

Math 155 Prerequisite Review Handout

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Contents

1	Basic Mathematical Operations	2
1.1	Examples	2
1.2	Exercises	3
2	Variables, Parameters, and Functions	4
2.1	Examples	4
2.2	Exercises	5
3	Manipulating Expressions that Contain Variables	6
3.1	The Distributive Property and Division	6
3.2	A More Complicated Distribution Problem	7
3.3	Factoring	8
3.4	Exercises	9
4	Review Problems	11

1 Basic Mathematical Operations

Recall the basic operations of addition, subtraction, multiplication, and division. Consider evaluating the following expression:

$$2 + 3 \times 5$$

Do we add 2 and 3 first or do we multiply 3 and 5? Depending on which we do first we get different answers, 25 and 17 respectively. It is for this reason that we have the order of operations, recall that the basic mathematical operations are performed in the following order:

1. **P**arentheses - first perform operations that are in parentheses
2. **E**xponents
3. **M**ultiplication and **D**ivision
4. **A**ddition and **S**ubtraction

If an expression contains two operations of the same "rank" then the operations are performed from left to right. For example

$$20 \div 5 \times 3 = 4 \times 3 = 12$$

Try performing the multiplication operation first, do you get the same answer?

The order of operations described above can be easily remembered with the abbreviation **PEMDAS**, which you can remember by the simple phrase, "Please Excuse My Dear Aunt Sally." We finish this section with the following examples:

1.1 Examples

Example 1. *We simplify the following expression:*

$$\begin{aligned} &4 \times (2 + 5)^2 - 10 \\ &= 4 \times (7)^2 - 10 \\ &= 4 \times 49 - 10 \\ &= 196 - 10 \\ &= 186 \end{aligned} \tag{1}$$

Example 2. How do we simplify the following:

$$\frac{4+5}{3} + 4 \times 5$$

The "fraction bar" is treated as if everything in the numerator (top) and denominator (bottom) of the fraction is in parentheses so,

$$\frac{4+5}{3} + 4 \times 5 = \frac{(4+5)}{(3)} + 4 \times 5$$

Thus we simplify the expression as follows:

$$\begin{aligned} & \frac{4+5}{3} + 4 \times 5 \\ &= \frac{9}{3} + 4 \times 5 \\ &= 3 + 4 \times 5 \\ &= 3 + 20 \\ &= 23 \end{aligned} \tag{2}$$

1.2 Exercises

Directions: Simplify the following expressions.

1. $4 + 6 \times 11 - 18$

2. $(4 + 6) \times (11 - 18)$

3. $(10 - 9) \times 3 \times 5 - (7 + 4)$

4. $\frac{10+4}{7} + 5 \times 10$

5. $10 - \frac{10+6}{3+1} + 2$

2 Variables, Parameters, and Functions

This section is based on Adler 1.2

Definition 3. 1. A **variable** is a symbol that represents a value in an algebraic expression, this value can change within the situation or experiment.

2. A **parameter** is a symbol that represents a value that is fixed for a given situation or experiment.

3. The number (or parameter) that is multiplied by a variable is called the **coefficient**.

4. A **function** is a mathematical object that takes an input and associates it to a unique output.

5. The input to a function is called the **independent variable** and the set of all inputs a function can accept is called the **domain**.

6. The output of a function is called the **dependent variable** and the set of all outputs of a function is called the **range**.

Note: Page 9 of the Adler textbook has a nice diagram that describes the relationship between a functions and its domain and range.

You can think of a variable as number that will be added later. So when a variable appears in an expression the same "PEMDAS" order of operations applies, as well as the associative property, commutative property, and the distributive property. All of these properties should be familiar to you from pervious courses, however, we will review the distributive property in one of the following sections.

Recall the **Vertical Line Test** to verify if a relation is a function, states that if the graph of a relation allows a vertical line to cross the graph at more than one point then the relation fails to be a function. See Adler page 12 for a nice picture of this.

We conclude this section with some examples:

2.1 Examples

Example 4.

$$3x^2 + x + 4$$

In this expression the variable is x and the coefficient on x^2 is 3, the coefficient on x is 1, and the **constant term** is 4.

Example 5. Based on Example 1.2.4 in Adler.

Suppose that you measure a fungus population, f , in 3 petri dishes, each dish having been stored at a different temperatures. For each dish the temperature was held constant, throughout the experiment. To determine the size of the population you measure the area of the petri dish the fungus covered. You found the following results:

time, t	f when $T=20$	f when $T=30$	f when $T=40$
0.0	1.00	1.00	1.00
1.0	1.16	1.56	1.15
2.0	1.34	1.99	1.30
3.0	1.45	2.64	1.45
4.0	1.56	3.25	1.60
5.0	1.69	4.59	1.75
6.0	1.82	6.24	1.90

So in this experiment t is the independent variable, f is the dependent variable, and T is a parameter representing temperature.

Now consider just the data in the last column. You might recognize that it all lies on the same line, in fact the line defined by the following function:

$$f(t) = t * 0.15 + 1$$

Question Can you identify the slope and y -intercept of the line? How large do you think that the fungus will be when $t = 7$?

2.2 Exercises

Directions: Identify the dependent variable, independent variable, and the value of the function at 3 and 9.

1. $f(x) = 3.24x + 5.7$
2. $g(t) = 7.56x^2 + 4.5x + 9.7$
3. $l(n) = 5.67x^3 + 8.5x + 8.45$

3 Manipulating Expressions that Contain Variables

Recall the **Distributive Property of Multiplication** states that

$$a(x + y) = ax + ay = (x + y)a$$

For example:

$$3(4 + 5) = 3 \cdot 4 + 3 \cdot 5 = 12 + 15 = 27$$

Similarly,

$$3(x + 5) = 3 \cdot x + 3 \cdot 5 = 3x + 15$$

It is when working with variable that we see the true power of the distributive property.

3.1 The Distributive Property and Division

Recall that multiplication and division are related, as inverse operations. So a natural question to ask is does the Distributive Property work for division and if so how? We aim to answer that question now.

Recall that

$$\frac{x}{a} = \frac{1}{a} \cdot x$$

For example,

$$\frac{x}{3} = \frac{1}{3} \cdot x$$

In this way we can view division as multiplication (by inverses). Using this view point we can apply the Distributive Property in the following way:

$$\frac{x + y}{a} = \frac{1}{a} \cdot (x + y) = \frac{x}{a} \cdot \frac{y}{a}$$

Example 6. *Let's try to write the following expression with only one x^2 and x term.*

$$\begin{aligned} & \frac{x^2 + x}{4} + x^2 + 2x \\ &= \frac{x^2}{4} + \frac{x}{4} + x^2 + 2x \\ &= \frac{5x^2}{4} + \frac{6x}{4} \end{aligned} \tag{3}$$

Example 7.

$$\begin{aligned} & \frac{8x^2 + 4}{4x} \\ &= \frac{8x^2}{4x} + \frac{4}{4x} \\ &= 2x + x^{-1} \end{aligned} \tag{4}$$

3.2 A More Complicated Distribution Problem

You are likely familiar with the term **"FOIL"** for multiplying expressions like:

$$(x + 3)(x + 2)$$

"FOIL" stands for **F**irst, **O**uter, **I**nner, **L**ast and helps us to remember how to use distribution to expand expressions such as the one above. This is an important technique that will be utilized frequently in this course, so let's do some examples beginning with the one above:

$$\begin{aligned} & (x + 3)(x + 2) \\ &= x^2 + 2x + 3x + 6 \\ &= x^2 + (2 + 3)x + 6 \\ &= x^2 + 5x + 6 \end{aligned} \tag{5}$$

Example 8.

$$\begin{aligned} & (4x + 5)(2x + 1) \\ &= 8x^2 + 4x + 10x + 5 \\ &= 8x^2 + (4 + 10)x + 5 \\ &= 8x^2 + 14x + 5 \end{aligned} \tag{6}$$

Example 9. Recall that we can even multiply expression like

$$\begin{aligned}(x^2 + 3)(x + 2) \\ = x^3 + 2x^2 + 3x + 6\end{aligned}\tag{7}$$

And expressions like

$$\begin{aligned}(x^2 + 3x + 2)(x + 2) \\ = x^3 + 2x^2 + 3x^2 + 6x + 2x + 4 \\ = x^3 + 6x^2 + 8x + 4\end{aligned}\tag{8}$$

Here we simply multiplied every term in the first set of parentheses by every term in the last set of parentheses, then we combined like terms.

3.3 Factoring

At times it is useful to reverse the process of distributing and/or "FOILING." Recall that the reversing of these processes is called factoring. This is an important concept for this class so lets consider the following examples:

Example 10.

$$\begin{aligned}x^2 + 5x + 6 \\ = (x + 2)(x + 3)\end{aligned}\tag{9}$$

Example 11.

$$\begin{aligned}4x^2 + 16x + 16 \\ = 4(x^2 + 4x + 4) \\ = 4(x + 2)(x + 2) \\ = 4(x + 2)^2\end{aligned}\tag{10}$$

Example 12. We can even factor out variables

$$\begin{aligned}2x^3 + 10x^2 + 12x \\ = 2x(x^2 + 5x + 6) \\ = 2x(x + 2)(x + 3)\end{aligned}\tag{11}$$

Recall that factoring is very useful when we want to solve for x when a function is set equal to 0 ($f(x) = 0$). This is also something that we will do a lot in this course so let's look at a quick example:

Example 13.

$$\begin{aligned} & x^2 + 5x + 6 = 0 \\ & = (x + 2)(x + 3) = 0 \\ \text{this implies} \quad & x + 2 = 0 \text{ or } x + 3 = 0 \\ \text{thus} \quad & x = -2 \text{ or } x = -3 \end{aligned} \tag{12}$$

Note: Some quadratic equations, that is equations of the form $ax^2 + bx + c$, do not factor easily (or do not factor in the Real Numbers). If you encounter such a quadratic then use the quadratic formula to solve for the roots. (Recall a root, r , of a function is an input to a function such that $f(r) = 0$.) We state the quadratic formula here for you reference.

The quadratic formula: The solutions of $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

3.4 Exercises

Directions: Write the following expressions with only one x^3, x^2, x, x^{-1} and constant terms.

1. $(x + 6)(x - 3)$

2. $(4x + 3)(8x + 1)$

3. $(x^2 + 3)(4x + 2)$

4. $(x^2 + 2x + 5)(x + 2)$

5. $\frac{4x^2 + 2x}{2} + x^2 + 2x$

$$6. \frac{12x^2 + 8x}{4} + \frac{5x^2 + 10x}{5}$$

$$7. \frac{30x^2 + 15x}{15} \cdot x + \frac{12x^2 + 4x}{4}$$

$$8. \frac{x^2 + x}{4} + \frac{x^2 + x}{2}$$

$$9. \frac{4x^2 + 8x}{4x} + \frac{4x^2 + 8x}{2}$$

$$10. \frac{4x^2 + 8}{4x} + \frac{4x^2 + 8x}{2}$$

Directions: Solve the following equations for the given independent variable.

$$1. x^2 + 9 + 18 = 0$$

$$2. x^2 - 2x + 1 = 0$$

$$3. (x^2 + 3)(4x + 2) = 0$$

$$4. 4x^3 - 20x^2 - 56x = 0$$

4 Review Problems

Here are some further review problems that correspond to both this handout and Adler Chapter 1.

Instructions The following problems should all be review. Please consider the following problems and review any material that you do not remember.

A. Find the equation of the line for the information given (i.e. points, slope, intercepts).

1. $(0, 0)$ and $(5, 5)$
2. $(1, 1)$ and $(-6, 6)$
3. $(0, 1)$ and $(4, 5)$
4. $(1, 2)$ and $(4, 8)$
5. slope: 4 and $(5, 6)$
6. slope: $\frac{1}{2}$ and $(4, 8)$
7. y-intercept: 4 and $(3, 3)$
8. y-intercept: 5 and slope: 2
9. y-intercept: $\frac{1}{2}$ and x-intercept: $\frac{1}{2}$
10. x-intercept: $\frac{1}{2}$ and slope: $\frac{5}{2}$

B. Find the Sum, Product, and Composition of the following functions. Then compute the value of each at $x = 2$ and $x = 5$.

1. $f(x) = x^2$ and $g(x) = x^2 + 3$
2. $p(x) = x^3$ and $t(x) = x^2 + 5x$
3. $f(x) = x^2 + x$ and $g(x) = x^2 + 3$
4. $b(x) = x^2$ and $d(x) = \sin(x)$
5. $f(x) = \ln(x)$ and $g(x) = x^2 + 3$

C. Graph the following functions and find the inverses if they exist.

1. $y = 3x + 4$
2. $f(x) = \frac{1}{4}x + 7$
3. $g(x) = x^2$

4. $h(x) = x^2 - 10$ with Domain: $x > 0$

D. Find a solution to the following systems of linear equations.

1. $f(x) = \frac{1}{4}x + 7$ and $g(x) = 3x + 4$

2. $g(x) = 3x + 4$ and $h(x) = 3x + 7$

3. $g(x) = 7x + 6$ and $j(x) = 3x + 7$

4. $f(x) = \frac{1}{4}x + 7$ and $h(x) = \frac{-1}{4}x + 7$

E. Graph the following functions using the rules of scaling and shifting

1. $f(x) = x^2$

2. $g(x) = x^2 - 7$

3. $h(x) = (x + 5)^2$

4. $j(x) = (x + 5)^2 - 7$

5. $k(x) = 3 \cdot f(x)$ ($f(x)$ appears in E.1.)

6. $l(x) = f(3x)$ ($f(x)$ appears in E.1.)

F. Does the following data line on a line? Why or why not?

1.

time	Mites on a lizard, y
0	1
1	5
2	10
3	15
4	20

2.

time	Mites on a lizard, y
0	10
2	12
4	14
6	15
8	16

G. Multiply the expressions.

1. $(x + 3)(x + 2)$

2. $(x + 3)(x - 3)$

3. $(x^2 + 3x)(x^3 + 2x + 5)$

H. Solve.

1. $x^2 - 4 = 0$

2. $x^2 + 9x + 9 = 0$

3. $3x^2 + 4x - 3 = 0$

4. $2x^2 + 5x - 10 = 0$

I. Simplify

1. $\frac{x^2-4}{x-2}$

2. $\frac{4x^2+6x}{2x}$

3. $\frac{4x^2+8x}{4} \cdot \frac{4x^2+6x}{2x}$

J. Graph the following piecewise defined functions.

1.

$$f(n) = \begin{cases} n^2 & \text{if } n \geq 0 \\ -n & \text{if } n \leq 0 \end{cases}$$

2.

$$f(n) = \begin{cases} 3n + 4 & \text{if } n \geq 2 \\ -n & \text{if } n < 2 \end{cases}$$

Other good quiz questions involve the volume, area, perimeter, and surface area equations on page 28 of your textbook. Also review the relations involving mass that appear on page 28 and examples on pages 48-50.