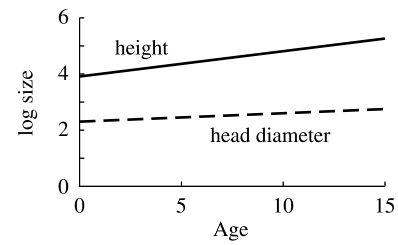
e. They start out with \$12,000, and the gambler ends up with 1/6, or \$2000.

21. a. $D(0) = 10.0, H(0) = 10.0, D(7.5) = 12.5, H(7.5) = 98.2, D(15) = 15.7, H(15) = 193.9$.

b.



c.



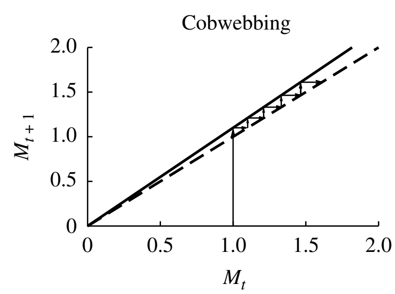
d. Doubling time is 23.1 for head diameter and 7.7 for height.

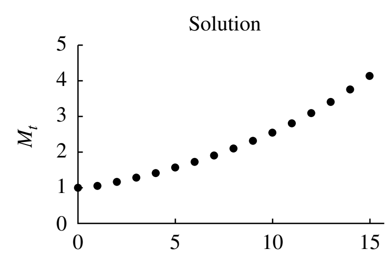
23. a. \$148,643.

b. In 16.88 yr, or in about 2012.

c. $M_{t+1} = 1.1M_t$.

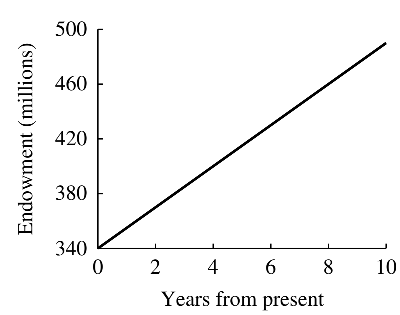
d. $M_t = 1.000001 \times 10^6 \cdot 1.1^t$ where t is measured in years before or after 1995.



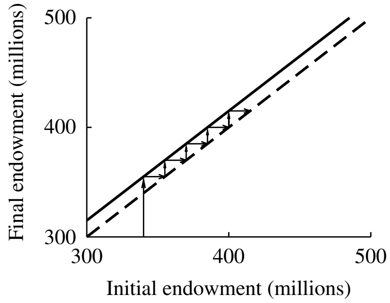


25. a. It will have \$460 million.

b.



c. $M_{t+1} = M_t + 15$.



d. I'd hire the second Texan—the money keeps piling up.

27. a. The distance a car moves in 2 seconds is 40 m.

b. There will be 25 vehicles per kilometer.

c. There will be $25 \cdot 72 = 1800$ vehicles passing a point in one hour, carrying 2700 people.

d. This oscillation has period 24 hours, amplitude of 900 (half the difference between the minimum of 900 and the maximum of 2700), a mean of 1800, and a phase of 8.0, with formula

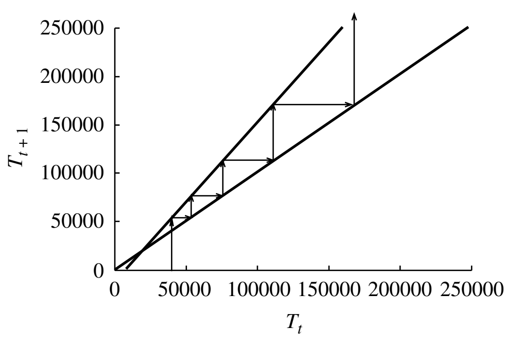
$$p(t) = 1800 + 900 \cos\left(\frac{t - 8.0}{24}\right).$$

29. a. $40{,}000 \cdot 1.6 - 10{,}000 = 54{,}000$.

b. $T_{t+1} = 1.6T_t - 10{,}000$.

c. Solving $T_{t+1} = T_t$ gives $T^* = 16{,}667$.

d.



e. This one will grow faster because traffic goes up by 60% rather than 50%. Once the numbers get really big, the 10,000 car reduction won't matter much.