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Solutions Exam 1 SP07

1. (20 pts) Answer the following questions:

(a) Evaluate $\lim_{b_t \to 10} b_{t+1}$ if $b_{t+1} = 0.75 b_t$.

$$\lim_{b_t \to 10} 0.75 b_t = 0.75(10) = 7.5$$

(b) Let $b_{t+1} = 0.75 b_t$. If $b_0 = 340$, write the solution to this discrete time dynamical system.

$$b_t = 340(0.75)^t$$

(c) Evaluate $\lim_{t \to 10} m_t$ if $m_t = 260(0.6)^t$.

$$\lim_{t \to 10} 260(0.6)^t = 260(0.6)^{10} = 1.5721$$

(d) Rewrite $m_t = 260(0.6)^t$ as an exponential function with base e.

$$0.6 = e^{\ln 0.6}$$

$$0.6^t = (e^{\ln 0.6})^t = e^{t \ln 0.6}$$

$$M_t = 260(0.6)^t = 260 e^{t \ln 0.6}$$

(e) Find the half-life of $m_t = 260(0.6)^t$.

$$\frac{1}{2} = e^{t_h \ln 0.6}$$

$$\ln \tfrac{1}{2} = \cancel{\ln e}^{t_h \ln 0.6}$$

$$\ln \tfrac{1}{2} = t_h \ln 0.6$$

$$\frac{\ln \frac{1}{2}}{\ln 0.6} = t_h$$

OR

$$\frac{1}{2} = 0.6^{t_h}$$

$$\ln \tfrac{1}{2} = \ln 0.6^{t_h}$$

$$\ln \tfrac{1}{2} = t_h \ln 0.6$$

$$\frac{\ln \frac{1}{2}}{\ln 0.6} = t_h$$

2. (14 points) Let $f(t) = 6 + 3\cos(t - \pi/4)$.

   (a) Fill in the following values of $f(t)$:

   i. Average = 6

   ii. Amplitude = 3

   iii. Period = 2π

   iv. Phase = $\pi/4$

   (b) Find the domain of $f(t)$.

   all real numbers or $(-\infty, \infty)$ or $-\infty < t < \infty$

   (c) Find the range of $f(t)$.

   $6 - 3 \leq f(t) \leq 6 + 3$ so $3 \leq f(t) \leq 9$

   (d) Evaluate $\lim_{t \to \pi/4} f(t)$

   $$\lim_{t \to \pi/4} 6 + 3\cos(t - \pi/4) = 6 + 3\cos(\pi/4 - \pi/4) = 6 + 3(1) = 9$$

3. (16 points) Answer the following questions:

   (a) Let $b = 3s^2 - 8$ for $s \geq 0$. Find the inverse function of b.

   $$b = 3s^2 - 8$$
   $$b + 8 = 3s^2$$
   $$\frac{b+8}{3} = s^2$$

   $\sqrt{\frac{b+8}{3}} = |s| = s$ since $s \geq 0$

   $$s = \sqrt{\frac{b+8}{3}}$$

   (b) Let $q = 3r + 2$ and let $r = \frac{q-2}{3}$.

   i. Compose q with r.

   $q \circ r = 3\left(\frac{q-2}{3}\right) + 2 = q - 2 + 2 = q$

   ii. Compose r with q.

   $r \circ q = \frac{(3r+2)-2}{3} = \frac{3r}{3} = r$

   iii. What can you conclude about the relationship between q and r?

   q and r are inverse functions

4. (22 points) Answer the following questions:

(a) Consider a population of crickets being cultured for feeding Nimata, the pet Tarantula. The per capita reproduction for the crickets is 11. Each month a new generation of crickets is produced. After reproduction, 6 crickets are removed to feed Nimata. Write an updating function to model this discrete time dynamical system.

$$c_{t+1} = 11c_t - 6$$

(b) Consider the following updating function: $x_{t+1} = 0.1x_t^2 - 0.2x_t + 1.6$ for $x_t \geq 0$. Find the equilibria.

$$x^* = 0.1{x^*}^2 - 0.2x^* + 1.6$$

$$0 = 0.1{x^*}^2 - 1.2x^* + 1.6$$

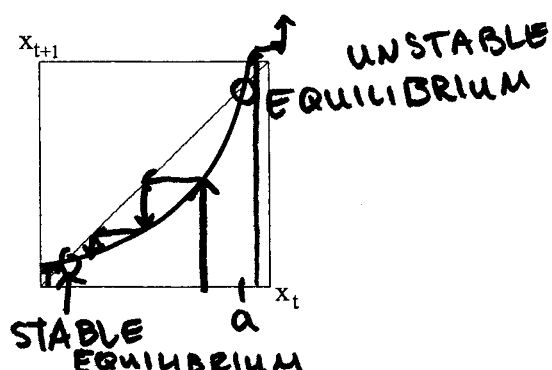
use quadratic formula or calculator:

$x^* = 1.528$

and

$x^* = 10.472$

(c) Cobweb the following graph to answer the questions below.



i. Identify the equilibria on the graph and classify them as stable or unstable.

ii. What is the long term behavior if $x_0 < \cancel{x^*}$? a

Long term behavior:

$x_t \rightarrow$ stable equilibrium

or x_t decreases, then levels out (at 1.528)

5. (12 points) Let V_{t+1} represent the voltage of the AV node in the Heart Model.

$$V_{t+1} = \begin{cases} e^{-\alpha\tau}V_t + u & \text{if } e^{-\alpha\tau}V_t \leq V_c \\ e^{-\alpha\tau}V_t & \text{if } e^{-\alpha\tau}V_t > V_c \end{cases}$$

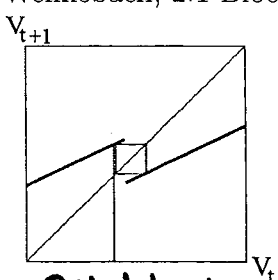
(a) Let $e^{-\alpha\tau} = .5$, $u = 10$ and $V_c = 22$. If $V_0 = 24$, will the heart beat? Why?

$e^{-\alpha\tau}V_0 = .5(24) = 12 < 22 = V_c$ so heart beats

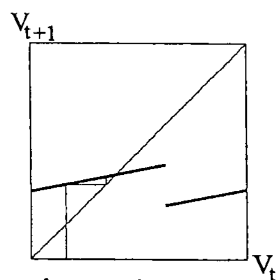
OR

$V_0 = 24 < \frac{V_c}{.5} = \frac{22}{.5} = 44$ so heart beats

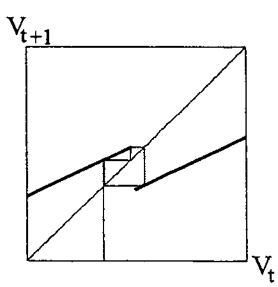
(b) Classify each cobweb diagram as representing a healthy heart, a heart with Wenkebach, 2:1 Block, 3:1 Block, or neither.



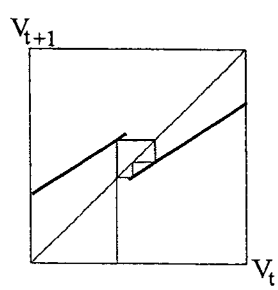
2:1 block



Healthy



Wenkebach



3:1 block

6. (16 points) Suppose the distance that Sonic the Hedgehog has rolled is given by the function $y(t) = 4t^2 - 3t + 1$ where y is in meters and t is in seconds.

   (a) Compute Sonic's average speed (average rate of change) between time t and time $t + \Delta t$.

$$\frac{y(t+\Delta t) - y(t)}{\Delta t} = \frac{4(t+\Delta t)^2 - 3(t+\Delta t) + 1 - (4t^2 - 3t + 1)}{\Delta t}$$

$$= \frac{4(t^2 + 2t\Delta t + \Delta t^2) - 3t - 3\Delta t + 1 - 4t^2 + 3t - 1}{\Delta t}$$

$$= \frac{4t^2 + 8t\Delta t + 4\Delta t^2 - 3\Delta t - 4t^2}{\Delta t}$$

$$= \frac{8t\Delta t + 4\Delta t^2 - 3\Delta t}{\Delta t} = \frac{\Delta t(8t + 4\Delta t - 3)}{\Delta t}$$

$$= 8t + 4\Delta t - 3$$

   (b) Using the result of Part 1, compute Sonic's velocity (instantaneous rate of change) as a function of t.

$$\text{IROC} = \lim_{\Delta t \to 0} \text{AROC} = \lim_{\Delta t \to 0} 8t + 4\Delta t - 3 = 8t + 4(0) - 3$$

$$= 8t - 3 = y'(t)$$

   (c) What is Sonic's velocity at time t=2?

$$y'(2) = 8(2) - 3 = 13$$