

# L'Hospital's Rule Solutions

1.  $\lim_{x \rightarrow 4} \frac{x^2 - 16}{x - 4}$  Form  $\frac{0}{0}$   
LR  $\lim_{x \rightarrow 4} 2x = 8$

2.  $\lim_{x \rightarrow 1} \frac{x-1}{x^2-1}$  Form  $\frac{0}{0}$   
LR  $\lim_{x \rightarrow 1} \frac{1}{2x} = \frac{1}{2}$

3.  $\lim_{x \rightarrow -2} \frac{2x^2 + x - 6}{x + 2}$  Form  $\frac{0}{0}$   
LR  $\lim_{x \rightarrow -2} \frac{4x+1}{1} = -7$

4.  $\lim_{x \rightarrow -3} \frac{x+3}{x^2+2x-3}$  Form  $\frac{0}{0}$   
LR  $\lim_{x \rightarrow -3} \frac{1}{2x+2} = -\frac{1}{4}$

5.  $\lim_{x \rightarrow 0} \frac{\sqrt{2x+4} - 2}{x}$  Form  $\frac{0}{0}$   
LR  $\lim_{x \rightarrow 0} \frac{\frac{1}{2}(2x+4)^{-1/2} \cdot 2}{1} = \lim_{x \rightarrow 0} \frac{1}{\sqrt{2x+4}} = \frac{1}{2}$

6.  $\lim_{x \rightarrow 0} \frac{3 - \sqrt{2x+9}}{2x}$  Form  $\frac{0}{0}$   
LR  $\lim_{x \rightarrow 0} \frac{-\frac{1}{2}(2x+9)^{-1/2} \cdot 2}{2} = \lim_{x \rightarrow 0} \frac{-1}{2\sqrt{2x+9}} = -\frac{1}{6}$

7.  $\lim_{x \rightarrow 0} \frac{\sin x}{x \cos x}$  Form  $\frac{0}{0}$   
LR  $\lim_{x \rightarrow 0} \frac{\cos x}{\cos x + x(-\sin x)} = 1$

$$8. \lim_{x \rightarrow 0} \frac{x \sin x}{1 - \cos x} \quad \text{Form } \frac{0}{0}$$

$$\text{LR} = \lim_{x \rightarrow 0} \frac{\sin x + x \cos x}{+ \sin x} \quad \text{Form } \frac{0}{0}$$

$$\text{LR} = \lim_{x \rightarrow 0} \frac{\cos x + \cos x + -x \sin x}{\cos x} = 2$$

$$9. \lim_{x \rightarrow 0} \frac{1 - \cos x}{x \tan x} \quad \text{Form } \frac{0}{0}$$

$$\text{LR} = \lim_{x \rightarrow 0} \frac{\sin x}{\tan x + x \sec^2 x} \quad \text{Form } \frac{0}{0}$$

$$\text{LR} = \lim_{x \rightarrow 0} \frac{\cos x}{\sec^2 x + \sec^2 x + x \cdot 2 \sec x (\sec x \tan x)}$$

$$= \frac{1}{2}$$

$$10. \lim_{x \rightarrow \pi/2} \frac{\sin(\pi/2 - x)}{\cos x} \quad \text{Form } \frac{0}{0}$$

$$\text{LR} = \lim_{x \rightarrow \pi/2} \frac{-\cos(\pi/2 - x)}{-\sin x} = \frac{-1}{-1} = 1$$

$$11. \lim_{x \rightarrow 0^+} \frac{\sqrt{x}}{\ln(x+1)} \quad \text{Form } \frac{0}{0}$$

$$\text{LR} = \lim_{x \rightarrow 0^+} \frac{\frac{1}{2} x^{-1/2}}{\frac{1}{x+1}} = \lim_{x \rightarrow 0^+} \frac{x+1}{2\sqrt{x}} = \infty$$

$$12. \lim_{x \rightarrow \infty} \frac{\ln x}{\sqrt{x}} \quad \text{Form } \frac{\infty}{\infty}$$

$$\text{LR} = \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{\frac{1}{2} x^{-1/2}} = \lim_{x \rightarrow \infty} \frac{2\sqrt{x}}{x}$$

$$= \lim_{x \rightarrow \infty} \frac{2}{\sqrt{x}} = 0$$

$$13. \lim_{x \rightarrow 0} \frac{2^x - 1}{3^x - 1} \quad \text{Form } \frac{0}{0}$$

$$= \lim_{x \rightarrow 0} \frac{e^{x \ln 2} - 1}{e^{x \ln 3} - 1} = \lim_{x \rightarrow 0} \frac{e^{x \ln 2} - 1}{e^{x \ln 3} - 1}$$

$$\begin{aligned} & \text{LR} \\ & = \lim_{x \rightarrow 0} \frac{\ln 2 e^{x \ln 2}}{\ln 3 e^{x \ln 3}} = \frac{\ln 2}{\ln 3} \end{aligned}$$

$$\begin{aligned} 14. \quad & \lim_{x \rightarrow 0} \frac{2^{-x} - 1}{5^x - 1} \quad \text{Form } \frac{0}{0} \\ & = \lim_{x \rightarrow 0} \frac{e^{\ln 2^{-x}} - 1}{e^{\ln 5^x} - 1} = \lim_{x \rightarrow 0} \frac{e^{-x \ln 2} - 1}{e^{x \ln 5} - 1} \\ & \stackrel{\text{LR}}{=} \lim_{x \rightarrow 0} \frac{-\ln 2 e^{-x \ln 2}}{\ln 5 e^{x \ln 5}} = \frac{-\ln 2}{\ln 5} \end{aligned}$$

$$\begin{aligned} 15. \quad & \lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2} \quad \text{Form } \frac{0}{0} \\ & \stackrel{\text{LR}}{=} \lim_{x \rightarrow 0} \frac{e^x - 1}{2x} \quad \text{Form } \frac{0}{0} \\ & \stackrel{\text{LR}}{=} \lim_{x \rightarrow 0} \frac{e^x}{2} = \frac{1}{2} \end{aligned}$$

$$\begin{aligned} 16. \quad & \lim_{x \rightarrow 0} \frac{e^x - 1 - x - x^2/2}{x^3} \quad \text{Form } \frac{0}{0} \\ & \stackrel{\text{LR}}{=} \lim_{x \rightarrow 0} \frac{e^x - 1 - x}{3x^2} \quad (\text{Form } \frac{0}{0}) \quad \stackrel{\text{LR}}{=} \lim_{x \rightarrow 0} \frac{e^x - 1}{6x} \\ & (\text{Form } \frac{0}{0}) \quad \stackrel{\text{LR}}{=} \lim_{x \rightarrow 0} \frac{e^x}{6} = \frac{1}{6} \end{aligned}$$

$$\begin{aligned} 17. \quad & \lim_{x \rightarrow \infty} \frac{(\ln x)^2}{x^2} \quad (\text{Form } \frac{\infty}{\infty}) \stackrel{\text{LR}}{=} \lim_{x \rightarrow \infty} \frac{2(\ln x)(1/x)}{2x} \\ & = \lim_{x \rightarrow \infty} \frac{2 \ln x}{2x^2} \quad (\text{Form } \frac{\infty}{\infty}) \stackrel{\text{LR}}{=} \lim_{x \rightarrow \infty} \frac{1/x}{2x} \\ & = \lim_{x \rightarrow \infty} \frac{1}{2x^2} = 0 \end{aligned}$$

$$\begin{aligned} 18. \quad & \lim_{x \rightarrow \infty} \frac{x^4}{e^x} \quad (\text{Form } \frac{\infty}{\infty}) \stackrel{\text{LR}}{=} \lim_{x \rightarrow \infty} \frac{4x^3}{e^x} \quad (\text{Form } \frac{\infty}{\infty}) \\ & \stackrel{\text{LR}}{=} \lim_{x \rightarrow \infty} \frac{12x^2}{e^x} \quad \stackrel{\text{LR}}{=} \lim_{x \rightarrow \infty} \frac{24x}{e^x} \quad \stackrel{\text{LR}}{=} \lim_{x \rightarrow \infty} \frac{24}{e^x} = 0 \end{aligned}$$