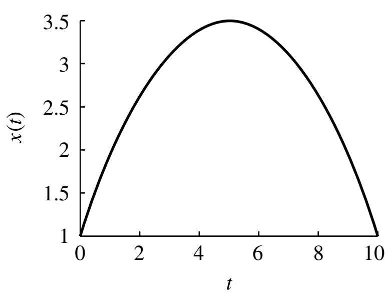
Chapter 4

Section 4.1, page 348

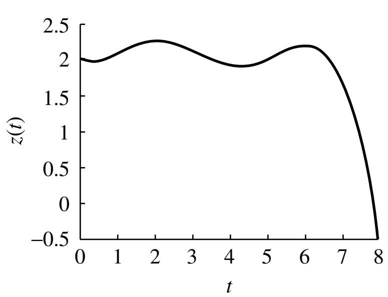
1. This is a pure-time differential equation because the rate of change depends only on t.

3. This is a pure-time differential equation because the rate of change depends only on t.

5.



7.

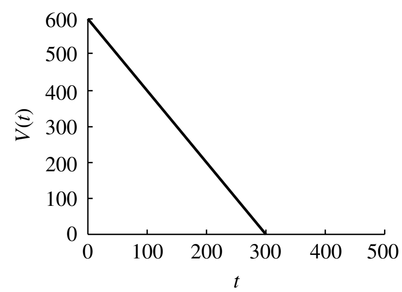


9. Taking the derivative, we find that $\frac{dx}{dt} = t$, as required by the differential equation. The initial condition must have been $x(0) = 1$.

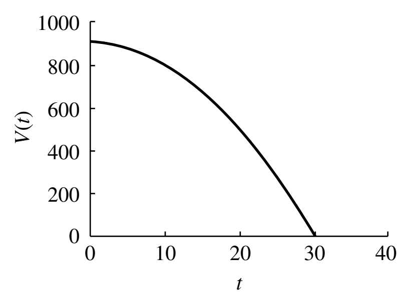
11. Taking the derivative, we find that $\frac{dy}{dt} = 8e^{2t}$. Because $y = 4e^{2t}$, we can rewrite this as $\frac{dy}{dt} = 2 \cdot 4e^{2t} = 2y$, as required by the differential equation. The initial condition must have been $y(0) = 4$.

13. With $\Delta t = 1$, $\hat{x}(1) = 1 + x'(0) \cdot 1 = 1 + 0 \cdot 1 = 1$. With $\Delta t = 0.5$, the first step is $\hat{x}(0.5) = 1 + x'(0) \cdot 0.5 = 1 + 0 \cdot 0.5 = 1$. The second step is $\hat{x}(1) = 1 + x'(0.5) \cdot 0.5 = 1 + 0.5 \cdot 0.5 = 1.25$. The exact answer, using the solution $x(t) = 1 + \frac{t^2}{2}$, is $x(1) = 1.5$.

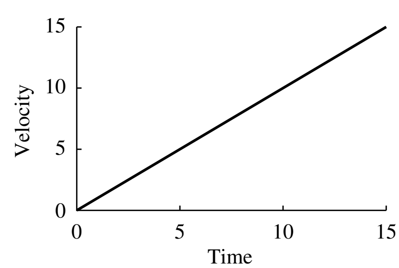
15. The differential equation is $\frac{dV}{dt} = -2$. Because it starts at 600, the solution is $V(t) = 600 - 2t$ because this function has derivative -2. The solution stops making sense after $t = 300$, when the volume becomes negative.



17. The differential equation is $\frac{dV}{dt} = -2t$. Because it starts at 900, the solution is $V(t) = 900 - t^2$ because this function has derivative $-2t$. The solution stops making sense after $t = 30$ when the volume becomes negative.



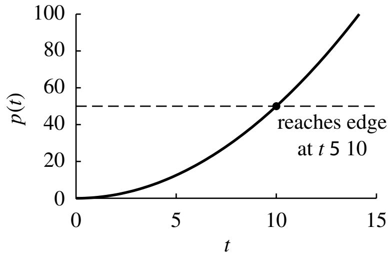
19. a.



b. $\frac{dp}{dt} = t$.

c. The function $p(t) = t^2$ has derivative $2t$, so we divide by 2 to guess $p(t) = t^2/2$.

d.



e. It reaches the other side when $p(t) = 50$, or when $t^2/2 = 50$. This has a solution of 10 min.

21. This cell follows the differential equation $\frac{dV}{dt} = -2$ with initial condition $V(0) = 600$. Then

$$\hat{V}(10) = V(0) + V'(0)\Delta t = 600 - 2 \cdot 10 = 580$$
$$\hat{V}(20) = \hat{V}(10) + V'(10)\Delta t = 580 - 2 \cdot 10 = 560$$
$$\hat{V}(30) = \hat{V}(20) + V'(20)\Delta t = 560 - 2 \cdot 10 = 540$$
$$\hat{V}(40) = \hat{V}(30) + V'(30)\Delta t = 540 - 2 \cdot 10 = 520$$

The solution we guessed was $V(t) = 600 - 2t$, and $V(40) = 520$. In this case, Euler's method gives exactly the right answer.

23. This cell follows the differential equation $\frac{dV}{dt} = -2t$ with initial condition $V(0) = 900$. Then

$$\hat{V}(10) = V(0) + V'(0)\Delta t = 900 - 2 \cdot 0 \cdot 10 = 900$$
$$\hat{V}(20) = \hat{V}(10) + V'(10)\Delta t = 900 - 2 \cdot 10 \cdot 10 = 700$$
$$\hat{V}(30) = \hat{V}(20) + V'(20)\Delta t = 700 - 2 \cdot 20 \cdot 10 = 300$$

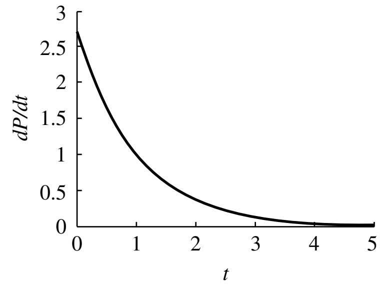
The solution we guessed was $V(t) = 900 - t^2$, and $V(30) = 0$. Euler's method is way off.

25. This snail follows the differential equation $\frac{dp}{dt} = t$ with initial condition $p(0) = 0$. Then

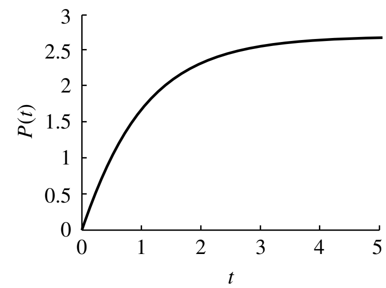
$$\hat{p}(2) = p(0) + p'(0)\Delta t = 0 + 0 \cdot 2 = 0$$
$$\hat{p}(4) = \hat{p}(2) + p'(2)\Delta t = 0 + 2 \cdot 2 = 4$$
$$\hat{p}(6) = \hat{p}(4) + p'(4)\Delta t = 4 + 4 \cdot 2 = 12$$
$$\hat{p}(8) = \hat{p}(6) + p'(6)\Delta t = 12 + 6 \cdot 2 = 24$$
$$\hat{p}(10) = \hat{p}(8) + p'(8)\Delta t = 24 + 8 \cdot 2 = 40$$

The solution we guessed was $p(t) = \frac{t^2}{2}$, and $p(10) = 50$. Euler's method is in the right ball park.

27. The derivative of e^{-t+1} is $-e^{-t+1}$. We need to multiply by -1, so the derivative of $-e^{-t+1}$ is e^{-t+1}. If $P(t) = -e^{-t+1}$, then $P(0) = -e$, which is e too low. We need to add e. $P(t) = e - e^{-t+1}$. The derivative is $\frac{dP}{dt} = e^{-t+1}$ and $P(0) = e - e = 0$. This checks.



a



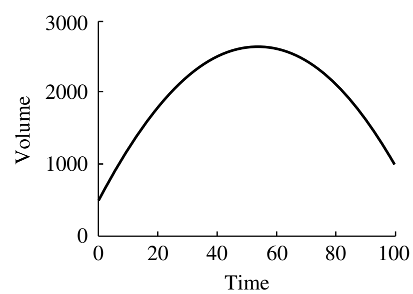
b

29. a. You could measure position with a map even if your speedometer was broken. b. You could measure speed but not position if your speedometer worked but you were lost.

31. You could measure total sodium with a destructive device that separated out sodium. You could measure sodium entering a cell, but not total sodium, if you could track the change in charge.

Section 4.2, page 358

1.



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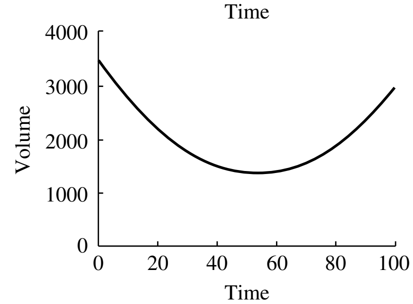

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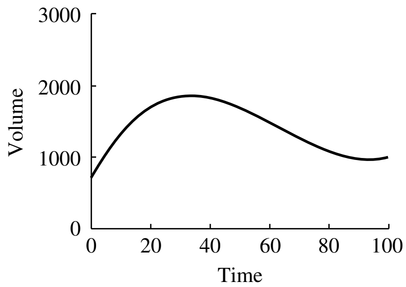


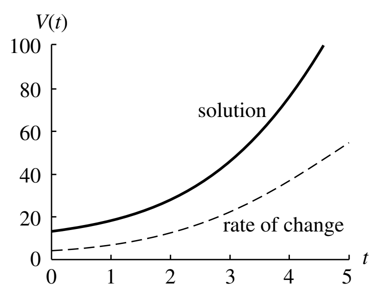
7. $\int 7x^2 dx = \frac{7x^3}{3} + c.$

9. $\int 72t + 5dt = 36t^2 + 5t + c.$

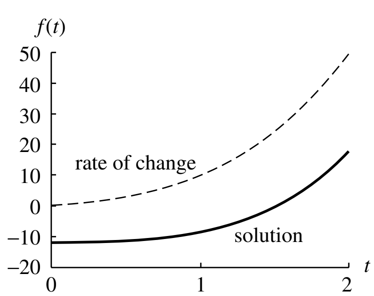
11. $\int \frac{5}{x^3} dx = \frac{-5x^{-2}}{2} + c.$

13. $\int \frac{2}{\sqrt[3]{t}} + 3dt = 3t^{2/3} + 3t + c.$

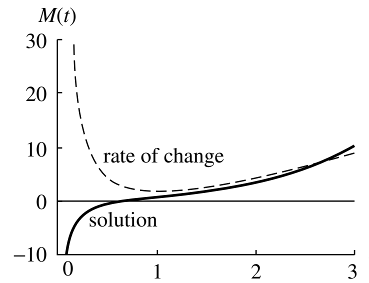
15. Integrating, we find that $V(t) = 2t^3/3 + 5t + c$. Substituting the initial condition, we get $V(1) = 2/3 + 5.0 + c = 19.0$ so $c = 40/3$. The solution is $V(t) = 2t^3/3 + 5t + 40/3$.



17. $f(t) = 1.25t^4 + 2.5t^2 + c$. Substituting the initial condition, we get $f(0) = c = -12.0$ so $c = -12$. The solution is $f(t) = 1.25t^4 + 2.5t^2 - 12.0$.



19. $M(t) = \frac{t^3}{3} - \frac{1}{t} + c$. Substituting the initial condition, we get $M(3) = \frac{27}{3} - \frac{1}{3} + c = 10.0$, so $c = \frac{4}{3}$.



21. Let $F(x) = \int f(x)dx = \frac{x^3}{3} + c$, and let $G(x) = \int g(x)dx = \frac{x^4}{4} + c$. The product of the functions is $f(x)g(x) = x^5$, with integral $\int x^5 dx = \frac{x^6}{6} + c \neq F(x)G(x)$.

23. a. $V_1(t) = t^2 + 5.0t + 10.0$, $V_2(t) = 2.5t^2 + 2.0t + 10.0$.

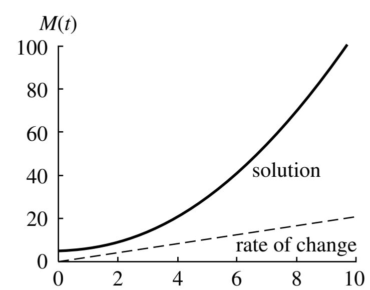
b. $\frac{dV}{dt} = 7.0t + 7.0$ with initial condition $V(0) = V_1(0) + V_2(0) = 20$.

c. $V(t) = 3.5t^2 + 7.0t + 20.0$. This is indeed the sum of $V_1(t)$ and $V_2(t)$.

25. a. The units of α must be $\frac{\text{grams}}{\text{day}^2}$.

b. $M(t) = t^2 + c$. Substituting the initial conditions, we have $M(0) = c = 5.0$ so $M(t) = t^2 + 5.0$.

c.



d. The mass increases more and more quickly.

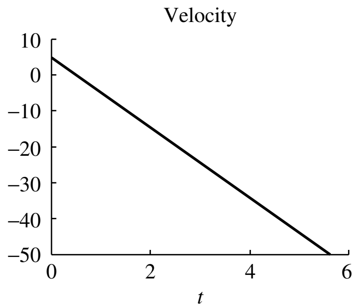
27. a. $v(t) = -9.8t + 5.0$, $p(t) = -4.9t^2 + 5.0t + 100$.

b. The maximum height is when $v(t) = 0$, or at $t \approx 0.51$. The position is 101.27 m.

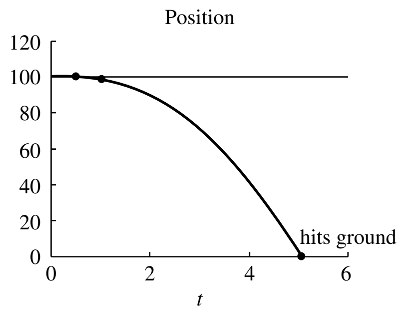
c. It passes the robot when $p(t) = 100$, or at $t = 0$ and $t \approx 1.02$. The velocity is -5.0 m/s.

d. It will take 5.06 s to hit the ground and will be moving at -44.55 m/s (which is about 99.65 mph in the downward direction).

e.



a



b

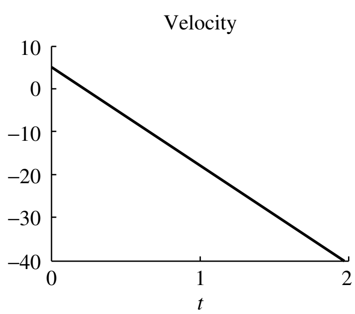
29. a. $v(t) = -22.88t + 5.0$, $p(t) = -11.44t^2 + 5.0t + 100$.

b. The maximum height is when $v(t) = 0$, or at $t \approx 0.22$. The position is 100.55 m.

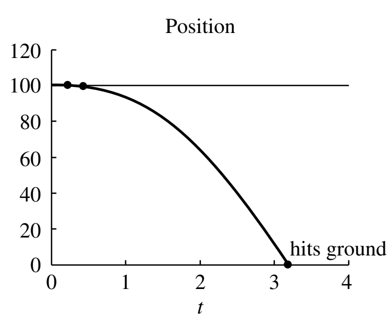
c. It passes the robot when $p(t) = 100$, or at $t = 0$ and $t \approx 0.44$. The velocity is -5.0 m/s.

d. It will take 3.18 s to hit the ground and will be moving at -67.83 m/s (which is about 151.7 mph in the downward direction).

e.

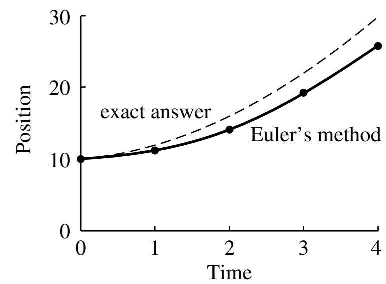


a

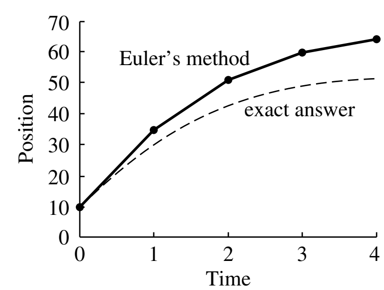


b

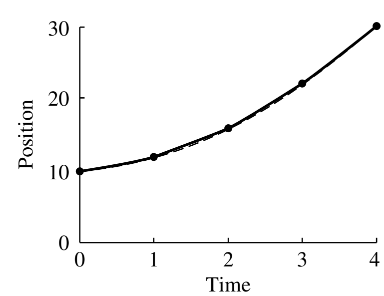
31. With Euler's method, $\hat{p}(1) = 11.0$, $\hat{p}(2) = 14.0$, $\hat{p}(3) = 19.0$, $\hat{p}(4) = 26.0$. The velocities fall on the line $v(t) = 1.0 + 2.0t$. Therefore, position satisfies the differential equation $\frac{dp}{dt} = 1.0 + 2.0t$. This has solution $p(t) = 1.0t + t^2 + 10.0$ when $p(0) = 10$. At $t = 4$, the exact solution is $p(t) = 30.0$.



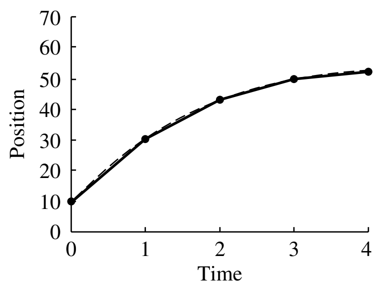
33. With Euler's method, $\hat{p}(1) = 35.0$, $\hat{p}(2) = 51.0$, $\hat{p}(3) = 60.0$, $\hat{p}(4) = 64.0$. The velocities fall on the quadratic $v(t) = (t - 5.0)^2 = t^2 - 10.0t + 25.0$. The position satisfies the differential equation $\frac{dp}{dt} = t^2 - 10t + 25$. This has solution $p(t) = \frac{t^3}{3} - 5.0t^2 + 25.0t + 10.0$ when $p(0) = 10$. At $t = 4$, the exact solution is $p(t) \approx 51.333$.



35. We estimate the velocity to be 2.0 during the first minute, 4.0 during the second, 6.0 during the third, and 8.0 during the fourth. Then $\hat{p}(1) = 12.0$, $\hat{p}(2) = 16.0$, $\hat{p}(3) = 22.0$, $\hat{p}(4) = 30.0$. At $t = 4$, this matches the exact solution of $p(t) = 30.0$.



37. We estimate the velocity to be 20.5 during the first minute, 12.5 during the second, 6.5 during the third, and 2.5 during the fourth. Then $\hat{p}(1) = 30.5$, $\hat{p}(2) = 43.0$, $\hat{p}(3) = 49.5$, $\hat{p}(4) = 52.0$. At $t = 4$, the exact solution is $p(t) \approx 51.333$, so this is much closer.



Section 4.3, page 369

1. $\frac{-3}{z} + \frac{z^3}{9} + c$.

3. $e^x + \ln(|x|) + c$.

5. $-2\cos(x) + 3\sin(x) + c$.

7. 1. Define a new variable $y = \frac{x}{5}$.

2. $\frac{dy}{dx} = \frac{1}{5}$.

3. $dy = \frac{dx}{5}$.

4. To write dx in terms of dy, solve to find $dx = 5dy$ and write

$$\int 3e^{\frac{x}{5}}\,dx = \int 15e^y\,dy$$

5. Integrate, finding

$$\int 15e^y\,dy = 15e^y + c$$

6. Put everything back in terms of x,

$$\int 3e^{\frac{x}{5}}\,dx = 15e^{\frac{x}{5}} + c$$

The derivative of this result is $3e^{\frac{x}{5}}$.

9. Use the substitution $s = 1 + \frac{t}{2}$, finding

$$\int \left(1 + \frac{t}{2}\right)^4 dt = 2\frac{\left(1 + \frac{t}{2}\right)^5}{5} + c$$

The derivative of this result is $\left(1 + \frac{t}{2}\right)^4$.

11. Use the substitution $y = 4 + t$, finding

$$\int \frac{1}{4+t}dt = \ln(|4+t|) + c.$$

The derivative of this result is $\frac{1}{4+t}$.

13. Substitute $y = 1 + e^x$, finding $dy = e^x\,dx$ so

$$\int \frac{e^x}{1+e^x}\,dx = \int \frac{1}{1+y}\,dy$$
$$= \ln(1+y) + c = \ln(1+e^x) + c$$

15. Using the substitution $z = 1 + y^2$, we find that the integral is $\frac{2}{3}(1+y^2)^{3/2} + c$.

17. Using the substitution $y = \cos(\theta)$, we find that the integral is $-\ln(|\cos(x)|)$.

19. Using the substitution $y = \ln(x)$, we find that the integral is $\ln(|\ln(x)|) + c$.

21. Let $u(x) = x$ and $\frac{dv}{dx} = e^{2x}$; then $\frac{du}{dx} = 1$ and $v(x) = \frac{e^{2x}}{2}$ (using the substitution $y = 2x$ to eliminate the constant 2 in the power). Then,

$$\int xe^{2x}dx = \frac{xe^{2x}}{2} - \int \frac{e^{2x}}{2}dx = \frac{xe^{2x}}{2} - \frac{e^{2x}}{4} + c$$

where we again used the substitution $y = 2x$ in the last step. Checking, we get

$$\frac{d\left(\frac{xe^{2x}}{2} - \frac{e^{2x}}{4} + c\right)}{dx} = \frac{e^{2x}}{2} + xe^{2x} - \frac{e^{2x}}{2} = xe^{2x}$$

23. Pick $u(x) = x^2$ and $\frac{dv}{dx} = e^x$. Then $v(x) = e^x$ and $\frac{du}{dx} = 2x$, giving

$$\int x^2e^x dx = x^2e^x - \int 2xe^x dx = x^2e^x - 2xe^x + 2e^x + c$$

where we used the result from Example 4.3.9. Checking yields

$$\frac{d(x^2e^x - 2xe^x + 2e^x + c)}{dx} = x^2e^x + 2xe^x - 2xe^x - 2e^x + 2e^x$$
$$= x^2e^x.$$

25. Exercise 2.10.25 gives $\frac{d\tan^{-1}(x)}{dx} = \frac{1}{1+x^2}$. If we break up the integral by setting $u(x) = \tan^{-1}(x)$ and $\frac{dv}{dx} = 1$, we find

$$\int \tan^{-1}(x)dx = x\tan^{-1}(x) - \int \frac{x}{1+x^2}dx$$
$$= x\tan^{-1}(x) - \frac{1}{2}\ln(1+x^2) + c$$

where we used the substitution $y = 1 + x^2$ in the last step.

27. Set $u(x) = x$ and $\frac{dv}{dx} = e^x$. Then $v(x) = \int e^x dx = e^x + c$ and $\frac{du}{dx} = 1$, giving

$$\int xe^x = x(e^x + c) - \int (e^x + c)dx$$
$$= xe^x + cx - e^x - cx + c_1 = xe^x - e^x + c_1$$

where c_1 is a new arbitrary constant. The original arbitrary constant c canceled, giving the previous answer.

29. Set $u(x) = \sin(x)$ and $\frac{dv}{dx} = e^x$. Then $v(x) = e^x$ and $\frac{du}{dx} = \cos(x)$, giving

$$\int e^x\sin(x)dx = e^x\sin(x) - \int e^x\cos(x)dx$$

To evaluate this new integral, set $u(x) = \cos(x)$ and $\frac{dv}{dx} = e^x$. Then $v(x) = e^x$ and $\frac{du}{dx} = -\sin(x)$, giving

$$\int e^x \cos(x)dx = e^x \cos(x) + \int e^x \sin(x)dx$$

Substituting into the original equation gives

$$\int e^x \sin(x)dx = e^x \sin(x) - \int e^x \cos(x)dx$$
$$= e^x \sin(x) - e^x \cos(x) - \int e^x \sin(x)dx$$

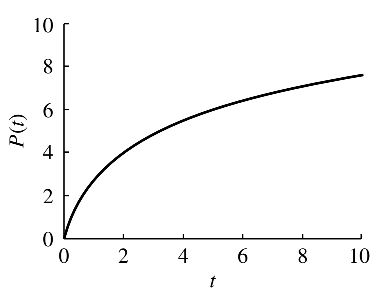
We can now solve for the original integral, getting

$$2\int e^x \sin(x)dx = e^x \sin(x) - e^x \cos(x) + c$$

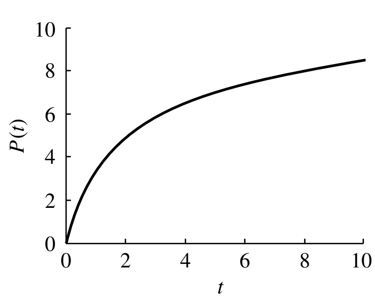
Dividing by 2 gives the result

$$\int e^x \sin(x)dx = \frac{e^x \sin(x) - e^x \cos(x)}{2} + c$$

31. Integrating, we get $P(t) = 2.5\ln(1 + 2.0t) + c$. Substituting the initial condition, we find $c = 0$. The amount of product increases to infinity, but it does so rather slowly. In fact, $P(10) = 7.61$ and $P(100) = 13.26$. This increases to infinity because the rate of chemical production decreases to 0 rather slowly.



33. Integrating yields $P(t) = 2.5\ln(1 + t) - 2.5e^{-t} + c$. Substituting the initial condition, we find $c = 2.5$. The limit is infinity as t approaches infinity. In fact, $P(10) \approx 8.49$ and $P(100) \approx 14.04$. The leading behavior of the rate of chemical production decreases to 0 rather slowly, like $\frac{1}{1+t}$.



35. Let $g(t) = (t + t^2)e^{-2t}$. To find the time when the toad grows fastest, we compute

$$g'(t) = (1 - 2t^2)e^{-2t}$$

This is zero when $1 - 2t^2 = 0$, or $t = \frac{\sqrt{2}}{2} \approx 0.71$. At this point, $g(0.71) \approx 0.293$. To find the size at $t = 1$, let $u(t) = t + t^2$ and $\frac{dv}{dt} = e^{-2t}$. Then $\frac{du}{dt} = (1 + 2t)$ and $v(t) = -\frac{e^{-2t}}{2}$. Therefore,

$$\int_0^1 (t + t^2)e^{-2t} = -(t + t^2)\frac{e^{-2t}}{2}\Big|_0^1 + \int_0^1 (1 + 2t)\frac{e^{-2t}}{2}$$

To integrate the second piece, let $u(t) = 1 + 2t$ and $\frac{dv}{dt} = \frac{e^{-2t}}{2}$. Then $\frac{du}{dt} = 2$ and $v(t) = -\frac{e^{-2t}}{4}$. Therefore,

$$\int_0^1 (1 + 2t)e^{-2t} = -(1 + 2t)\frac{e^{-2t}}{4}\Big|_0^1 + \int_0^1 \frac{e^{-2t}}{2}$$
$$= -(1 + 2t)\frac{e^{-2t}}{4}\Big|_0^1 - \frac{e^{-2t}}{4}\Big|_0^1$$

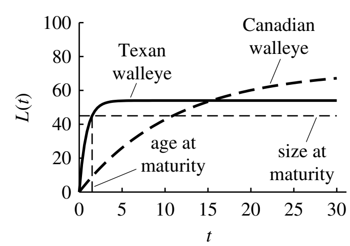
Evaluating each term gives $M(1) = 2e^{-2} + \frac{1}{2} \approx 0.23$. This is about 20% smaller than the toad would be if it grew at its maximum rate the entire time.

37.
a. $L(t) = 54.0(1 - e^{-1.19t})$.

b. The limit is 54.0 cm.

c. These walleye reach maturity when $L(t) = 45$, or at $t = 1.5$ yr.

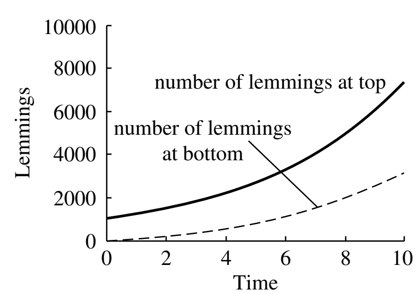
d. These walleye reach maturity much more quickly, but then they more or less stop growing.



39.
a. $\frac{dB}{dt} = 100e^{0.2t}$.

b. Integrating, we get $B(t) = 500e^{0.2t} + c$. The initial condition implies that $c = -500$, so $B(t) = 500e^{0.2t} - 500$.

c.



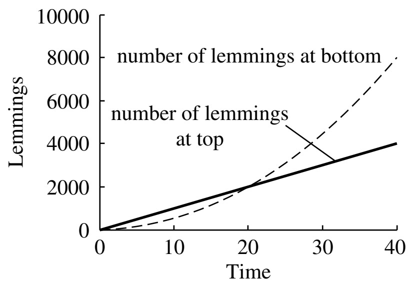
d. The limit is 0.5. There are about half as many lemmings at the bottom as at the top.

41. a.

$$\frac{dB}{dt} = 10t.$$

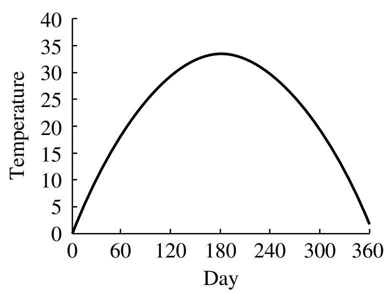
b. Integrating, we get $B(t) = 5t^2 + c$. The initial condition implies that $c = 0$, so $B(t) = 5t^2$.

c.



d. The limit of the ratio is infinite. This is probably because the lemmings at the top are reproducing quite slowly.

43. a.



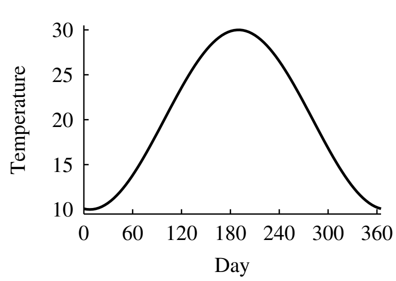
b. The solution is

$$\begin{aligned} L(t) &= \int 0.000001t(365 - t)dt \\ &= \int 0.000365t - 0.000001t^2 dt \\ &= 0.0001825t^2 - 0.00000033t^3 + c \end{aligned}$$

Using the initial condition $T(0) = 0.1$, we find $c = 0.1$. The size at $t = 30$ is then 0.25525 cm.

c. Using the initial condition $T(150) = 0.1$, we find that the constant is $c = -2.88125$, and $L(180) = 10.8775$. This bug is a lot bigger because the temperature is warmer.

45. a.



b. The solution is

$$\begin{aligned} L(t) &= \int 0.02 + 0.01\cos\left(\frac{2\pi(t - 190.0)}{365}\right) dt \\ &= 0.02t + 0.01\sin\left(\frac{2\pi(t - 190.0)}{365}\right)\frac{365}{2\pi} + c \end{aligned}$$

Using the initial condition $T(0) = 0.1$, we find $c = 0.0252$. At $t = 30$, $L(30.0) = 0.4058$.

c. Using the initial condition $T(150) = 0.1$, we find $c = -2.531$. After 30 days, at $t = 180$, $L(180.0) = 0.969$. This bug is bigger.

Section 4.4, page 378

1. $1 + 1/2 + 1/3 + 1/4 + 1/5 = 2.383$.

3. $1 + 1/4 + 1/9 + 1/16 + 1/25 = 1.464$.

5. $\Delta t = 0.4$, $t_0 = 0$, $t_1 = 0.4$, $t_2 = 0.8$, $t_3 = 1.2$, $t_4 = 1.6$, $t_5 = 2.0$.

7. $\Delta t = 0.2$, $t_0 = 2.0$, $t_1 = 2.2$, $t_2 = 2.4$, $t_3 = 2.6$, $t_4 = 2.8$, $t_5 = 3.0$.

9.

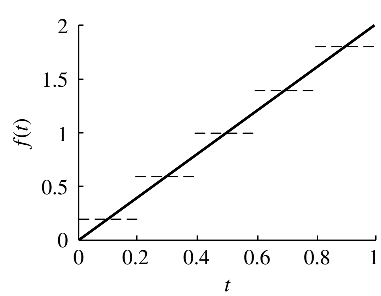
	Left-hand estimate			Right-hand estimate		
Time	Rate	Influx	Net influx	Rate	Influx	Net influx
0.2	0.0	0.00	0.00	0.4	0.08	0.08
0.4	0.4	0.08	0.08	0.8	0.16	0.24
0.6	0.8	0.16	0.24	1.2	0.24	0.48
0.8	1.2	0.24	0.48	1.6	0.32	0.80
1.0	1.6	0.32	0.80	2.0	0.40	1.20

11. Left-hand estimate is 1.92, right-hand estimate is 3.52.

13. $I_l = \sum_{i=0}^{4} 2t_i \cdot 0.2$, $I_r = \sum_{i=1}^{5} 2t_i \cdot 0.2$.

15. $I_l = \sum_{i=0}^{4} t_i^2 \cdot 0.4$, $I_r = \sum_{i=1}^{5} t_i^2 \cdot 0.4$.

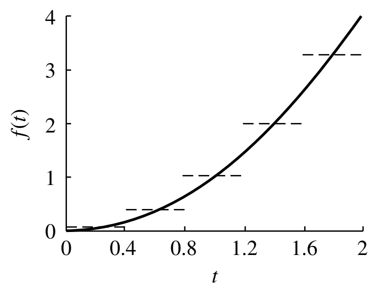
17. a.



b. $I_a = \sum_{i=0}^{4}(t_i + t_{i+1}) \cdot 0.2$.

c. The sum is 1.0.

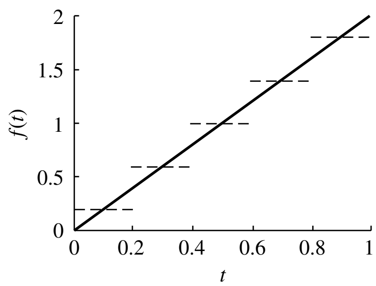
19. a.



b. $I_a = \sum_{i=0}^{4} \frac{t_i^2 + t_{i+1}^2}{2} \cdot 0.4.$

c. The sum is 2.72.

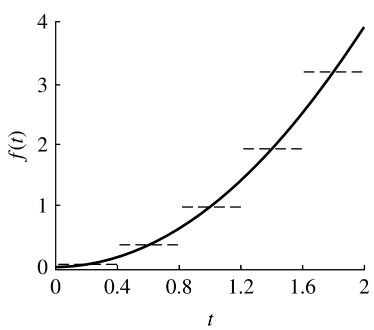
21. a.



b. $I_a = \sum_{i=0}^{4} (t_i + t_{i+1}) \cdot 0.2.$

c. The sum is 1.0.

23. a.



b. $I_a = \sum_{i=0}^{4} \left(\frac{t_i + t_{i+1}}{2}\right)^2 \cdot 0.4.$

c. The sum is 2.64.

25. Let B_i represent the number of offspring in year i; then the total number of offspring T is $T = \sum_{i=1}^{5} B_i = 2+3+5+4+1 = 15$.

27. The total number of offspring T is $T = \sum_{i=0}^{6} B_i = \sum_{i=0}^{6} i(6-i) = 0+5+8+9+8+5+0 = 35$.

29. $\hat{V}(0.2) = V(0) + 2\cdot 0.0 \cdot 0.2 = 0.0$, $\hat{V}(0.4) = \hat{V}(0.2) + 2\cdot 0.2 \cdot 0.2 = 0.08$, $\hat{V}(0.6) = \hat{V}(0.4) + 2\cdot 0.4\cdot 0.2 = 0.24$, $\hat{V}(0.8) = \hat{V}(0.6) + 2\cdot 0.6 \cdot 0.2 = 0.48$, $\hat{V}(1.0) = \hat{V}(0.8) + 2\cdot 0.8\cdot 0.2 = 0.8$. Euler's method gives exactly the same answer as the left-hand estimate, because both methods make the same approximation: assume that the rate during an interval (or between two steps) is equal to the rate at the beginning of the interval.

31. $\hat{V}(0.4) = V(0) + 0.0^2 \cdot 0.4 = 0.0$, $\hat{V}(0.8) = \hat{V}(0.4) + 0.4^2 \cdot 0.4 = 0.064$, $\hat{V}(1.2) = \hat{V}(0.8) + 0.8^2\cdot 0.4 = 0.32$, $\hat{V}(1.6) = \hat{V}(1.2) + 1.2^2 \cdot 0.4 = 0.896$, $\hat{V}(2.0) = \hat{V}(1.6) + 1.6^2 \cdot 0.4 = 1.92$. Euler's method gives the same answer as the left-hand estimate.

33. $I_l = (127.0 + 118.0 + 113.0 + 112.0 + 116.0)\cdot 2.0 = 1172$
$I_r = (118.0 + 113.0 + 112.0 + 116.0 + 125.0)\cdot 2.0 = 1168$

35. The method of Exercises 4.4.17–4.4.20 applied to all the measurements gives 1167 as the average of the left-hand and right-hand measurements.

37. The modified left-hand estimate might fill in the last measured value for each of the NA's, giving $12\cdot 2 + 16 + 17\cdot 4 + 13 = 121$. The total number counted in the four years is 58, so we would estimate 116 with this method. This gives a lower answer because there were three years with NA's following the highest measurement of 17 in 1993.

39. We fill in the NA's using the next measurement, finding $34\cdot 2 + 40\cdot 3 + 31 + 37\cdot 2 = 293$. The total number counted in the four years is 142, so we estimate 284 with this method.

Section 4.5, page 390

1. $\int_0^1 2t\,dt = t^2|_0^1 = 1.0$. This is close to the answer to Exercise 4.4.9 and matches the answer to Exercise 4.4.17.

3. $\int_0^2 t^2 dt = \left.\frac{t^3}{3}\right|_0^2 = 2.667$. This is pretty close to the answer of 2.72 in Exercise 4.4.19.

5. $\left.\frac{7x^3}{3}\right|_0^1 = \frac{7}{3}$.

7. $36t^2 + 5t|_{-1}^2 = 123$.

9. $\left.\frac{-5x^{-2}}{2}\right|_1^2 = \frac{15}{8}$.

11. $\int_1^8 \left(\frac{2}{\sqrt[3]{t}} + 3\right) dt = 3t^{2/3} + 3t|_1^8 = 30$.

13. $\left.\frac{-3}{z} + \frac{z^3}{9}\right|_2^3 \approx 2.61$.

15. $e^x + \ln(|x|)|_1^4 \approx 53.27$.

17. $-2\cos(x) + 3\sin(x)|_0^{\pi} = 4$.

19. After using the substitution $y = \frac{x}{5}$, we find $\int_0^5 3e^{\frac{x}{5}}\,dx = 15e^{\frac{x}{5}}|_0^5 = 15e - 15 \approx 25.77$.

21. Use the substitution $s = \frac{t}{2}$, finding

$$\int_0^4 \left(1 + \frac{t}{2}\right)^4 dt = 2\left.\frac{\left(1+\frac{t}{2}\right)^5}{5}\right|_0^4 = 96.8$$

23. Use the substitution $y = 4 + t$, and find $\int_{-3}^{0} \frac{1}{4+t}\,dt = \ln(|4 + t|)|_{-3}^{0} = \ln(4)$.

25. $\int_1^2 g(t)\,dt = \frac{7}{3}$, $\int_2^3 g(t)\,dt = \frac{19}{3}$, and $\int_1^3 g(t)\,dt = \frac{26}{3}$, which is equal to $\frac{7}{3} + \frac{19}{3}$.

27. $\int_1^2 L(t)dt = 1.875$, $\int_2^3 L(t)dt \approx 0.347$, $\int_1^3 L(t)dt \approx 2.222 = 1.875 + 0.347$.

29. $\int_1^2 F(t)dt \approx 5.36$, $\int_2^3 F(t)dt \approx 13.10$, $\int_1^3 F(t)dt \approx 18.46 = 5.36 + 13.10$.

31. $\int_0^x s^2\,ds = \frac{s^3}{3}\Big|_0^x = \frac{x^3}{3}$. But $\frac{d}{dx}\frac{x^3}{3} = x^2 = f(x)$. It worked.

33. Use the substitution $s = \frac{t}{2}$ to find

$$\int_{-1}^{x}\left(1+\frac{t}{2}\right)^4 dt = 2\frac{\left(1+\frac{t}{2}\right)^5}{5}\Bigg|_{-1}^{x} = \frac{2}{5}\left(1+\frac{t}{2}\right)^5 - \frac{1}{80}$$

But $\frac{d}{dx}\left(\frac{2}{5}\left(1+\frac{t}{2}\right)^5 - \frac{1}{80}\right) = \left(1+\frac{x}{2}\right)^4$ if we use the chain rule.

35. The solution is $p(t) = -4.9t^2 - 5.0t + 200$. Change of position $p(5) - p(1) = 52.5 - 190.1 = -137.6$. The definite integral is

$$\begin{aligned}\int_1^5 -9.8t - 5.0\,dt &= -4.9t^2 - 5.0t\big|_1^5\\ &= -4.9\cdot 5^2 - 5.0\cdot 5 - -4.9\cdot 1^2 - 5.0\cdot 1\\ &= -137.6\end{aligned}$$

37. Solution is $L(t) = 59.0 - 54.0e^{-1.19t}$. The amount grown is $L(1.5) - L(0.5) = 27.72$, which is equal to $\int_{0.5}^{1.5} 64.3e^{-1.19t}\,dt$.

39. Solution is $P(t) = 2.5\ln(1+2.0t) + 2.0$. The amount produced is $P(10) - P(5) \approx 1.616$, which matches $\int_5^{10} \frac{5}{1+2.0t}\,dt$.

41. $p(3) - p(1) = -49.2$, $p(5) - p(3) = -88.4$. These add to -137.6.

43. $L(1.0) - L(0.5) = 13.36$, $L(1.5) - L(1.0) \approx 7.37$. These add to approximately 20.72.

45. $P(7.5) - P(5.0) \approx 0.937$, $P(10.0) - P(7.5) \approx 0.680$. These add to approximately 1.616.

47. a. The velocity follows $\frac{dv}{dt} = 12.0$, which has solution $v(t) = 12.0t$ if $v(0) = 0$. Then $\frac{dp}{dt} = 12.0t$ is a differential equation for position. This has solution $p(t) = 6.0t^2$ using the initial condition $p(0) = 0$.

b. $v(10) = 120$ and $p(10) = 600$.

c. The velocity follows $\frac{dv}{dt} = -9.8$, which has solution $v(t) = -9.8t + 218$ if $v(10) = 120$. Then $\frac{dp}{dt} = -9.8t + 218$ is a differential equation for position. This has solution $p(t) = -4.9t^2 + 218t - 1090$ using the initial condition $p(10) = 600$.

d. The maximum height is reached when $v = 0$ or when $-9.8t + 218 = 0$. This happens at $t = 22.24$. The height is then 1334 m. It rises more after running out of fuel, because the acceleration of gravity is weaker than the acceleration of the engine.

e. It hits the ground when $p(t) = 0$, or at $t = 38.75$. The velocity is -161.75 m/s.

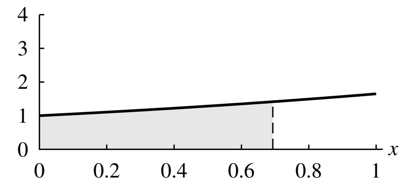
Section 4.6, page 398

1. $\int_0^3 3x^3\,dx = \frac{3x^4}{4}\Big|_0^3 = 60.75$.

3. Use the substitution $y = x/2$. Then $dx = 2dy$ and the limits of integration are $y = 0$ to $y = \frac{\ln(2)}{2}$.

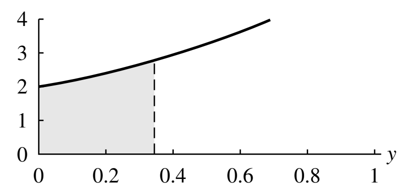
$$\int_0^{\ln(2)} e^{x/2}dx = \int_0^{\frac{\ln(2)}{2}} 2e^y dy = 2\left(\sqrt{2}-1\right) \approx 0.828$$

Area before substitution



a

Area after substitution

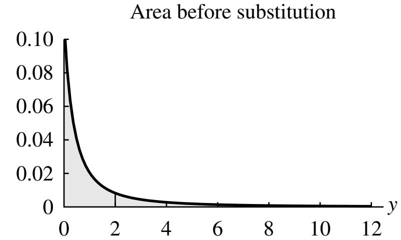


b

5. Set $z = (3+4y)$. Then $dy = dz/4$, and the limits of integration are from $z = 3$ to $z = 11$:

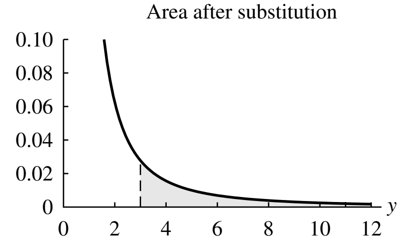
$$\int_0^2 (3+4y)^{-2}dy = \int_3^{11}\frac{z^{-2}}{4}\,dz = \frac{-z^{-1}}{4}\Bigg|_3^{11} \approx 0.061.$$

Area before substitution



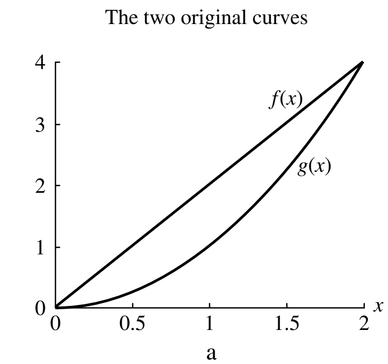
a

Area after substitution

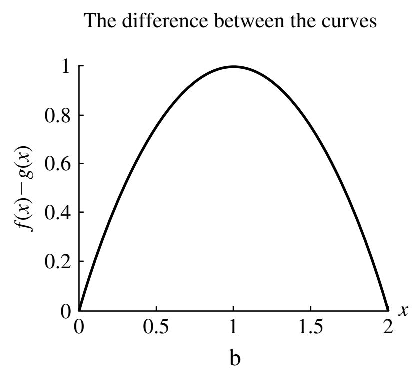


b

7. a.



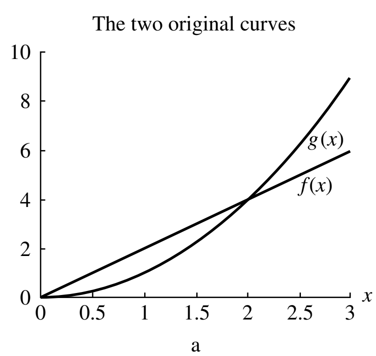
b.



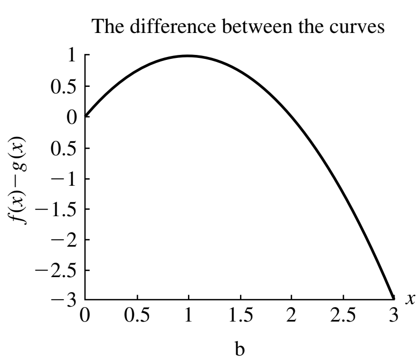
c. $2x - x^2 > 0$ for all $0 \le x \le 2$. Therefore,

$$\int_0^2 2x - x^2\,dx = x^2 - \frac{x^3}{3}\bigg|_0^2 = \left(4 - \frac{8}{3}\right) \approx 1.33$$

9. a.



b.



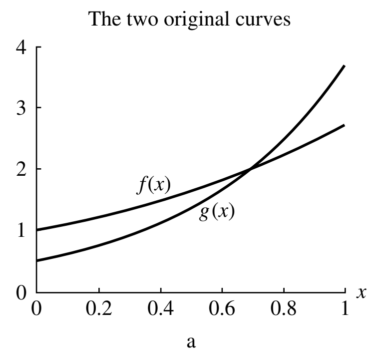
c. $2x - x^2 > 0$ for $0 \le x < 2$, but $2x - x^2 < 0$ for $2 < x \le 4$. Therefore,

$$\int_0^4 |\,2x - x^2\,|\,dx = \int_0^2 (2x - x^2)\,dx - \int_2^4 (x^2 - 2x)\,dx$$

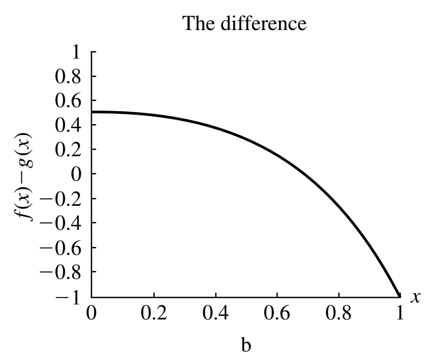
$$= x^2 - \frac{x^3}{3}\bigg|_0^2 + \frac{x^3}{3} - x^2\bigg|_2^4$$

$$= \frac{4}{3} + \left(9 - 9 - \frac{8}{3} + 4\right) = \frac{8}{3}$$

11. a.



b.



c. $e^x - \frac{e^{2x}}{2} > 0$ if $x < \ln(2)$. Therefore,

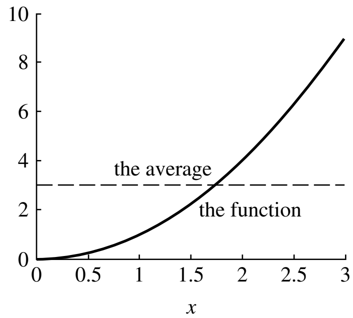
$$\int_0^1 \left|e^x - \frac{e^{2x}}{2}\right| dx = \int_0^{\ln(2)} \left(e^x - \frac{e^{2x}}{2}\right) dx$$

$$- \int_{\ln(2)}^1 \left(\frac{e^{2x}}{2} - e^x\right) dx$$

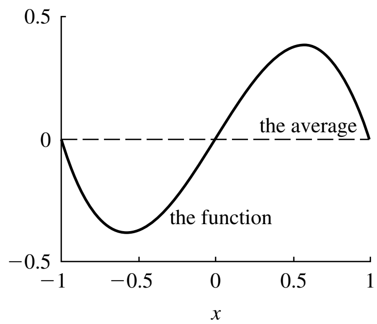
$$= \left(e^x - \frac{e^{2x}}{4}\right)\bigg|_0^{\ln(2)} - \left(\frac{e^{2x}}{4} - e^x\right)\bigg|_{\ln(2)}^1$$

$$= -e + \frac{e^2}{4} + \frac{5}{4} \approx 0.378$$

13. $\int_0^3 x^2 = \frac{x^3}{3}\Big|_0^3 = 9$. The average is the integral divided by the width of the interval, or $9/3 = 3$.



15. $\int_{-1}^{1} x - x^3 = \frac{x^2}{2} - \frac{x^4}{4}\Big|_{-1}^{1} = 0$. The average is the integral divided by the width of the interval, or $0/2 = 0$.



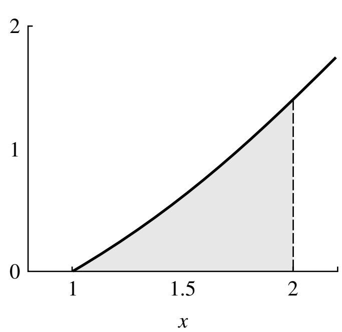
17. We need to compute

$$\text{area} = \int_1^2 x \ln(x) dx$$

Let $u(x) = \ln(x)$ and $\frac{dv}{dx} = x$; then $\frac{du}{dx} = \frac{1}{x}$ and $v(x) = \frac{x^2}{2}$. Then

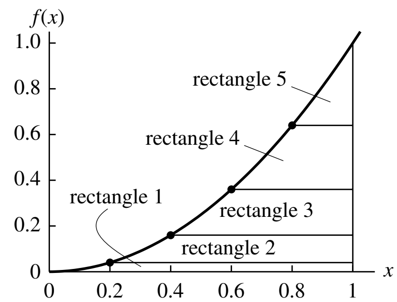
$$\begin{aligned}\int_1^2 x \ln(x) dx &= u(x)v(x) - \int v(x)\frac{du}{dx} dx \\ &= \frac{x^2 \ln(x)}{2}\Big|_1^2 - \int_1^2 \frac{x}{2} dx \\ &= \frac{x^2 \ln(x)}{2}\Big|_1^2 - \frac{x^2}{4}\Big|_1^2 \\ &= 2\ln(2) - \frac{1}{2}\ln(1) - 1 + \frac{1}{4} \\ &= 2\ln(2) - \frac{3}{4} \approx 0.636\end{aligned}$$

The function itself increases from $g(1) = 0$ to $g(2) = 2\ln(2) \approx 1.386$.



The area is about half the area of a rectangle with width 1 and height 1.386, as is consistent with the region being nearly triangular.

19. a.



b. Rectangle 1: lower size estimate is $0.8 \cdot f(0.2) = 0.0032$; upper size estimate is $1.0 \cdot f(0.2) = 0.004$. Rectangle 2: lower size estimate is $0.6 \cdot (f(0.4) - f(0.2)) = 0.072$; upper size estimate is $0.8 \cdot (f(0.4) - f(0.2)) = 0.096$. Rectangle 3: lower size estimate is $0.4 \cdot (f(0.6) - f(0.4)) = 0.08$; upper size estimate is $0.6 \cdot (f(0.6) - f(0.4)) = 0.08$. Rectangle 4: lower size estimate is $0.2 \cdot (f(0.8) - f(0.6)) = 0.056$; upper size estimate is $0.4 \cdot (f(0.8) - f(0.6)) = 0.112$. Rectangle 5: lower size estimate is 0.0; upper size estimate is $0.2 \cdot (f(1.0) - f(0.8)) = 0.072$.

c. Lower estimate is 0.211; upper estimate is 0.364.

d. A rectangle at height y goes from the point where $x^2 = y$, or $x = \sqrt{y}$, to $x = 1$. Its length is $1 - \sqrt{y}$.

e. Area $= \int_0^1 (1 - \sqrt{y})\, dy$.

f. $\int_0^1 (1 - \sqrt{y})\, dy = \left(y - \frac{2y^{3/2}}{3}\right)\Big|_0^1 = \frac{1}{3}$. It checks.

21. A little ring at radius r will have area approximately equal to $2\pi r \Delta r$ because it looks like a curved rectangle with length equal to the perimeter $2\pi r$ and width Δr. If we pick n rings with width Δr (so that $n\Delta r = 1$), the total area will be approximated by the Riemann sum $\sum_{i=0}^{n} 2\pi r \Delta r$. In the limit, this is the definite integral $\int_0^1 2\pi r\, dr = \pi r^2|_0^1 = \pi$.

23.

$$\begin{aligned}l(6) - l(3) &= \int_1^6 \frac{1}{x} dx - \int_1^3 \frac{1}{x} dx = \int_3^6 \frac{1}{x} dx \\ &= \int_1^2 \frac{1}{y} dy = l(2)\end{aligned}$$

where we set $y = \frac{x}{3}$, so $dy = \frac{dx}{3}$ and the limits of integration go from 1 to 2.

25. $\int_1^{10^2} \frac{1}{x}\, dx = l(10^2)$. Substituting $y = \sqrt{x}$, we find that

$$\frac{dy}{dx} = \frac{1}{2}x^{-1/2} = \frac{1}{2}\frac{y}{x}$$

Then the integrand becomes

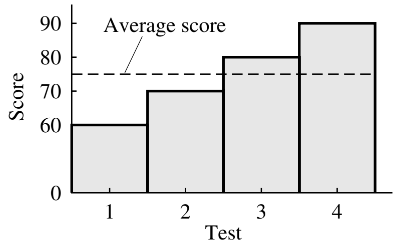
$$\frac{1}{x}\, dx = \frac{2}{y}\, dy$$

and the limits of integration go from 1 to 10. Hence

$$\int_1^{10^2} \frac{1}{x}\,dx = \int_1^{10} \frac{2}{y}\,dy = 2l(10)$$

This matches the law of logs.

27.



The average is $(60+70+80+90)/4 = 75$. The function is

$$f(x) = \begin{cases} 60 & \text{for } 0 \le x < 1 \\ 70 & \text{for } 1 \le x < 2 \\ 80 & \text{for } 2 \le x < 3 \\ 90 & \text{for } 3 \le x < 4 \end{cases}$$

$$\text{total score} = \int_0^4 f(x)dx = 60+70+80+90 = 300$$

$$\text{average score} = \frac{\text{total score}}{\text{width of interval}} = \frac{300}{4} = 75$$

29. $\int_0^{15} g(t)\,dt = 9281.25$. Divide this answer by 15 h to find 618.75 L/h.

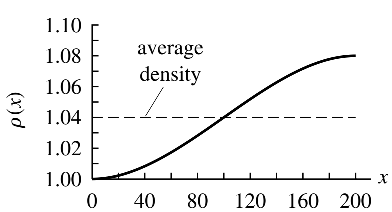
31. $\int_0^{24} g(t)\,dt = 6912.0$. Over the full 24 h, the average rate is 288.0 L/h.

33. a. The critical points are at $x = 0$ and $x = 200$ (both endpoints). Because $\rho(0) = 1.0$ and $\rho(200) = 1.08$, the first is the minimum and the second is the maximum.

b. $\int_0^{200} \rho(x)dx = 208$ g.

c. The average is 1.04 g/cm, which lies right between the minimum and the maximum.

d.



35. a. Let x represent distance along the strand. Then the formula for the line giving the number of A's can be found by finding the slope as

$$\text{slope} = \frac{300-150}{4.7 \times 10^3} = 3.19 \times 10^{-2}$$

Using the point $A(0) = 150$, we find $A(x) = 150 + 3.19 \times 10^{-2}x$. Similarly, $C(x) = 350 - 3.19 \times 10^{-2}x$ and $G(x) = 220 + 2.13 \times 10^{-2}x$. Because $A(x) + C(x) + G(x) + T(x) = 1000$,

$$T(x) = 1000 - A(x) - C(x) - G(x) = 280 - 2.13 \times 10^{-2}x$$

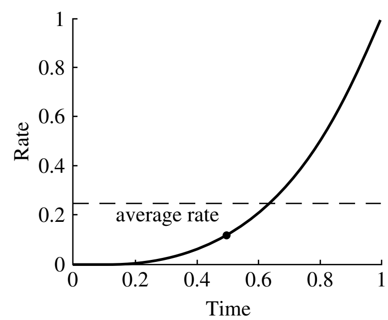
b. The totals can be found by integrating:

$$\int_0^{4.7 \times 10^3} A(x) = 1.057 \times 10^6$$

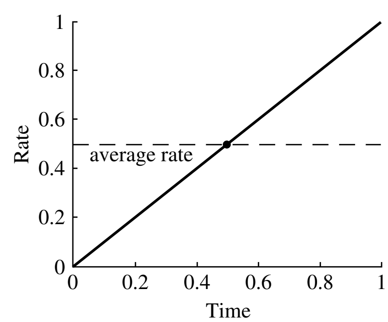
The total number of C's is 1.292×10^6, the total number of G's is 1.269×10^6, and the total number of T's is 1.055×10^6.

c. The mean numbers per thousand are: A, 224; C, 275; G, 270; T, 230.

37. $\int_0^1 t^3\,dt = 0.25\text{ cm}^3$. The average rate is $0.25\text{ cm}^3/\text{s}$. The rate at time 0.5 is 0.125, less than the average rate during the first second. This seems to be because the function is concave up (has positive second derivative).



39. $\int_0^1 t\,dt = 0.5\text{ cm}^3$. The average rate is $0.5\text{ cm}^3/\text{s}$. The average matches the rate at the average time, probably because this function is linear.



Section 4.7, page 409

1. We know that e^{-x} is faster because exponential functions are faster than power functions. With L'Hôpital's rule,

$$\frac{e^{-x}}{\frac{1}{x}} = \frac{x}{e^x}$$

Using L'Hôpital's rule, we find that the limit of this quotient (which is an indeterminate form because both the numerator and the denominator approach infinity) is

$$\lim_{x\to\infty} \frac{x}{e^x} = \lim_{x\to\infty} \frac{1}{e^x} = 0$$

e^{-x} does indeed approach 0 faster than $\frac{1}{x}$ does as x approaches infinity.

3. The power in the denominator of $\frac{1}{x^2}$ is larger, so it approaches infinity more quickly. Algebraically,

$$\frac{\frac{1}{x^2}}{\frac{1}{x}} = \frac{x}{x^2} = \frac{1}{x}$$

which approaches infinity. Therefore, $\frac{1}{x^2}$ does approach infinity more quickly than $\frac{1}{x}$ does as x approaches 0.

5. $\int_0^\infty e^{-3t}\,dt = 1/3$.

7. Does not converge because the integrand approaches 0 too slowly.

9. Using the substitution $u = 1 + 3x$, we find that this integral is 2/3.

11. Diverges because the power is not less than 1.

13. $\int_0^{0.001} \frac{1}{\sqrt[3]{x}}\,dx = 0.015$.

15. Near 0, the leading behavior of the denominator is $\sqrt[3]{x}$. The function then acts like $1/\sqrt[3]{x}$, which converges on the interval from 0 to 1.

17. For large x, the leading behavior of the denominator is x^3. The function then acts like $1/x^3$, which converges on the interval from 1 to infinity. The whole integral converges.

19. For $x < 1$, $\sqrt[3]{x} > x^3$, so

$$\int_0^1 \frac{1}{\sqrt[3]{x} + x^3}\,dx < \int_0^1 \frac{1}{2\sqrt[3]{x}}\,dx = 3x^{2/3}\Big|_0^1 = 3$$

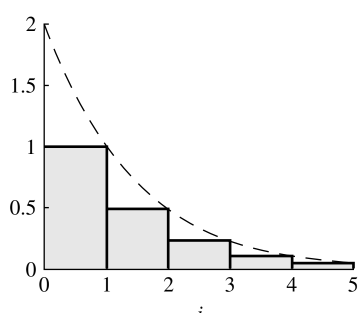
The value is less than 3.

21. For $x > 1$, $x^3 > \sqrt[3]{x}$, so

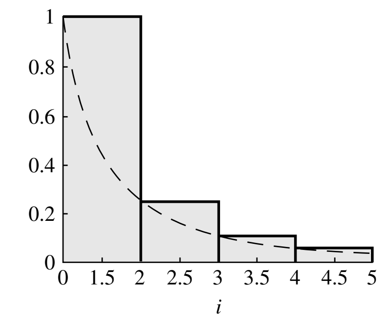
$$\int_1^\infty \frac{1}{\sqrt[3]{x} + x^3}\,dx < \int_1^\infty \frac{1}{2x^3}\,dx = -x^{-2}\Big|_1^\infty = 1$$

The value is less than 1.

23. The sum corresponds to the area under the blocks. Because the blocks lie totally below the curve, and the area under the curve is finite, the sum must also be finite. A bit of experimentation shows that the sum is exactly equal to 2. Integration shows that the area under the curve is 2.885.



25. The sum corresponds to the area under the blocks. Although the blocks do not lie totally below the curve, if we removed the first block (area 1) and shifted them all over by one, they would lie below the curve. The area under the curve is finite, so the sum must also be finite. (Amazingly, the sum is equal to $\pi^2/6$.)



27. The differential equation is

$$\frac{dV}{dt} = \frac{100}{(1+t)^2}$$

The solution is

$$V(t) = 500 + \int_0^t \frac{100}{(1+s)^2}\,ds = 500 + \frac{-100}{1+s}\Big|_0^t = 600 - \frac{100}{1+t}$$

This rule could be followed indefinitely, and the limit of the volume is 600.

29. The differential equation is

$$\frac{db}{dt} = \frac{1000}{(2+3t)^{.75}}$$

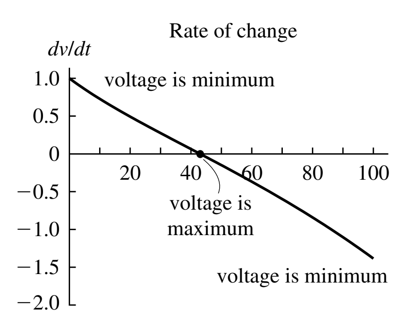
The solution is

$$\begin{aligned} b(t) &= 1.0 \times 10^6 + \int_0^t \frac{1000}{(2+3s)^{.75}}\,ds \\ &\approx 1.0 \times 10^6 + 1333(2+3s)^{0.25}\Big|_0^t \\ &= 1.0 \times 10^6 + 1333(2+3t)^{0.25} - 1333 2^{0.25} \\ &\approx 9.98 \times 10^5 + 1333(2+3t)^{0.25} \end{aligned}$$

This rule could not be followed indefinitely. The population would reach 2.0×10^6 when $t = 1.06 \times 10^{11}$. It grows very slowly.

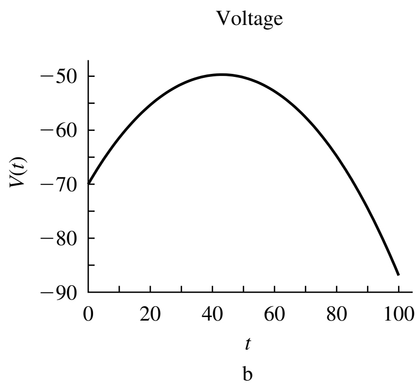
Supplementary Problems, page 410

1. a.



a

b.



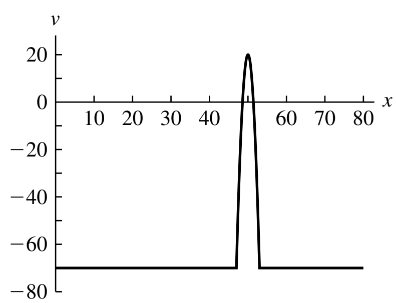
c. Solving with the indefinite integral, we get

$$v(t) = t + 50\ln(1 + 0.02t) - 100e^{0.01t} + c$$

Substituting the initial condition, we have that $c = 30$. Therefore, $v(100) = 100 + 50\ln(3.0) - 100e^1 + 30 = -86.9$.

3. a. At a speed of $10\frac{\text{m}}{\text{s}} = 1000\frac{\text{cm}}{\text{s}}$, it takes $t = 80$ cm/(1000 cm/s) $= 0.08$ s to reach your hand. Similarly, your elbow seems to be 50 cm from the brain, so it takes the signal 0.05 s to get there.

b.



c. average $= \frac{1}{6}\int_{47}^{53} -70.0 + 10.0(9.0 - (x - 50.0)^2)dx$. Substituting $y = x - 50$, we have that $dy = dx$ and limits of integration from $y = -3$ to $y = 3$, or

$$\begin{aligned}\text{average} &= \frac{1}{6}\int_{-3}^{3} -70.0 + 10.0(9.0 - y^2)dy \\ &= \frac{1}{6}\int_{-3}^{3} 20.0 - 10.0y^2 dy \\ &= \frac{1}{6}\left(20.0y - \frac{10.0}{3}y^3\right)\Big|_{-3}^{3} \\ &= \frac{1}{6}\left(\left(60.0 - \frac{10.0}{3}3^3\right)\right. \\ &\quad \left. - \left(-60.0 - \frac{10.0}{3}(-3)^3\right)\right) = -10\end{aligned}$$

d. The total voltage along the 6-cm piece is −60. Along the rest (74 cm) the total is $74 \cdot (-70) = -5180$. The total along the whole thing is $-5180 - 60 = -5240$, so the average is $-5240/80 = -65.6$.

5. a. The second is a pure-time differential equation. The first might describe a population of bacteria that are autonomously reproducing, and the second might describe a population being supplemented from outside at an ever-increasing rate.

b. $\hat{b}(0.1) = b(0) + b'(0) \cdot 0.1 = 1 + 2 \cdot 0.1 = 1.2$. Similarly, $\hat{B}(0.1) = B(0) + B'(0) \cdot 0.1 = 1 + 2 \cdot 0.1 = 1.2$.

c. $\hat{b}(0.2) = \hat{b}(0.1) + 2\hat{b}(0.1) \cdot 0.1 = 1.2 + 2 \cdot 1.2 \cdot 0.1 = 1.44$. Similarly, $\hat{B}(0.2) = \hat{B}(0.1) + B'(0.1) \cdot 0.1 = 1.2 + 1.2 \cdot 0.1 = 1.32$.

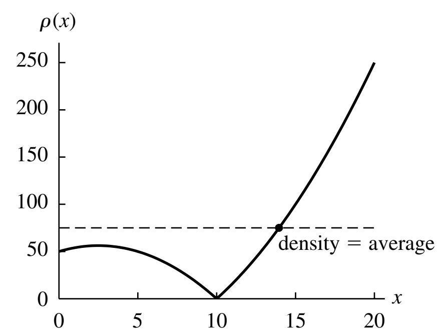
7. a. The volume increases until time $t = 2$ and decreases thereafter.

b. The volume is a maximum when the rate of change is 0, or at $t = 2$.

c. Left-hand estimate $= V(0) \cdot 1 + V(1) \cdot 1 + V(2) \cdot 1 = 7$, and right-hand estimate $= V(1) \cdot 1 + V(2) \cdot 1 + V(3) \cdot 1 = -2$.

d. $V(3) = \int_0^3 4 - t^2 dt = 4t - t^3/3|_0^3 = 3$.

9. a. The function hits 0 when $x = 10$. The derivative is 0 at $x = 2.5$, where the density is 56.25. At the endpoints, we have densities of 50 (at $x = 0$) and 250 (at $x = 20$). The maximum is thus at $x = 20$, with the minimum at $x = 10$.



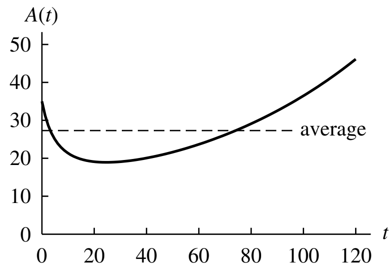
b. Taking into account the absolute values, we find that the total number is

$$\begin{aligned}\text{total} &= \int_0^{20} \left| -x^2 + 5x + 50 \right| dx \\ &= \int_0^{10} -x^2 + 5x + 50dx + \int_{10}^{20} x^2 - 5x - 50dx \\ &= -x^3/3 + 5x^2/2 + 50x|_0^{10} + x^3/3 \\ &\quad - 5x^2/2 - 50x|_{10}^{20} = 1500\end{aligned}$$

c. The average density is 75.0.

d. Shown in figure for part **a.**

11. a. $A'(t) = -15/(2 + .3t)^2 + 0.125e^{0.125t}$, so $A'(0) = -3.625$. Also, $A(0) = 35.0$ and $A(120) = 46.13$. The graph thus begins decreasing and then increases. The maximum is at $t = 120$.



b. $\displaystyle\int_0^{120} A(t)dt = \frac{50}{0.3}\ln(2+0.3t) + 800e^{0.0125t}|_0^{120} = 3276$

c. The average is $3276/120 = 27.3$.

d. The minimum value must be less than 27.3.

e. RHE $= \sum_{i=1}^{6} A(20i)20$. I suspect the estimate is high.