Chapter 2

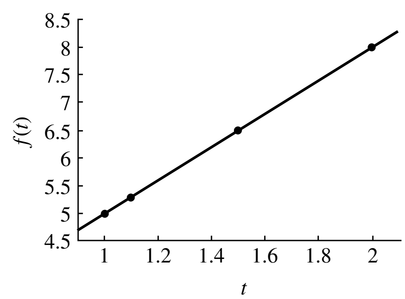
Section 2.1, page 143

1. With $\Delta t = 1.0$, $\Delta f = f(2.0) - f(1.0) = 3.0$, so $\frac{\Delta f}{\Delta t} = 3.0$.
With $\Delta t = 0.5$, $\Delta f = f(1.5) - f(1.0) = 1.5$, so $\frac{\Delta f}{\Delta t} = 3.0$.
With $\Delta t = 0.1$, $\Delta f = f(1.1) - f(1.0) = 0.3$, so $\frac{\Delta f}{\Delta t} = 3.0$.
With $\Delta t = 0.01$, $\Delta f = f(1.01) - f(1.0) = 0.03$, so $\frac{\Delta f}{\Delta t} = 3.0$.

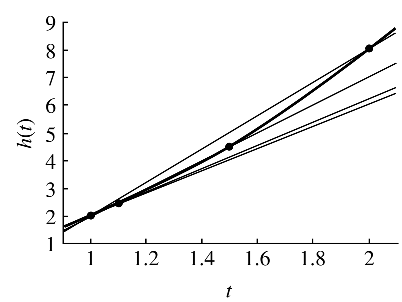
3. With $\Delta t = 1.0$, $\Delta h = h(2.0) - h(1.0) = 6.0$, so $\frac{\Delta h}{\Delta t} = 6.0$.
With $\Delta t = 0.5$, $\Delta h = h(1.5) - h(1.0) = 2.5$, so $\frac{\Delta h}{\Delta t} = 5.0$.
With $\Delta t = 0.1$, $\Delta h = h(1.1) - h(1.0) = 0.42$, so $\frac{\Delta h}{\Delta t} = 4.2$.
With $\Delta t = 0.01$, $\Delta h = h(1.01) - h(1.0) = 0.0402$, so $\frac{\Delta h}{\Delta t} = 4.02$.

5. With $\Delta t = 1.0$, $\Delta G = G(1.0) - G(0.0) \approx 6.389$, so $\frac{\Delta G}{\Delta t} \approx 6.389$. With $\Delta t = 0.5$, $\Delta G = G(0.5) - G(0.0) \approx 1.718$, so $\frac{\Delta G}{\Delta t} \approx 3.436$. With $\Delta t = 0.1$, $\Delta G = G(0.1) - G(0.0) \approx 0.221$, so $\frac{\Delta G}{\Delta t} \approx 2.21$. With $\Delta t = 0.01$, $\Delta G = G(0.01) - G(0.0) \approx 0.0202$, so $\frac{\Delta G}{\Delta t} \approx 2.02$.

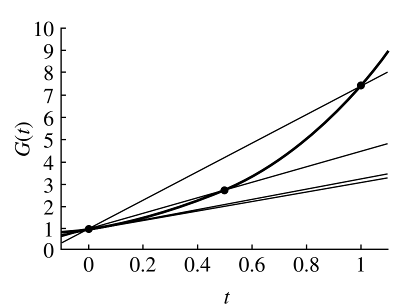
7. Each secant line is $f_s(t) = 2 + 3t$.



9. The coordinates of the base point are (1, 2), so the secant lines are as follows: with $\Delta t = 1.0$, $h_s(t) = 2 + 6(t-1)$; with $\Delta t = 0.5$, $h_s(t) = 2 + 5(t-1)$; with $\Delta t = 0.1$, $h_s(t) = 2 + 4.2(t-1)$; with $\Delta t = 0.01$, $h_s(t) = 2 + 4.02(t-1)$.



11. The coordinates of the base point are (0, 1), so the secant lines are as follows: with $\Delta t = 1.0$, $G_s(t) = 1 + 6.389t$; with $\Delta t = 0.5$, $G_s(t) = 1 + 3.436t$; with $\Delta t = 0.1$, $G_s(t) = 1 + 2.21t$; with $\Delta t = 0.01$, $G_s(t) = 1 + 2.02t$.



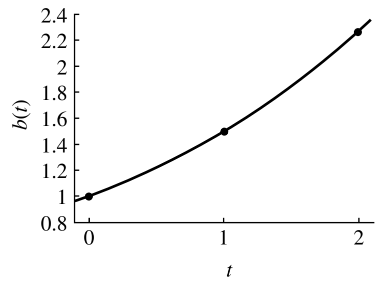
13. The slope is 3, so the tangent line is $\hat{f}(t) = 2 + 3t$.

15. It looks like the slopes are getting close to 4.0, so the tangent line is $\hat{h}(t) = 2 + 4(t - 1)$.

17. It looks like the slopes are getting close to 2.0, so the tangent line is $\hat{G}(t) = 1 + 2t$.

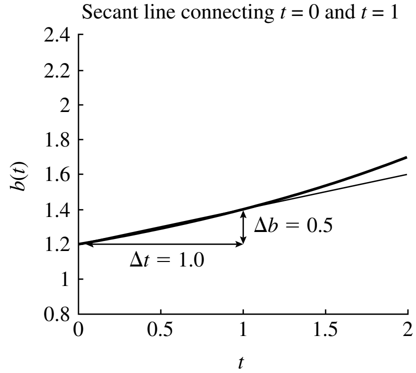
19. The derivative of $g(t)$, the slope of the tangent line.

21. a. $b(0) = 1.0$, $b(1.0) = 1.5$, $b(2.0) = 2.25$.



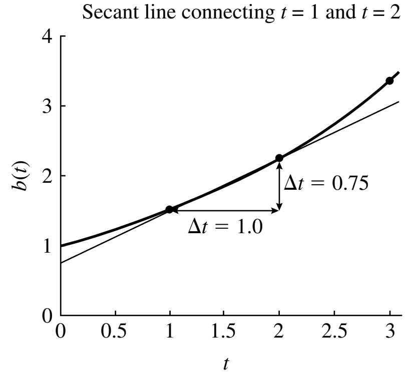
b. $\Delta b = 1.5 - 1.0 = 0.5$, so $\Delta b/\Delta t = 0.5$.

Secant line connecting $t = 0$ and $t = 1$



c. $\Delta b = 2.25 - 1.5 = 0.75$, so $\Delta b/\Delta t = 0.75$.

Secant line connecting $t = 1$ and $t = 2$



23. a. $\Delta b = 1.5^{1.0} - 1.0 = 0.5$, and $\Delta b/\Delta t = 0.5$.

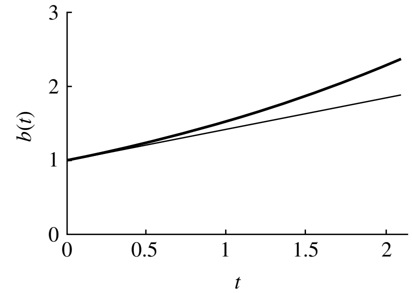
b. $\Delta b = 1.5^{0.1} - 1.0 = 0.0413$, and $\Delta b/\Delta t = 0.414$.

c. $\Delta b = 1.5^{0.01} - 1.0 = 0.00406$, and $\Delta b/\Delta t = 0.406$.

d. $\Delta b = 1.5^{0.001} - 1.0 = 0.000405$, and $\Delta b/\Delta t = 0.405$.

e. The limit looks like 0.405.

f.



25. a. The slope is $(5 \cdot 1.0^2 - 0.0)/1.0 = 5.0$.

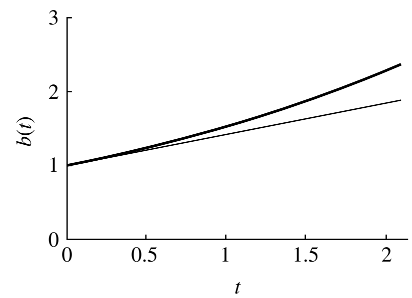
b. The slope is $(5 \cdot 0.1^2 - 0.0)/0.1 = 0.5$.

c. The slope is $(5 \cdot 0.01^2 - 0.0)/0.01 = 0.05$.

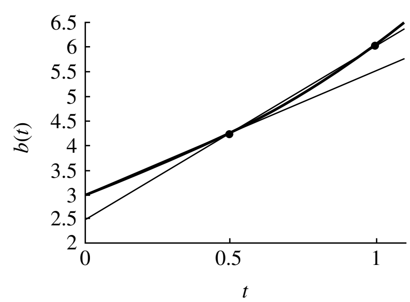
d. The slope is $(5 \cdot 0.001^2 - 0.0)/0.001 = 0.005$.

e. The slope gets close to 0.

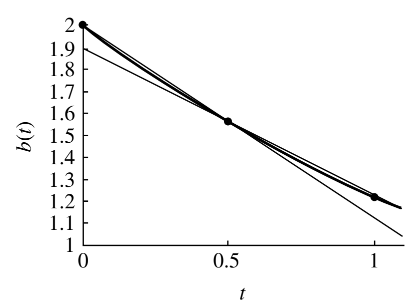
f.



27. During the first hour, 3.0 bacteria/h. During the first half-hour, 2.485 bacteria/h. During the second half-hour, 3.515 bacteria/h. Changes faster during the second half-hour.



29. During the first hour, −0.79 bacteria/h. During the first half-hour, −0.88 bacteria/h. During the second half-hour, −0,69 bacteria/h. Changes more rapidly during the first half-hour.



31.

t	$b(t)$	Change, Δb	Average Rate of Change, $\frac{\Delta b}{\Delta t} \approx \frac{db}{dt}$	Per Capita Rate of Change $\frac{\frac{db}{dt}}{b(t)}$
0.0	1.0	—	—	—
1.0	2.0	1.0	1.0	0.5
2.0	4.0	2.0	2.0	0.5
3.0	8.0	4.0	4.0	0.5

$$\frac{db}{dt} = 0.5b(t)$$

33.

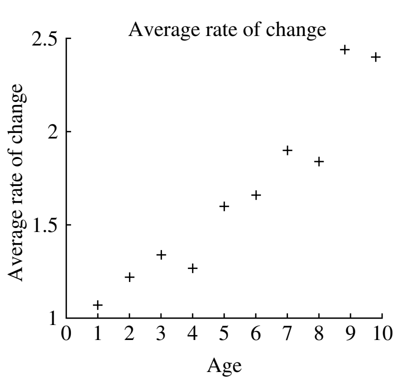
t	$b(t)$	Change, Δb	Average Rate of Change, $\frac{\Delta b}{\Delta t} \approx \frac{db}{dt}$	Per Capita Rate of Change $\frac{\frac{db}{dt}}{b(t)}$
0.0000	1.0000	—	—	—
0.0100	1.0070	0.00696	0.69556	0.69075
0.0200	1.0140	0.00700	0.70039	0.69075
0.0300	1.0210	0.00705	0.70526	0.69075
0.0400	1.0281	0.00710	0.71017	0.69075
0.0500	1.0353	0.00715	0.71511	0.69075

$$\frac{db}{dt} = 0.6907b(t)$$

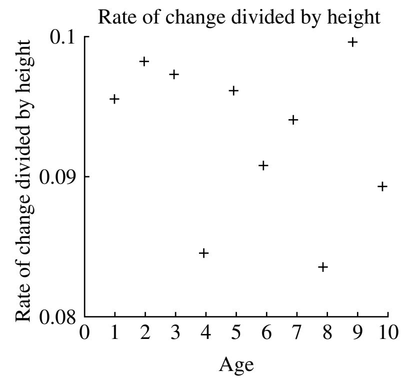
35. a. Using the previous measurement as the point, we find

Age	Rate of Change (m/year)	Rate Divided by Height
1	1.07	0.0957
2	1.22	0.0984
3	1.34	0.0975
4	1.27	0.0846
5	1.60	0.0963
6	1.66	0.0909
7	1.90	0.0942
8	1.84	0.0836
9	2.44	0.0998
10	2.40	0.0894

b.



c.



d. Both seem to jump around a bit, but the tree increases its height by about 9% per year. The approximate differential equation is

$$\frac{dh}{dt} = 0.09h$$

37. Getting 5% interest corresponds to multiplying the principle by 1.05, giving $1.05 \cdot \$1000 = \$1050.00$.

39. $\frac{5}{12} \approx 0.417$. We must multiply the principle by 1.00417 twelve times, for $(1.00417)^{12} \cdot \$1000 \approx \$1051.16$, for an extra \$1.16.

41. Breaking into intervals of length Δt, we get

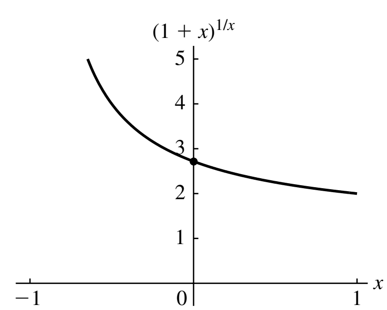
$$\lim_{\Delta t \to 0} (1 + 0.05\Delta t)^{\frac{1}{\Delta t}} \$1000$$

It looks like the limit is \$1051.27.

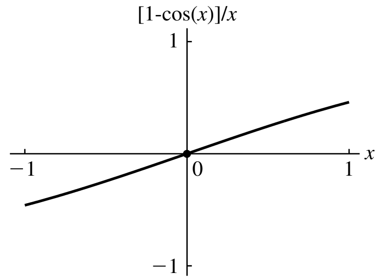
43. With yearly compounding, you'd get $2.0 \cdot \$1000 = \$2000.00$. With monthly compounding, you'd get $\frac{100}{12} \approx 8.33$ for each of 12 months, giving $1.0833^{12} \cdot \$1000 \approx \$2613.00$. With daily compounding, you'd get $\frac{100}{365} \approx 0.274$ for each of 365 days, giving $1.00274^{365} \cdot \$1000 \approx \$2714.60$. The compound interest is more important because the principal changes much more during the year.

Section 2.2, page 153

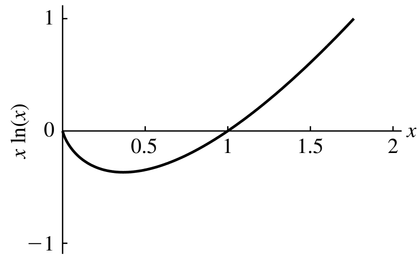
1. At $x = 0.1$, value is 2.594; at $x = 0.01$ it is 2.705; at $x = 0.001$ it is 2.717; and at $x = 0.0001$ it is 2.718. The limit seems to be e.



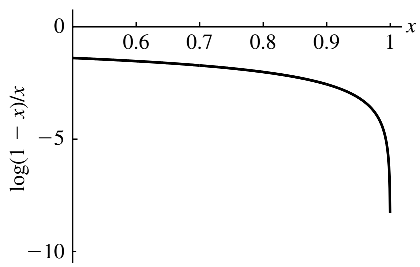
3. At $x = 0.1$, value is 0.499; at $x = 0.01$ it is 0.005; at $x = 0.001$ it is 0.0005. The limit seems to be 0.



5. $0.1\ln(0.1) = -0.230, 0.01\ln(0.01) = -0.046, 0.001\ln(0.001) = -0.006$. It seems to approach 0.



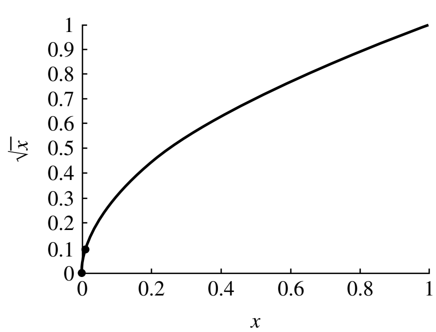
7. Denoting the function by $h(x)$, we find that $h(0.9) = -2.558$, $h(0.99) = -4.652$, $h(0.999) = -6.915$, $h(0.9999) = -9.211$. This seems to go to negative infinity.



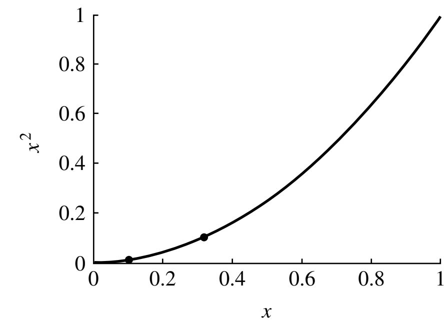
9. This will be 5 times the previous limit, or $5e = 13.59$, using Theorem 2.2c.

11. This will be the product of the two earlier limits, or $e \cdot 0 = 0$, using Theorem 2.2b.

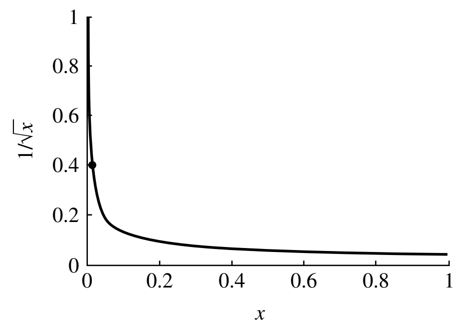
13. $f_1(x) < 0.1$ if $x < 0.01$, $f_1(x) < 0.01$ if $x < 0.0001$. This approaches 0 slowly because tiny values of the input are required to produce small values of the output. Graphically, the value of the function does not become small until x is very small.



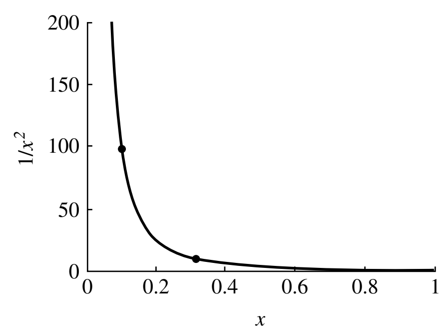
15. $f_3(x) < 0.1$ if $x < 0.316$, $f_3(x) < 0.01$ if $x < 0.1$. This approaches 0 quickly because small values of the input produce tiny values of the output. Graphically, the value of the function becomes small even when x is not that small.



17. $g_1(x) > 10$ if $x < 0.01$. $g_1(x) > 100$ if $x < 0.0001$. This approaches infinity slowly because tiny values of the input are required to produce large values of the output. Graphically, the value of the function becomes large only when x is very small.



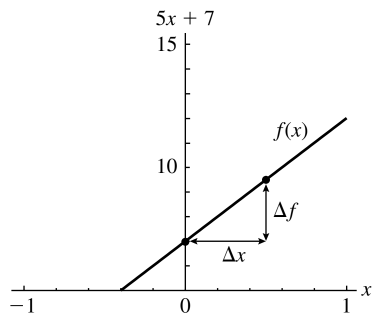
19. $g_3(x) > 10$ if $x < 0.316$. $g_3(x) > 100$ if $x < 0.1$. This approaches infinity quickly because small values of the input produce huge values of the output. Graphically, the value of the function becomes large even when x is not too small.



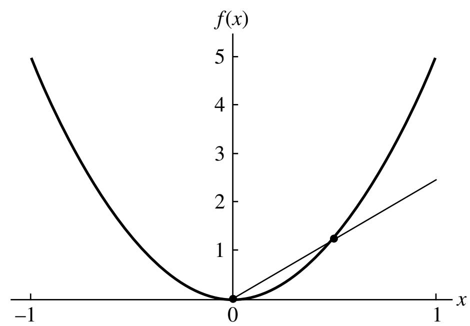
21. Both the left- and the right-hand limits are 1.

23. The left-hand limit is about 0.1, and the right-hand limit is about 0.02.

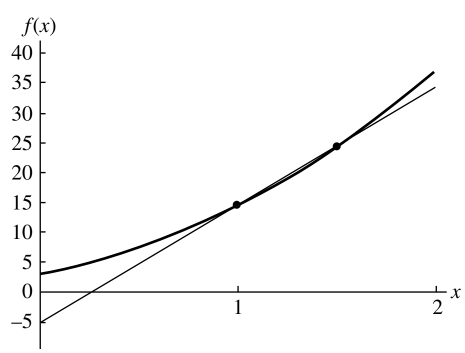
25. $\Delta f = 5\Delta x$, so the average rate of change is 5 (unless $\Delta x = 0$). The limit is also 5.



27. $\Delta f = 5\Delta x^2$, so the average rate of change is $5\Delta x$ (unless $\Delta x = 0$). The secant becomes flatter and flatter as Δx becomes smaller, so the limit must be 0.



29. $\Delta f = 5(1+\Delta x)^2 + 7(1+\Delta x) - 12$, so the average rate of change is $17 + 5\Delta x$ (unless $\Delta x = 0$). The limit is 17.



31. a. $\lim_{T\to 0} V(T) = 1.0\ \text{cm}^3$.

b. $V(2.0) = 5.0$.

c. $V(1.0) = 2.0$.

d. We need to solve $V(T) = 1.01$. But $1 + T^2 = 1.01$ if $T^2 = 0.01$, or $T = 0.1$ K.

33. The average rate of change between $t = 0$ and $t = \Delta t$ is

$$\frac{b(\Delta t) - b(0)}{\Delta t} = \frac{\Delta t + \Delta t^2 - 0}{\Delta t} = 1 + \Delta t$$

So Δt would have to be less than 0.01 for the average rate of change to be within 1% of the instantaneous rate of change.

35. The average rate of change between $t = 0$ and $t = \Delta t$ is

$$\frac{b(\Delta t) - b(0)}{\Delta t} = \frac{e^{\Delta t} - 1}{\Delta t}$$

Plugging in small values of Δt, I found that the average rate of change is 1.01 when $\Delta t = 0.02$.

37. a. This would cost only \$5.

b. This would cost \$50.

c. This would cost \$500.

39. a. The difference in temperature is 6.25°C. It looks like a \$1 device would be good enough.

b. The difference in temperature is 0.11°C. This would cost about \$10 to detect.

c. The difference in temperature is 0.00077°C. This would cost over \$1000 to detect.

41. a. The pressure is 11 atmospheres, and measuring it would cost \$121.

b. The pressure is 101 atmospheres, and measuring it would cost \$10,201.

c. The pressure is 501 atmospheres, and measuring it would cost \$251,001.

Section 2.3, page 163

1. This is a linear function and is continuous everywhere.

3. This is constructed as the quotient of the continuous exponential function and a continuous linear function. It is guaranteed to be continuous everywhere that the denominator is not equal to 0. The only potential trouble point is $x = -1$.

5. This is a composition of the natural log with a linear function divided by a polynomial. The theorems guarantee continuity except at $z = 1$, where we are taking the natural log of 0. At $z = 0$, where the denominator is 0, the logarithm is not defined.

7. This is a composition of the continuous cosine function with a continuous linear function $\left(x - \frac{\pi}{2}\right)$ and is continuous everywhere.

9. This is the quotient of the constant 1 by the function $(1 - w)^4$, which is a polynomial. This is guaranteed to be continuous except where the denominator is 0, or when $w = 1$.

11. $l(5) = 31$, $l(5.1) = 31.5$, $l(5.01) = 31.05$, $l(4.9) = 30.5$, $l(4.99) = 30.95$.

13. $f(0) = e^0/(0+1) = 1$. $f(0.1) = 1.0047$, $f(0.01) = 1.00005$, $f(-0.1) = 1.0054$, and $f(-0.01) = 1.00005$.

15. $g(2) = 0$. $g(2.1) = 0.022$, $g(2.01) = 0.0024$, $g(1.9) = -0.029$, $g(1.99) = -0.0025$.

17. $g(0)$ cannot be computed. In fact, $g(z)$ does not make sense for $z \le 1$.

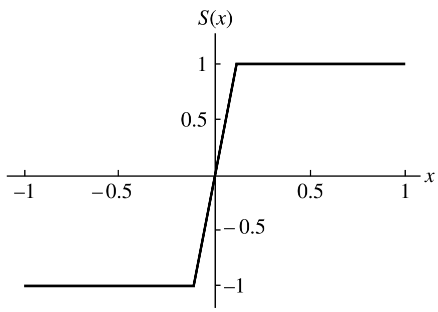
19. $r(1)$ cannot be computed. $r(1.1) = 1.0 \times 10^4$, $r(1.01) = 1.0 \times 10^8$, $r(0.9) = 1.0 \times 10^4$, $r(0.99) = 1.0 \times 10^8$. This limit is infinity.

21. $f(x) = 2.1$ if $x = 0.1$ and $f(x) = 1.9$ if $x = -0.1$, so $-0.1 \le x \le 0.1$.

23. $f(x) = 1.1$ if $x = \sqrt{1.1} = 1.049$ and $f(x) = 0.9$ if $x = 0.949$, so $0.949 \leq x \leq 1.049$.

25. Any value of $x \geq 0$ works, so x must be within 1 away from 1.

27. a.



b. The slope on the central part is 10.

$$S(x) = \begin{cases} -1 & \text{if } x \leq -0.1 \\ -1 + 10(x + 0.1) & \text{if } 0.1 < x < 0.1 \\ 1 & \text{if } x \geq 0.1 \end{cases}$$

c. The input would have to be between -0.01 and 0.01.

29.

$$4.8 < M < 5.2$$
$$4.8 < 2.0V < 5.2$$
$$2.4 < V < 2.6$$

V must be within 0.1 cm^3 of 2.5 cm^3 for M to be within 0.2 g of 5.0 g.

31.

$$0.95 < F < 1.05$$
$$0.95 < r^4 < 1.05$$
$$0.987 < r < 1.012$$

The radius must be within about 1% of 1.0 to guarantee a flow within 5%.

33. Between 4.5×10^8 and 5.5×10^8. Tolerance is 5.0×10^7.

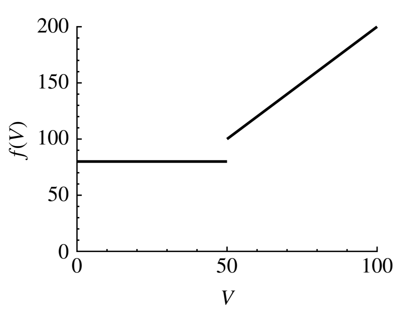
35. Between 8.789×10^5 and 10.742×10^5. Tolerance is 9.76×10^4.

37. Between 0.96 and 1.04 g/L. Tolerance is 0.04 g/L.

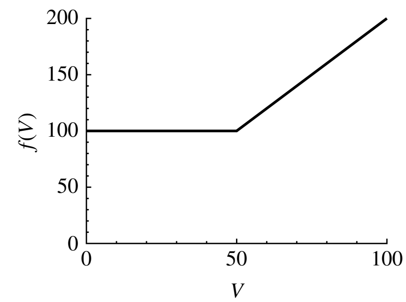
39. Between 491.52 and 532.48 g/L. Tolerance is 20.48 g/L.

41. Denote the function by f. Then

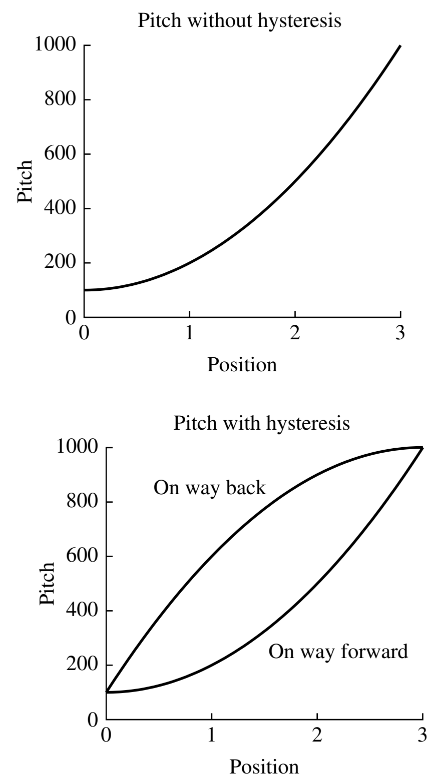
$$f(V) = \begin{cases} 80 & \text{if } V < 50 \\ 2V & \text{if } V \geq 50 \end{cases}$$



43. We need that $2V = V^*$ at $V = V_0 = 50$. In this case, $V^* = 100$.



45. a.



b. The second seems more likely. The pitch is usually different in different directions.

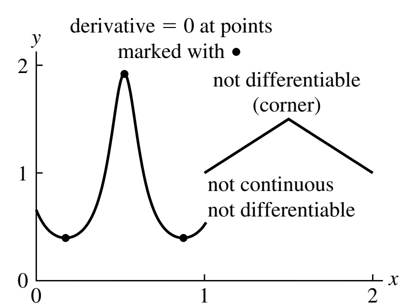
c. Both sound awful.

Section 2.4, page 171

1. $x^2 + 2x\Delta x + \Delta x^2$.

3. $9x^2 + 12x\Delta x + 4\Delta x^2$.

5.



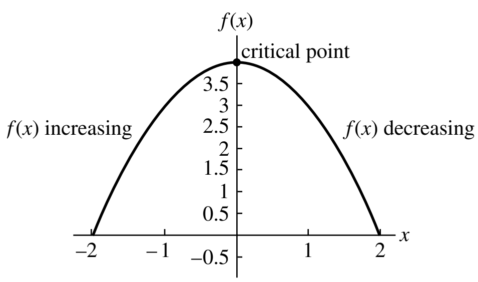
7. $M'(x) = 0.5$, $\dfrac{dM}{dx} = 0.5$. This function is increasing.

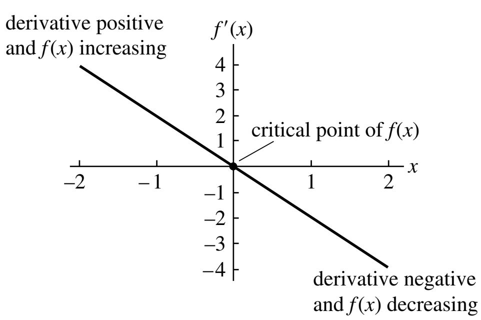
9. $g'(y) = -3$, $\dfrac{dg}{dy} = -3$. This function is decreasing.

11. $f(1) = 3$, $f(1 + \Delta x) = 3 - 2\Delta x - \Delta x^2$, $\Delta f = -2\Delta x - \Delta x^2$, so the slope of the secant is $\dfrac{\Delta f}{\Delta x} = -2 - \Delta x$ if $\Delta x \neq 0$. Taking the limit of this constant function by substituting $\Delta x = 0$, we find that the slope of the tangent is $f'(1) = -2$.

13. $f(x + \Delta x) = 4 - x^2 - 2x\Delta x - \Delta x^2$, $\Delta f = -2x\Delta x - \Delta x^2$, so the slope of the secant is $\dfrac{\Delta f}{\Delta x} = -2x - \Delta x$ if $\Delta x \neq 0$. Taking the limit, we find that the derivative is $f'(x) = -2x$, or $\dfrac{df}{dx} = -2x$.

15. The derivative is $f'(x) = -2x$, so the critical point occurs at $x = 0$. For $x < 0$, the derivative is positive, whereas for $x > 0$, it is negative. The function thus switches from increasing to decreasing at $x = 0$.





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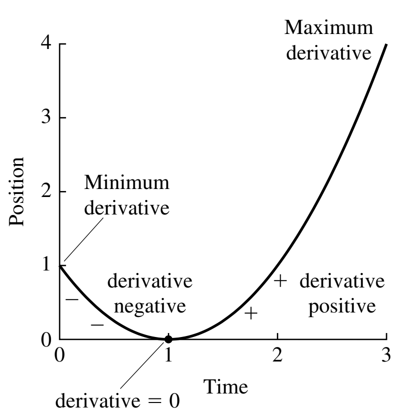

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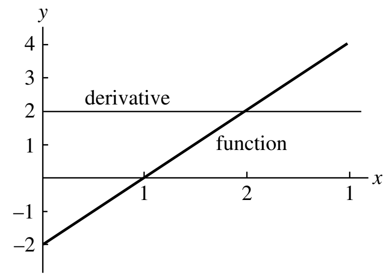

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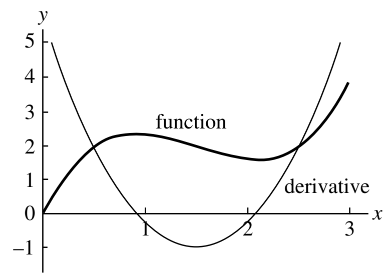
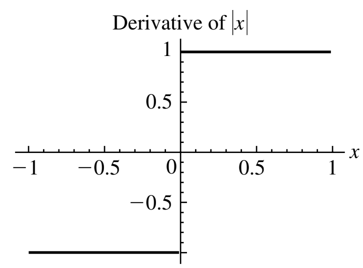
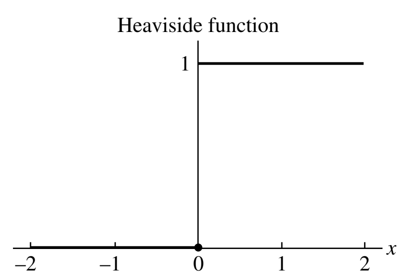


23. At $x = 0$, the slope of any secant with $\Delta x > 0$ is 1, whereas the slope of any secant with $\Delta x < 0$ is -1. The derivative wants to be both 1 and -1, which is impossible. From the graph, these slopes correspond to two possible tangent lines.



25. The slope of any secant with $\Delta x > 0$ is 0. With $\Delta x < 0$, the slope of the secant is $-1/\Delta x$, which has a limit of negative infinity. There is a sort of half-tangent line with equation $\hat{H}(x) = 1$ for positive Δx, but there is no candidate tangent line at all for negative Δx.



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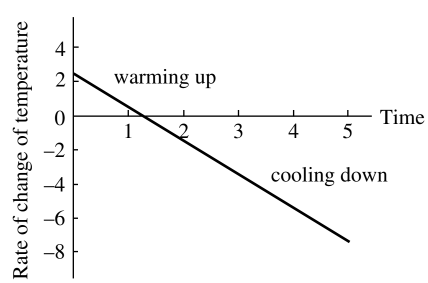

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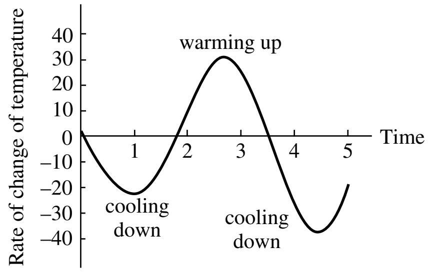
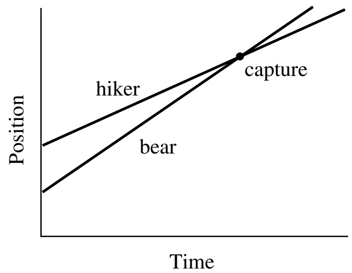


31.



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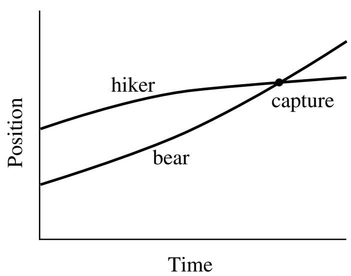


35. The time solves $M(t) = 0$, or $100 = \frac{9.78}{2}t^2$, or $t^2 = 20.44$ or $t = 4.52$ s. The speed is the derivative of the position, or $M'(t) = at = 9.78t$. The speed when it hits the ground is 44.2 m/s.

37. The time solves $M(t) = 0$, or $100 = \frac{22.88}{2}t^2$, or $t = 2.96$ s. The speed is the derivative of the position, or $M'(t) = at = 22.88t$. The speed when it hits the ground is 67.64 m/s.

Section 2.5, page 184

1. $5x^4$.

3. $0.2x^{-0.8}$.

5. ex^{e-1}.

7. $\frac{1}{e}x^{\frac{1}{e}-1}$.

9.

$$f'(x) = \frac{d(3x^2)}{dx} + \frac{d(3x)}{dx} + \frac{d1}{dx} \qquad \text{sum rule}$$

$$= 3\frac{d(x^2)}{dx} + 3\frac{dx}{dx} \qquad \text{constant product rule}$$

$$= 3 \cdot 2x + 3 \cdot 1 = 6x + 3 \qquad \text{power rule}$$

11.

$$g'(z) = \frac{d(3z^3)}{dz} + \frac{d(2z^2)}{dz} \qquad \text{sum rule}$$

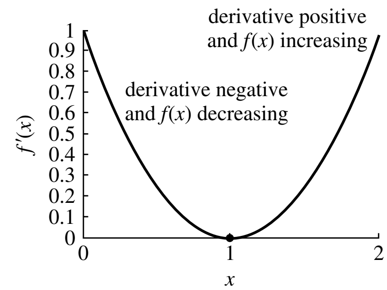
$$= 3\frac{d(z^3)}{dz} + 2\frac{dz^2}{dz} \qquad \text{constant product rule}$$

$$= 3 \cdot 3z^2 + 2 \cdot 2z = 9z^2 + 4z \qquad \text{power rule}$$

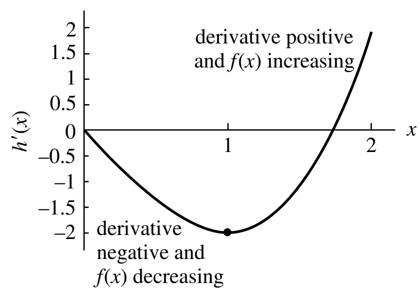
13. $x^3 + 3x^2 + 3x + 1$.

15. $8x^3 + 12x^2 + 6x + 1$.

17. $f'(x) = -2 + 2x$, which is negative for $x < 1$, zero at $x = 1$, and positive for $x > 1$.



19. $h'(x) = 3x^2 - 3$, which is negative for $x < 1$, zero at $x = 1$, and positive for $x > 1$.



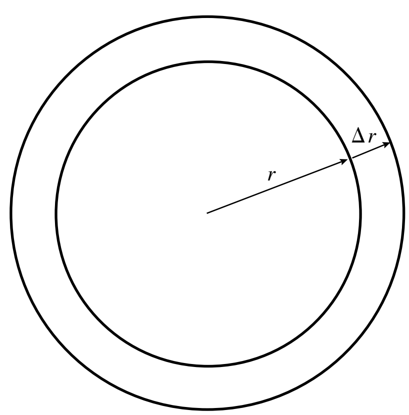
21. The constant 2 is the derivative of a linear function with slope 2. One such function is $2x$.

23. x^{15}.

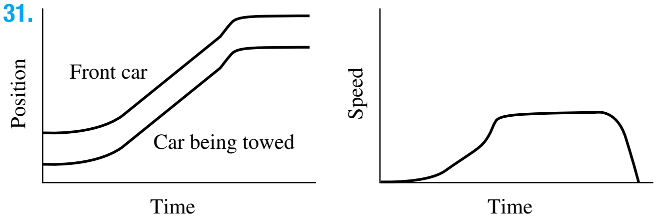
25. x^{-1}.

27. $A'(t) = 525t^2$, with units of people per year. New cases went up quickly.

29. $A'(r) = 2\pi r$. The units are centimeters. The derivative is equal to the perimeter of the circle, corresponding to the area of the little ring around the circle.



31.



33. Subtract the 10 mph for the passenger from the 80 mph for the train to get 70 mph.

35. Relative to passenger: 20 mph; relative to train: 10 mph; relative to ground: 90 mph.

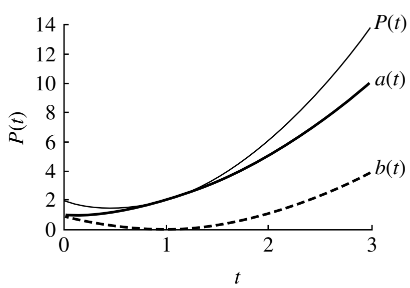
37. a. $P(t) = 2 - 2t + 2t^2$.

b. $a'(t) = 2t$ and $b'(t) = -2 + 2t$. The units are millions of bacteria per hour.

c. $P'(t) = -2 + 4t$, which is equal to $a'(t) + b'(t) = 2t - 2 + 2t$.

d. The population of a is always growing, the population of b shrinks before it grows, and the whole population also shrinks before it grows.

e.



39. The speed is $M'(t) = 10 - at = 10 - 9.78t$. The critical point is then when $t = 1.02$ s. Plugging in $M(1.02) = 105.11$. It hits the ground when $M(t) = 0$, or at $t = 5.66$. The speed is $M'(5.66) = 45.34$ m/s.

41. The speed is $M'(t) = 10 - at = 10 - 22.88t$. The critical point is then when $t = 0.44$ s. Plugging in $M(0.44) = 102.18$. It hits the ground when $M(t) = 0$, or at $t = 3.42$. The speed is $M'(3.42) = 68.38$ m/s.

43. $S(t) = 3t^2$ works. $S(1) = 3$ and $S(2) = 12$. That seems pretty fast.

45. $S(t) = t^6$ because $\frac{dS}{dt} = 6t^5 = 6\frac{S(t)}{t}$. $S(1) = 1$ and $S(2) = 64$. That is quite fast, because the derivative gets bigger the larger the organism is.

Section 2.6, page 192

1. $f'(x) = 2 \cdot (-3x + 2) + (-3) \cdot (2x + 3) = -12x - 5$.

3. $r'(y) = 5(y^2 - 1) + 2y(5y - 3)$.

5.

$$\begin{aligned}\frac{dh}{dx} &= \frac{d(x+2)}{dx}(2x+3)(-3x+2) \\ &\quad + (x+2)\frac{d(2x+3)(-3x+2)}{dx} \\ &= 1 \cdot (2x+3)(-3x+2) + (x+2) \cdot (-12x - 5) \\ &= (2x+3)(-3x+2) - (x+2)(12x+5)\end{aligned}$$

7. Set $u(x) = 1 + x$ and $v(x) = 2 + x$. Then $u'(x) = 1$ and $v'(x) = 2$. By the quotient rule,

$$f'(x) = \frac{(2+x) \cdot 1 - (1+x) \cdot 2}{(2+x)^2} = \frac{1}{(2+x)^2}$$

9. $g'(z) = \frac{-2z(-1 + 3z + z^3)}{(1 + 2z^3)^2}$.

11. Set $u(x) = 1 + x$ and $v(x) = (2 + x)(3 + x)$. Then $u'(x) = 1$ and $v'(x) = (2 + x) + (3 + x) = 5 + 2x$, by the product rule. Applying the quotient rule gives

$$\begin{aligned}F'(x) &= \frac{(2+x)(3+x) \cdot 1 - (1+x)(5+2x)}{(2+x)^2(3+x)^2} \\ &= \frac{1 - 2x - x^2}{(2+x)^2(3+x)^2}\end{aligned}$$

13. $\Delta f = 0.2$, $\Delta g = -0.3$, and $\Delta(fg) = -1.76$. This is equal to $g(x_0)\Delta f + f(x_0)\Delta g + \Delta f \Delta g = -0.2 - 1.5 - 0.06$. With $\Delta x = 0.01$, $\Delta f = 0.02$, $\Delta g = -0.03$, and $\Delta(fg) = -0.1706$. This is equal to $g(x_0)\Delta f + f(x_0)\Delta g + \Delta f \Delta g = -0.02 - 0.15 - 0.0006$. The last term is now much smaller than the rest.

15. Would get that

$$\frac{dx \cdot x^2}{dx} = 1 \cdot 2x$$

which is wrong.

17.

$$\frac{df(x) \cdot f(x)}{dx} = f'(x)f(x) + f'(x)f(x) = 2f'(x)f(x) > 0$$

because $f'(x) > 0$ and $f(x) > 0$. Therefore, $f(x)^2$ is increasing.

19. The power rule works for $n = 1$ because $x^1 = x$ is a linear function with derivative $1 = 1x^0$.

21. $\frac{dx^3}{dx} = x^2 \cdot \frac{dx}{dx} + \frac{dx^2}{dx} \cdot x = x^2 + 2x^2 = 3x^2$. It worked.

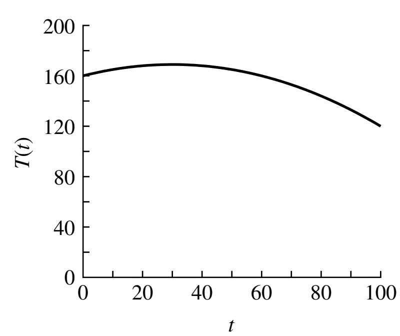
23. We denote the total mass by $T(t)$.

a. $T(t) = P(t)W(T) = (2.0 \times 10^6 + 2.0 \times 10^4 t)(80 - 0.5t)$.

b. $T'(t) = 6.0 \times 10^5 - 2.0 \times 10^4 t$.

c. $T'(t) = 0$ when $6.0 \times 10^5 - 2.0 \times 10^4 t = 0$ or when $t = 30$ yr. At time 30, the population is 2.6×10^6, the weight per person is 65 kg, and the total weight is 1.69×10^8 kg.

d.



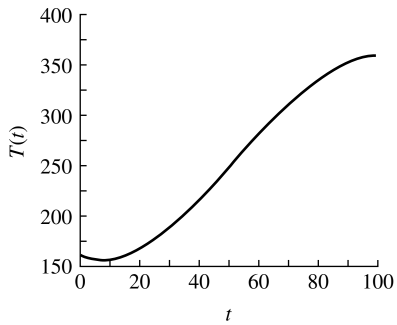
25. We denote the total mass by $T(t)$.

a. $T(t) = P(t)W(T) = (2.0 \times 10^6 + 1.0 \times 10^3 t^2)(80 - 0.5t)$.

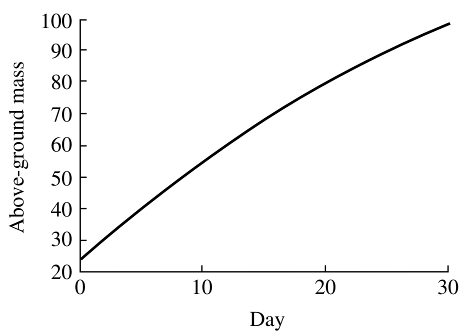
b. $T'(t) = 1.0 \times 10^6 + 1.6 \times 10^5 t - 1.5 \times 10^3 t^2$.

c. $T'(t) = 0$ at $t \approx 6.67$ and $t = 100.0$. At time 6.67, the population is 2.04×10^6, the weight per person is 76.67 kg, and the total weight is 1.56×10^8 kg. At time 100, the population is 1.2×10^7, the weight per person is 30.0 kg, and the total weight is 3.6×10^8 kg.

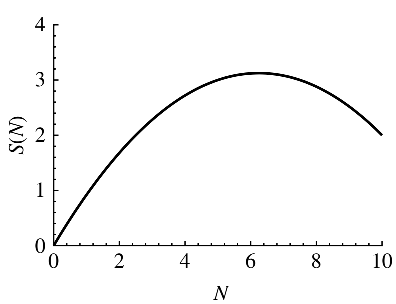
d.



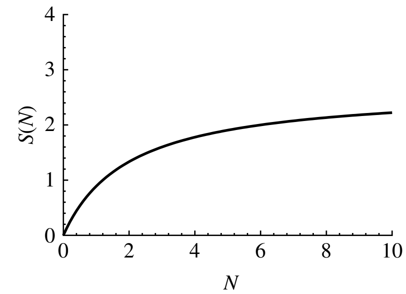
27. Total mass above ground is $(3.0t + 20)(1.2 - 0.01t)$. The derivative is $-0.06t + 3.4$. This switches from positive to negative at $t \approx 56.7$, and thus the above-ground mass increases during the entire first 30 days.



29. $S(1) = 0.92$, $S(5) = 3.0$, $S(10) = 2.0$. $S'(N) = 1 - 0.16N$. This bird does best by laying about 6 eggs.



31. $S(1) = 0.67$, $S(5) = 1.43$, $S(10) = 1.67$. $S'(N) = \dfrac{1}{(1 + 0.5N)^2}$ which is always positive. This bird does best by laying as many eggs as it can.

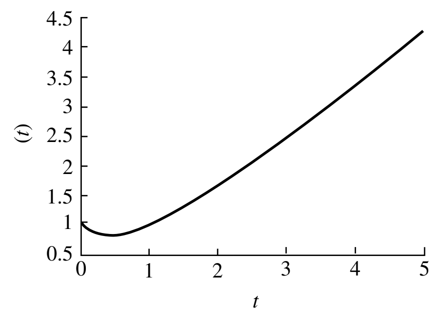


33. This simplifies to $\dfrac{2.4}{(1.2p + 2.0(1 - p))^2}$.

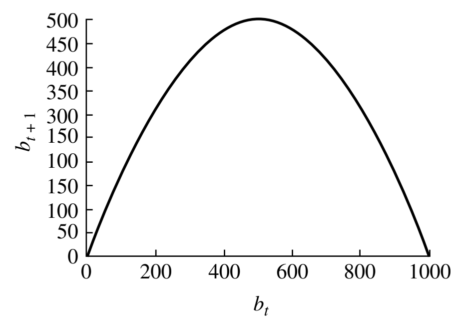
35. a. $\rho(t) = \dfrac{1 + t^2}{1 + t}$.

b. $\rho'(t) = \dfrac{t^2 + 2t - 1}{1 + t^2}$.

c. This is positive when $t^2 + 2t - 1 > 0$. This occurs for t larger than the solution of $t^2 + 2t - 1 = 0$, which can be found with the quadratic formula to be 0.414. After that, the density increases.



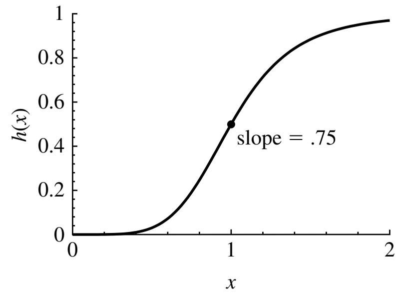
37. The final population is the product of the per capita reproduction and the initial population b_t, or $b_{t+1} = 2.0\left(1 - \dfrac{b_t}{1000}\right) b_t$. The derivative of $f(b) = 2.0b\left(1 - \dfrac{b}{1000}\right)$ is $f'(b) = 2.0 - \dfrac{4.0b}{1000}$, which is positive for $b < 500$ and negative for $b > 500$.



39. a. $h_3(0) = 0$, $h_3(1) = 0.5$, $h_3(2) \approx 0.89$.

b. $h_3'(x) = \dfrac{3x^2}{(1 + x^3)^2}$. $h_3'(0) = 0$ $h_3'(1) = 0.75$, $h_3'(2) \approx 0.148$.

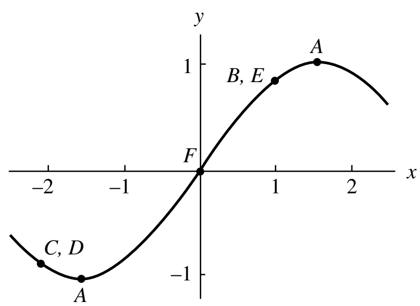
c.



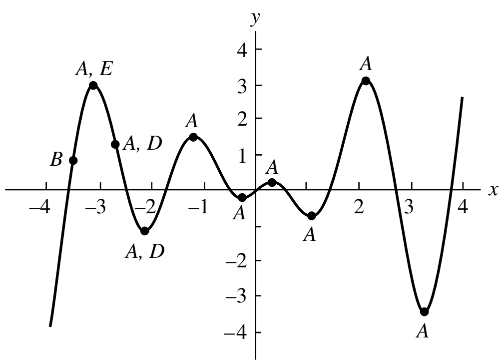
d. This is kind of a mushy response.

Section 2.7, page 201

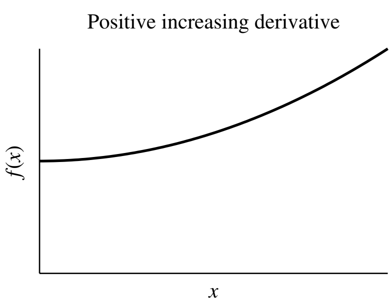
1.



3.



5.



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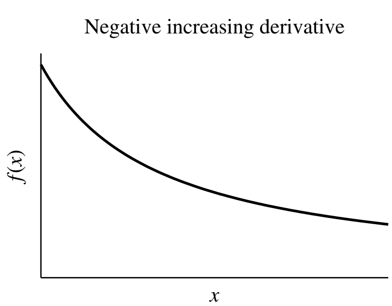


9. The point of inflection $x = 0$ has a negative third derivative because the second derivative changes from positive to negative values and is therefore decreasing.

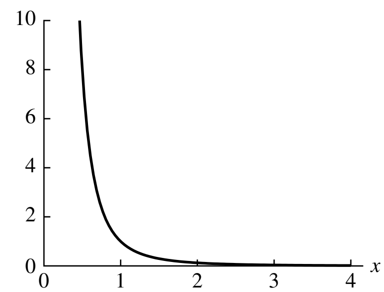
11. $s'(x) = -1 + 2x - 3x^2 + 4x^3$. $s''(x) = 2 - 6x + 12x^2$.

13. $h'(y) = 10y^9 - 9y^8$. $h''(y) = 90y^8 - 72y^7$.

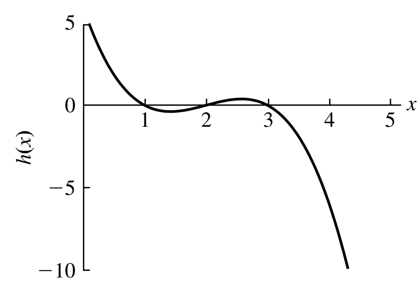
15. $F'(z) = (1+z)(2+z) + z(2+z) + z(1+z)$. $F''(z) = (1 + z) + (2+z) + z + (2+z) + z + (1+z) = 6z + 6$.

17. $f'(x) = \dfrac{2x - 2(3+x)}{4x^2} = \dfrac{-3}{2x^2}$. $f''(x) = \dfrac{-3}{x^3}$.

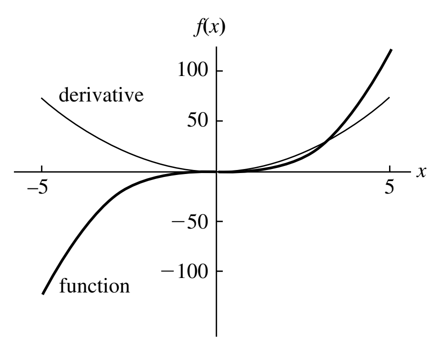
19. $f'(x) = -3x^{-4} < 0$, so the function is decreasing, $f''(x) = 12x^{-5} > 0$, so the function is always concave up.



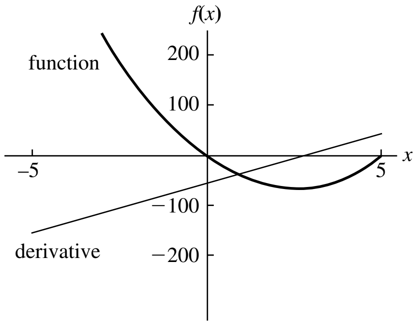
21. $h'(x) = -3x^2 + 12x - 11$, which has solutions at 2.577 and 1.422. $h''(x) = -6x + 12$, so the function is concave up for $x < 2$ and concave down for $x > 2$.



23. $f'(x) = 6x^2$. The first derivative is always positive. $f''(x) = 12x$. The second derivative is positive when $x > 0$ and negative when $x < 0$, producing a point of inflection at $x = 0$.



25. $f'(x) = 20x - 50$. The first derivative is positive when $x > 2.5$ and negative when $x < 2.5$. $f''(x) = 20$. The second derivative is always positive.



27. It is 0, because the degree will have been reduced to 0.

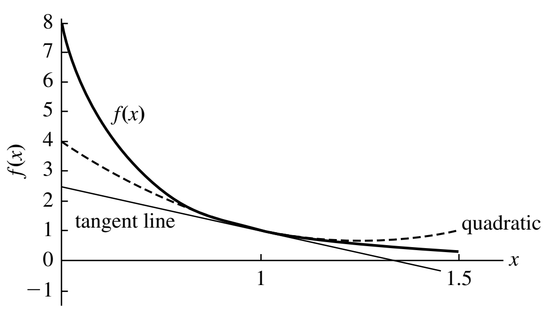
29. Positive.

31. The derivative is $f'(x) = -3x^{-4}$ and the second derivative is $f''(x) = 12x^{-5}$.

a. Because $f(1) = 1$ and $f'(1) = -3$, the tangent line is $\hat{f}(x) = 1 - 3(x - 1)$.

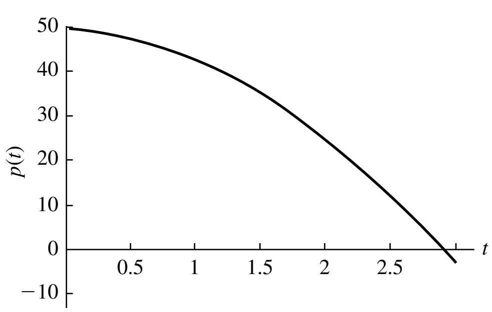
b. Because $f''(1) = 12$, the quadratic $\hat{\hat{f}}(x) = 1 - 3(x - 1) + 6(x - 1)^2$ matches both the first derivative and the second derivative at $x = 1$.

c.



33. a. Velocity is $p'(t) = -10.4t - 2.0$. Acceleration is $p''(t) = -10.4$.

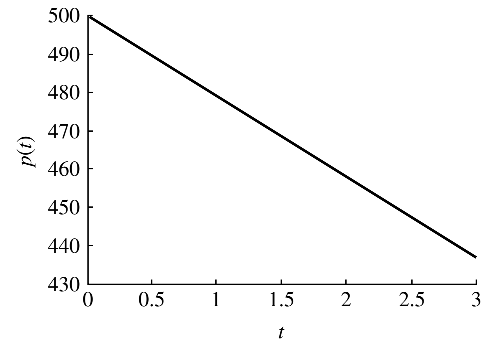
b. The position at $t = 3$ is less than 0, so the object has already hit the ground.



c. The tower was 50.0 m high, and the object was thrown downward at 2.0 m/s. The acceleration of gravity on Saturn is only slightly greater than that on Earth, probably because of Saturn's low density.

35. a. Velocity is $p'(t) = -0.65t - 20.0$. Acceleration is $p''(t) = -0.65$.

b.

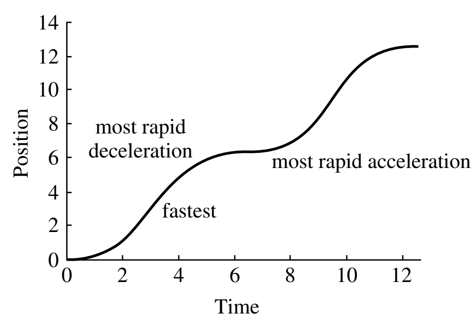


c. The tower was 500.0 m high, and the object was thrown downward at 20.0 m/s. The acceleration of gravity on Pluto is tiny, so the object is falling only slightly faster after 3 s than it was when it was thrown.

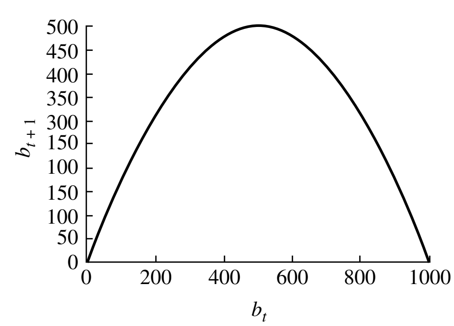
37. The second derivative is -20000, matching the graph, which is always concave down.

39. The second derivative is $160000 - 3000t$, which is positive for $t \leq 53.33$ and negative thereafter. This matches a graph that switches from being concave up to being concave down.

41.



43. The derivative of $f(b) = 2.0b\left(1 - \frac{b}{1000}\right)$ is $f'(b) = 2.0 - \frac{4.0b}{1000}$, and the second derivative is $-\frac{4.0}{1000}$, which is always negative. The graph is always concave down.

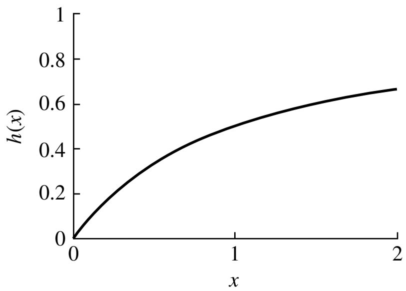


45. Using the quotient rule, we find that the first derivative is

$$h'(x) = \frac{1}{(1 + x)^2}$$

The second derivative can also be found with the quotient rule and product rule as

$$h''(x) = \frac{(1+x)^2 \cdot 0 - 1 \cdot ((1+x)+(1+x))}{(1+x)^4} = \frac{-2}{(1+x)^3}$$



This is always negative, and thus the function is concave down.

Section 2.8, page 209

1. $F'(x) = 2x + 4e^x$. $F''(x) = 2 + 4e^x$.

3. $f'(x) = (x^2 + 2x)e^x$. $f''(x) = (x^2 + 4x + 2)e^x$.

5. $g'(x) = \frac{e^x(x-1)}{x^2}$. $g''(x) = \frac{e^x(x^2 - 2x + 2)}{x^3}$.

7. $f'(x) = \frac{-x}{e^x}$. $f''(x) = \frac{x-1}{e^x}$.

9. $f'(x) = 1 + \frac{4}{x}$. $f''(x) = \frac{-4}{x^2}$.

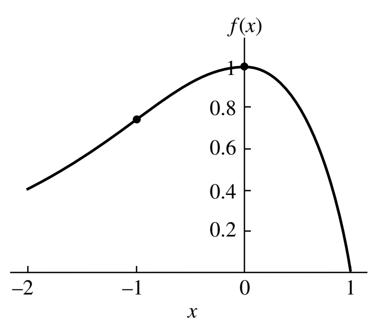
11. $g'(z) = \ln(z) + 1 + \frac{4}{z}$. $g''(z) = \frac{1}{z} - \frac{4}{z^2}$.

13. $F'(w) = e^w \ln(w) + \frac{e^w}{w}$. $F''(w) = e^w \ln(w) + 2\frac{e^w}{w} - \frac{e^w}{w^2}$.

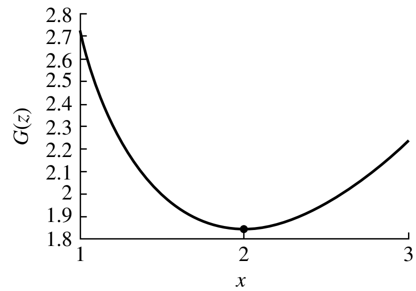
15. $\ln(x^2) = 2\ln(x)$, so $s'(x) = \frac{2}{x}$. $s''(x) = \frac{-2}{x^2}$.

17. $F'(z) = \frac{-(1+e^{-z})}{1+e^z}$.

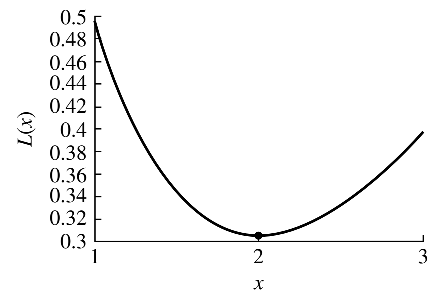
19. $f'(x) = -xe^x$, which is positive if $x < 0$ and negative if $x > 0$. $f''(x) = -(1+x)e^x$, which is positive if $x < -1$ and negative if $x > -1$. This function is increasing when $x < 0$ and is concave up when $x < -1$.



21. $G'(z) = \frac{e^z(z-2)}{z^3}$, which is negative if $z < 2$ and positive if $z > 2$. $G''(z) = \frac{e^z(z^2 - 4z + 6)}{z^4}$, which is always positive. This function is increasing when $z > 2$ and is always concave up.



23. $L'(x) = \frac{1}{2} - \frac{1}{x}$, which is negative if $x < 2$ and positive if $x > 2$. $L''(x) = \frac{1}{x^2}$, which is always positive. This function is increasing when $x > 2$ and is always concave up.



25. a.

$$\frac{df}{dx} = \lim_{h\to 0} \frac{5^{x+h} - 5^x}{h}$$

b.

$$\begin{aligned}\frac{df}{dx} &= \lim_{h\to 0} \frac{5^x 5^h - 5^x}{h} \\ &= \lim_{h\to 0} \frac{5^x(5^h - 1)}{h} \\ &= 5^x \left(\lim_{h\to 0} \frac{5^h - 1}{h}\right)\end{aligned}$$

c. For $h = 1$, $(5^1 - 5^0)/1 = 4.0$. For $h = 0.1$, $(5^{0.1} - 5^0)/0.1 = 1.746$. For $h = 0.01$, $(5^{0.01} - 5^0)/0.01 = 1.622$. For $h = 0.001$, $(5^{0.001} - 5^0)/0.001 = 1.611$. For $h = 0.0001$, $(5^{0.0001} - 5^0)/0.0001 = 1.609$.

d. $e^{1.609} = 4.998$, which is suspiciously close to 5.

27. $h'(x) = (x+2)e^x$, $h''(x) = (x+3)e^x$.

29. $h(x) = (x-1)e^x$ seems to work. $h'(x) = xe^x$, so the critical point is at $x = 0$. $h''(x) = (x+1)e^x$, so the point of inflection is at $x = -1$.

31. a. The definition is

$$\frac{d\ln(x)}{dx} = \lim_{h\to 0} \frac{\ln(1+h) - \ln(1)}{h} = \lim_{h\to 0} \frac{\ln(1+h)}{h}$$

b. The limit seems to be 1.0, meaning that the derivative is 1.0.

c. The derivative of $\ln(x)$ is $\frac{1}{x}$, which is 1.0 at $x = 1$.

33. The definition is

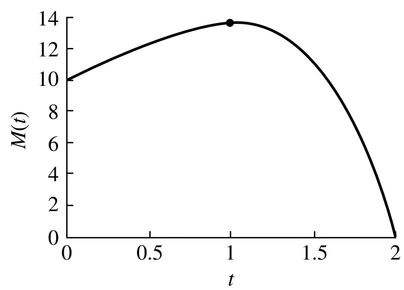
$$\frac{d\ln(x)}{dx} = \lim_{h\to 0} \frac{\ln(2+h) - \ln(2)}{h} = \lim_{h\to 0} \frac{\ln(1+h/2)}{h}$$

If we write $\Delta x = h/2$, then $h = 2\Delta x$ and the limit can be written

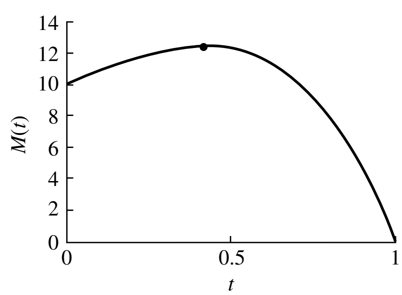
$$\lim_{h\to 0}\frac{\ln(1+h/2)}{h} = \lim_{\Delta x\to 0}\frac{\ln(1+\Delta x)}{2\Delta x} = \frac{1}{2}\lim_{\Delta x\to 0}\frac{\ln(1+\Delta x)}{\Delta x} = \frac{1}{2}$$

where we used the limit in Exercise 31.

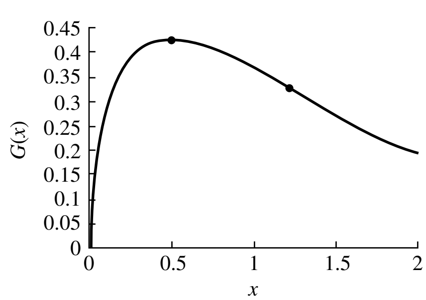
35. The total mass is $M(t) = 10\left(1-\frac{t}{2}\right)e^t$. $M'(t) = \frac{1-t}{2}10e^t = 5(1-t)e^t$, which is negative for $t > 1$, and $M''(t) = -5te^t$, which is always negative. The total mass is greater than the initial value at $t = 0$. It reaches zero at $t = 2$.



37. The total mass is $M(t) = 10(1-t^2)e^t$. $M'(t) = 10(1-2t-t^2)e^t$, which is negative for $t > \sqrt{2}-1$, and $M''(t) = 10(-1-4t-t^2)e^t$, which is always negative. The total mass is greater than the initial value at $t = \sqrt{2}-1$. It reaches zero at $t = 1$.



39. $G'(x) = \frac{1-2x}{2\sqrt{x}}e^{-x}$, which is positive for $x < 1/2$ and negative for $x > 1/2$. It also says that the curve is infinitely steep at $x = 0$. $G''(x) = \frac{4x^2-4x-1}{4x^{3/2}}e^{-x}$. Solving $4x^2-4x-1 = 0$ gives points of inflection at $\frac{1\pm\sqrt{2}}{2}$. Only the larger one is positive. Below this, the function is concave down, and above it, it is concave up.



41. This equation says that the rate of change of the population increases exponentially over time. The derivative of $b(t) = e^t$ is e^t, so it works. This solution is increasing.

43. This equation says that the rate of change of the population is equal to the negative of population size, meaning that it shrinks faster the larger it is. The derivative of $b(t) = e^{-t}$ is $-e^{-t}$, which is indeed the negative of $b(t)$. This solution is decreasing.

Section 2.9, page 220

1. $g(x) = f(h(x))$ where $h(x) = 1+3x$ and $f(h) = h^2$. Then

$$g'(x) = f'(h(x))h'(x) = 2h\cdot 3 = 6(1+3x)$$

3. $f_1(t) = h(g(t))$ where $g(t) = 1+3t$ and $h(g) = g^{30}$. Then $g'(t) = 3$ and $h'(g) = 30g^{29}$. Therefore, $f_1'(t) = 3\cdot 30g^{29} = 90(1+t)^{29}$.

5. This is a quotient of two compositions. The numerator is $(1+3x)^2$ with derivative $6(1+3x)$. The denominator is $(1+2x)^3$ with derivative $6(1+2x)^2$. By the quotient rule, the derivative is $r'(x) = \frac{6(1+2x)^3(1+3x) - 6(1+3x)^2(1+2x)^2}{(1+2x)^6} = \frac{-6x(1+3x)}{(1+2x)^4}$.

7. $F(z) = g(h(z))$ where $h(z) = 1+\frac{2}{1+z}$ and $g(h) = h^3$. $h'(z) = -2(1+z)^{-2}$, found using the chain rule. Then

$$F'(z) = g'(h(z))h'(z) = 3h^2\cdot\left(-2(1+z)^{-2}\right) = -6\frac{\left(1+\frac{2}{1+z}\right)^2}{(1+z)^2}$$

9. Set $f(x) = g(h(x))$ where $g(h) = e^h$ and $h(x) = -3x$. Then

$$f'(x) = g'(h(x))h'(x) = e^{h(x)}\cdot(-3) = -3e^{-3x}$$

11. $g'(y) = \frac{1}{1+y}$.

13. $G'(x) = 16xe^{x^2}$.

15. $L'(x) = \frac{1}{x\ln(x)}$.

17. With the quotient rule, set $u(x) = 1$ and $v(x) = 1+e^x$. Then

$$H'(x) = \left(\frac{u}{v}\right)'(x) = \frac{v(x)u'(x) - u(x)v'(x)}{v^2(x)} = \frac{(1+e^x)\cdot 0 - 1\cdot e^x}{(1+e^x)^2} = -\frac{e^x}{(1+e^x)^2}$$

With the chain rule, set $H(x) = r(p(x))$ where $p(x) = 1+e^x$ and $r(p) = 1/p$. Then $p'(x) = e^x$ and $r'(p) = \frac{-1}{p^2}$. By the chain rule,

$$H'(x) = r'(p)p'(x) = \frac{-1}{p^2}\cdot e^x = \frac{-e^x}{(1+e^x)^2}$$

19. With law 1 of logs, $\ln(3x) = \ln(3) + \ln(x)$. Then

$$\frac{d\ln(3x)}{dx} = \frac{d\ln(3)}{dx} + \frac{d\ln(x)}{dx} = 0 + \frac{1}{x} = \frac{1}{x}$$

With the chain rule, set $f(x) = 3x$ and $l(f) = \ln(f)$. Then

$$(l \circ f)'(x) = l'(f(x))f'(x) = \frac{1}{f}\cdot 3 = \frac{1}{3x}\cdot 3 = \frac{1}{x}$$

The two answers check.

21. $F(x) = 1 + 4x + 4x^2$ and $F'(x) = 4 + 8x$. With the chain rule, we think of F as the composition $F(x) = g(h(x))$ where $h(x) = 1 + 2x$ and $g(h) = h^2$. Then $h'(x) = 2$ and $g'(h) = 2h$, so $F'(x) = g'(h)h'(x) = 2h\cdot 2 = 4(1 + 2x)$. Multiplying out, we find that this matches the result found directly.

23. $F'(x) = 3x^2$. We can write $F(x) = e^{3\ln(x)}$, a composition $f(g(x))$ where $f(g) = e^g$ and $g(x) = 3\ln(x)$. Then $f'(g) = e^g$ and $g'(x) = 3/x$, so

$$F'(x) = f'(g)g'(x) = e^g\cdot\frac{3}{x} = e^{3\ln(x)}\frac{3}{x} = x^3\frac{3}{x} = 3x^2$$

25. The inverse is $f^{-1}(y) = \frac{y-1}{3}$ with derivative $(f^{-1})'(y) = 1/3$. Using the theorem, $f'(x) = 3$ for any x, so $f'(f^{-1}(y)) = 3$ and $(f^{-1})'(y) = 1/3$.

27. The inverse is $h^{-1}(y) = (y-2)^{1/3}$ with derivative $(h^{-1})'(y) = \frac{1}{3}(y-2)^{-2/3}$. Using the theorem, $h'(x) = 3x^2$, so $h'(h^{-1}(y)) = 3(y-2)^{2/3}$ and $(h^{-1})'(y) = \frac{1}{3}(y-2)^{-2/3}$.

29. The inverse is $q^{-1}(y) = \frac{-1+\sqrt{4+y}}{2}$. $(q^{-1})'(y) = \frac{1}{\sqrt{4+y}}$. Using the theorem, $q'(x) = 1 + 2x$. So $q'(q^{-1}(y)) = \sqrt{4+y}$ and $(q^{-1})'(y) = \frac{1}{\sqrt{4+y}}$.

31. $x^n = e^{n\ln(x)}$.

33. $f'(x) = \frac{-x}{\sqrt{1-x^2}}$.

35. $B(M) = 0.5M$, so $B'(M) = 0.5$. $M(W) = 5W + 2$, so $M'(W) = 5$. Therefore, the derivative of the composition is $B'(M)M'(W) = 2.5$.

37. $F'(V) = 0.4$ and $V'(I) = 10I$, so the derivative of the composition is $F'(V)V'(I) = 4I$.

39. $Q'(t) = -0.000122\cdot 6.0\times 10^{10}e^{-0.000122t} = -7.32\times 10^6 e^{-0.000122t}$.

41. The derivative is $\frac{db}{dt} = 300e^{3t} = 3\cdot 100e^{3t} = 3b(t)$. This solution is increasing.

43. The derivative is $\frac{db}{dt} = 6e^{2t} = 1 + 2(3e^{2t} - 0.5) = 1 + 2b(t)$. This solution is increasing.

Section 2.10, page 230

1. $f'(x) = 2x\sin(x) + x^2\cos(x)$.

3. $h'(x) = \cos^2(\theta) - \sin^2(\theta)$.

5. $F'(z) = -2\sin(2z - 1)$.

7. $f'(x) = -\sin(x)e^{\cos(x)}$.

9. As in the text, we can use the fact that $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$ to find that the derivative is $\sec^2(\theta) = \cos(\theta)^{-2}$. With the chain rule, the second derivative is $\frac{2\sin(\theta)}{\cos(\theta)^3}$, which can be written $2\tan(\theta)\sec(\theta)^2$.

11. The first derivative is found with the chain rule.

$$\frac{d\sec(\theta)}{d\theta} = \frac{d\cos(\theta)^{-1}}{d\theta} = \sin(\theta)\cos(\theta)^{-2}$$

This can be rewritten as $\sec(\theta)\tan(\theta)$. We can find the second derivative with the product rule as $\sec(\theta)\tan(\theta)^2 + \sec(\theta)^3$.

13. The double angle formula is $\cos(2\theta) = \cos(\theta)\cos(\theta) - \sin(\theta)\sin(\theta)$. Taking the derivative yields

$$\begin{aligned}\frac{d}{d\theta}\cos(2\theta) &= -\sin(\theta)\cos(\theta) - \sin(\theta)\cos(\theta)\\ &\quad -\sin(\theta)\cos(\theta) - \sin(\theta)\cos(\theta)\\ &= -4\sin(\theta)\cos(\theta) = -2\sin(2\theta)\end{aligned}$$

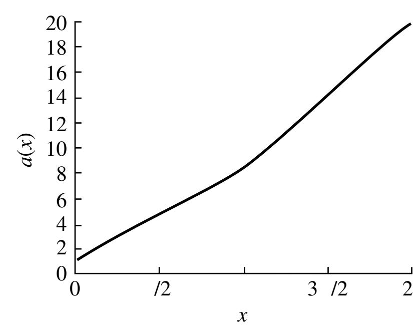
This is just what the chain rule gives.

15. The angle addition formula says that $\cos(\theta + \phi) = \cos(\theta)\cos(\phi) - \sin(\theta)\sin(\phi)$. Taking the derivative yields

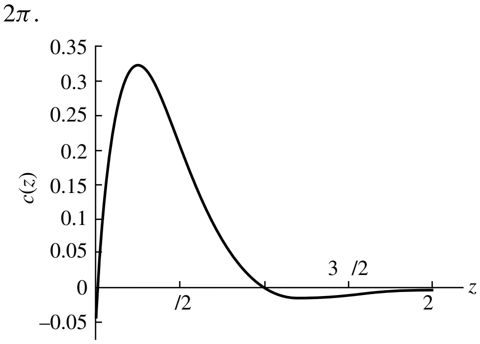
$$\begin{aligned}\frac{d}{d\theta}\cos(\theta+\phi) &= -\sin(\theta)\cos(\phi) - \cos(\theta)\sin(\phi)\\ &= -\sin(\theta+\phi)\end{aligned}$$

This matches the result with the chain rule.

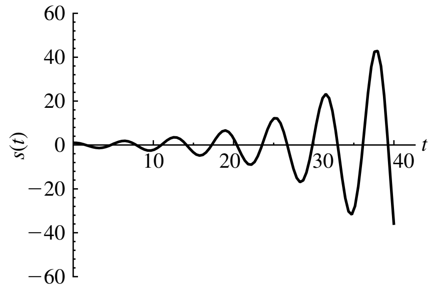
17. $a'(x) = 3 - \sin(x)$. $a(0) = 1$, $a'(0) = 3$, $a(\pi/2) = 3\pi/2$, $a'(\pi/2) = 2$, $a(\pi) = 3\pi - 1$, $a'(\pi) = 3$. The function is increasing, but with a slight slowing at around $x = \pi$.



19. $c'(z) = (\cos(z) - \sin(z))/e^z$. $c(0) = 0$, $c'(0) = 1$, $c(\pi/2) = e^{-\pi/2}$, $c'(\pi/2) = -e^{-\pi/2}$, $c(\pi) = 0$, $c'(\pi) = -e^{-\pi}$. The function zips up to a maximum at around $x = \pi/2$, dips down to zero at $x = \pi$, becomes negative, and then returns to zero at $x = 2\pi$.



21. $s'(t) = 0.2e^{0.2t}\cos(t) - e^{0.2t}\sin(t)$. $s(0) = 1$, $s'(0) = 0.2$, $s(\pi/2) = 0$, $s'(\pi/2) = -1.369$, $s(\pi) = -1.874$, $s'(\pi) = -0.375$. This oscillation expands exponentially.



23. The derivative of $\sin(x)$ is $\cos(x)$. Therefore,

$$(\sin^{-1})'(x) = \frac{1}{\cos(\sin^{-1}(x))}$$

Using the identity, $\cos(x) = \sqrt{1 - \sin^2(x)}$, so

$$\cos(\sin^{-1}(x)) = \sqrt{1 - \sin^2(\sin^{-1}(x))} = \sqrt{1 - x^2}$$

Therefore, the derivative is $(\sin^{-1})'(x) = \frac{1}{\sqrt{1 - x^2}}$.

25. The derivative of $\tan(x)$ is $\sec^2(x)$. Therefore,

$$(\tan^{-1})'(x) = \frac{1}{\sec^2(\tan^{-1}(x))}$$

Using the identity,

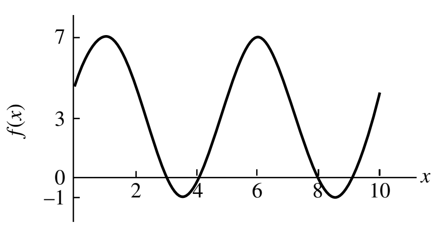
$$\sec^2(\tan^{-1}(x)) = 1 + \tan^2(\tan^{-1}(x)) = 1 + x^2$$

Therefore, the derivative is $(\tan^{-1})'(x) = \frac{1}{1 + x^2}$.

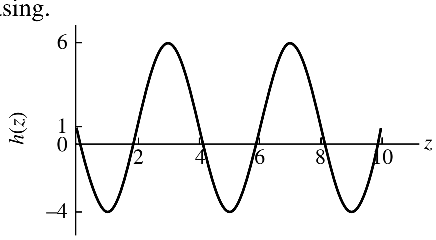
27. The derivative is $s'(t) = \cos(t)$.

29. The fourth derivative of each piece repeats.

31. $f'(x) = -\frac{8.0\pi}{5.0}\sin(2\pi\frac{x - 1.0}{5.0})$. $f'(0) = -\frac{8.0\pi}{5.0}\sin(-2\pi/5) > 0$, which is consistent with the fact that this oscillation is increasing at $x = 0$.



33. $h'(z) = -\frac{5.0\pi}{2.0}\sin(2\pi\frac{z - 3.0}{4.0})$. $h'(0) = -\frac{5.0\pi}{2.0}\sin(-3\pi/2) < 0$, which is consistent with the fact that the curve begins by decreasing.



35.

$$p'(t) = -\frac{2\pi}{T}\sin\left(\frac{2\pi t}{T}\right), \quad p''(t) = -\left(\frac{2\pi}{T}\right)^2\cos\left(\frac{2\pi t}{T}\right)$$

We thus need

$$\left(\frac{2\pi}{T}\right)^2 = \frac{0.1}{1.0} = 0.1$$

This has solution $T = \sqrt{0.1}2\pi$. This weaker spring oscillates more slowly with a larger period.

37. The derivative of the sum is

$$-0.2 \cdot \frac{2\pi}{28}\sin\left(\frac{2\pi(t - 16)}{28}\right) - 0.3 \cdot 2\pi\sin(2\pi(t - 0.583))$$

39. This says that acceleration has two pieces: one proportional to the negative of displacement (the $-2x$ term) and one proportional to the negative of velocity (the $2\frac{dx}{dt}$ term).

$$\frac{dx}{dt} = -e^{-t}\cos(t) - e^{-t}\sin(t), \quad \frac{d^2x}{dt^2} = 2e^{-t}\sin(t)$$

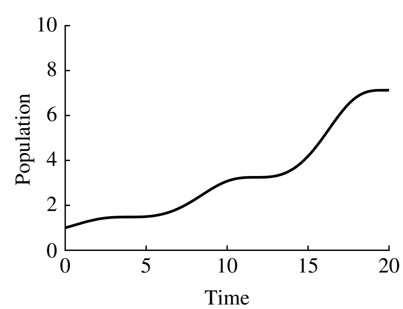
But

$$-2x - 2\frac{dx}{dt} = -2e^{-t}\cos(t) - 2(-e^{-t}\cos(t) - e^{-t}\sin(t))$$
$$= 2e^{-t}\sin(t) = \frac{d^2x}{dt^2}$$

This is indeed a solution.

41. This population is growing, but per capita production oscillates.

43.



45. $b'(t) = (0.1 + 0.1A\cos(0.8t))b(t) = 0.1(1 + A\cos(0.8t))b(t)$.

47. a. $t = \pi$, June 21, 16.5 h.

b. $t = \pi/2$, March 21, 12.0 h.

c. $L'(t) = 4.5\sin(t)$ equal to 0 at $t = 0$ and $t = \pi$, when days are shortest or longest.

d. The day length changes most rapidly at $t = \pi/2$ and $t = 3\pi/2$, or on March 21 and September 21. This should be easiest for the plant to detect.

Supplementary Problems, page 233

1. We can substitute because the function is continuous. The limit is 1.

3. This increases to infinity.

5. This increases to infinity.

7. $F'(y) = 4y^3 + 10y$.

9. $H'(c) = \dfrac{2c(1+c)}{(1+2c)^2}$ for $c \neq -1/2$.

11. $b'(y) = -0.75y^{-1.75}$ for $y \geq 0$.

13. $g'(x) = 60x(4 + 5x^2)^5$.

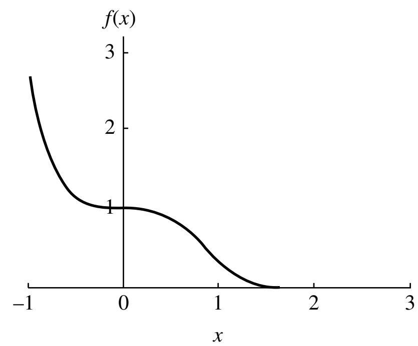
15. $s'(t) = 3/t$.

17. $s'(x) = -3e^{-3x+1} + 5/x$.

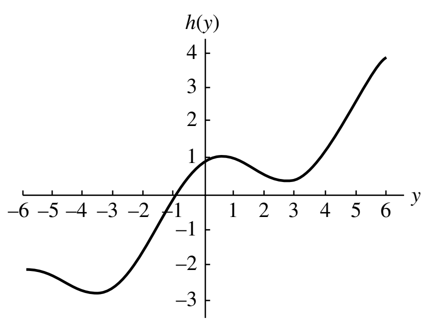
19. $f'(t) = (\cos(t) - \sin(t))e^t$. The point $t = \pi/4$ is a critical point.

21. $h'(y) = \dfrac{2(y-2)}{(1+y)^4}$. $h(y)$ is increasing when $y > 2$.

23. $f'(x) = -3x^2 e^{-x^3}$, which is 0 at $x = 0$. $f''(x) = (9x^4 - 6x)e^{-x^3}$, which is 0 at $x = 0$ and $x \approx 0.873$.

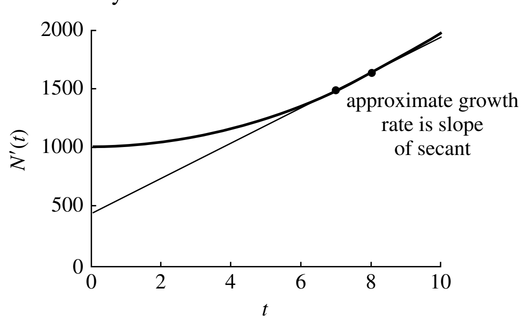


25. $h'(y) = -\sin(y) + \dfrac{1}{2}$, which is 0 at $y = \dfrac{\pi}{6} + 2n\pi$ and $y = \dfrac{t\pi}{6} + 2n\pi$ for any integer n. $h''(y) = -\cos(y)$, which is 0 at $y = \dfrac{\pi}{2} + n\pi$ for any integer n.



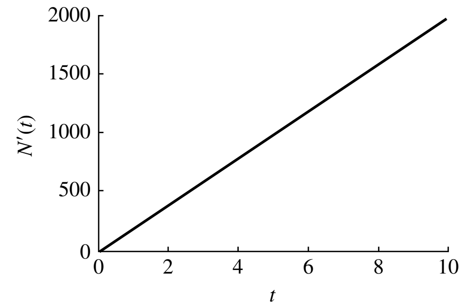
27. a. The 1000 has units of individuals, and the 10 has units of individuals/yr^2.

b.



c. The population is 1490 after 7 yr and 1640 after 8 yr. The approximate growth rate is 150 during this year.

d. The derivative is $20t$.

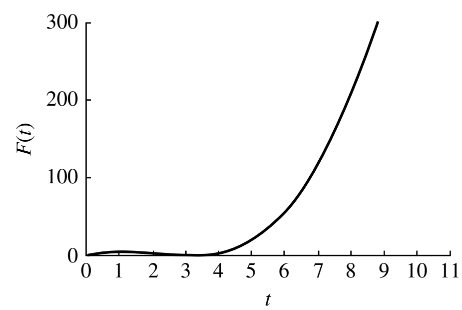


e. The per capita rate of growth is $\dfrac{20t}{(1000 + 10t^2)}$. It starts at 0 and increases for a while. Eventually, however, it will decrease.

29. a. $F'(t) = 3t^2 - 12t + 9$ is the derivative, with units of factoids per week.

b. With the quadratic formula, we find that $F'(t) = 0$ when $t = 1$ or $t = 3$. Because $F'(t)$ is a parabola that goes up, it must be negative between times 1 and 3. This is the time when knowledge is being lost.

c.

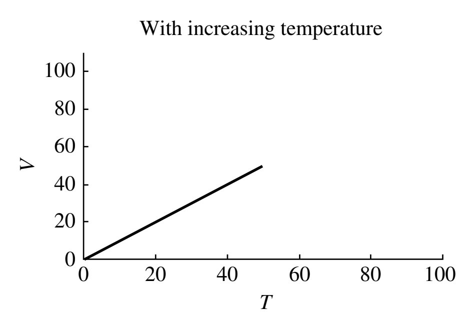


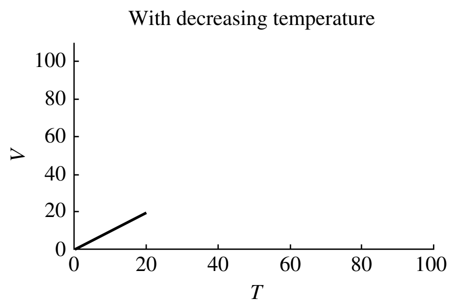
31. a. $p_0 = 0.333$.

b. p_0 would have to be between 0.243 and 0.428, or within about 0.1.

c. It would get smaller and smaller also.

33. a.

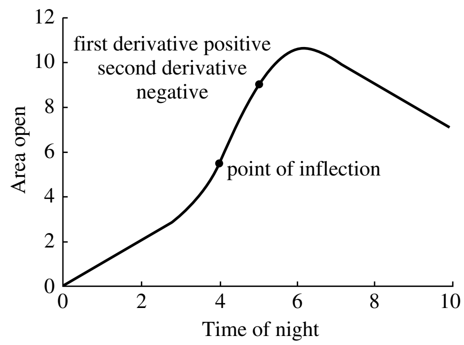




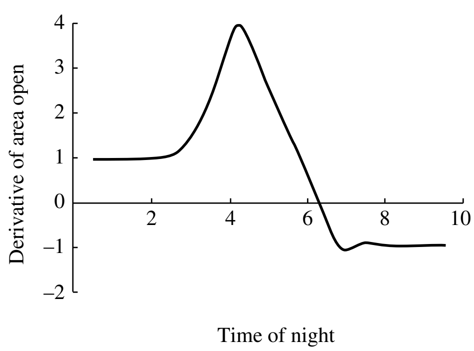
b. $\lim_{T\to 50^+} V_i(T) = 100$. $\lim_{T\to 50^-} V_i(T) = 50$. $\lim_{T\to 50^+} V_d(T) = \lim_{T\to 50^-} V_d(T) = 100$.

c. $\lim_{T\to 20^+} V_i(T) = \lim_{T\to 20^-} V_i(T) = 20$. $\lim_{T\to 20^+} V_d(T) = 100$. $\lim_{T\to 20^-} V_d(T) = 20$.

35. a.



b.



c.