

NAME: \_\_\_\_\_

SECTION OR TIME YOU HAVE CLASS: \_\_\_\_\_

INSTRUCTOR: \_\_\_\_\_

**Instructions:** The exam is closed book and closed notes. You may use an approved calculator, but be sure to show your work on each problem for full credit. Work that is crossed out or erased will not be graded. Turn in any scratch paper that you use during the exam. You will have one hour and 45 minutes to work on the exam.

Problem	Points	Score
1	20	
2	14	
3	16	
4	22	
5	12	
6	16	
Total	100	

1. (20 pts) Answer the following questions:

(a) Evaluate  $\lim_{b_t \rightarrow 10} b_{t+1}$  if  $b_{t+1} = 0.75b_t$ .

(b) Let  $b_{t+1} = 0.75b_t$ . If  $b_0 = 340$ , write the solution to this discrete time dynamical system.

(c) Evaluate  $\lim_{t \rightarrow 10} m_t$  if  $m_t = 260(0.6)^t$ .

(d) Rewrite  $m_t = 260(0.6)^t$  as an exponential function with base  $e$ .

(e) Find the half-life of  $m_t = 260(0.6)^t$ .

2. (14 points) Let  $f(t) = 6 + 3 \cos(t - \pi/4)$ .

(a) Fill in the following values of  $f(t)$ :

- i. Average =
- ii. Amplitude =
- iii. Period =
- iv. Phase =

(b) Find the domain of  $f(t)$ .

(c) Find the range of  $f(t)$ .

(d) Evaluate  $\lim_{t \rightarrow \pi/4} f(t)$

3. (16 points) Answer the following questions:

(a) Let  $b = 3s^2 - 8$  for  $s \geq 0$ . Find the inverse function of  $b$ .

(b) Let  $q = 3r + 2$  and let  $r = \frac{q-2}{3}$ .

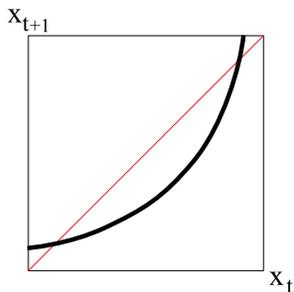
- i. Compose  $q$  with  $r$ .
- ii. Compose  $r$  with  $q$ .
- iii. What can you conclude about the relationship between  $q$  and  $r$ ?

4. (22 points) Answer the following questions:

(a) Consider a population of crickets being cultured for feeding Nimata, the pet Tarantula. The per capita reproduction for the crickets is 11. Each month a new generation of crickets is produced. After reproduction, 6 crickets are removed to feed Nimata. Write an updating function to model this discrete time dynamical system.

(b) Consider the following updating function:  $x_{t+1} = 0.1x_t^2 - 0.2x_t + 1.6$  for  $x_t \geq 0$ . Find the equilibria.

(c) Cobweb the following graph to answer the questions below.



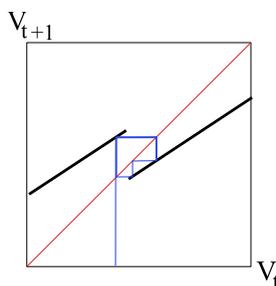
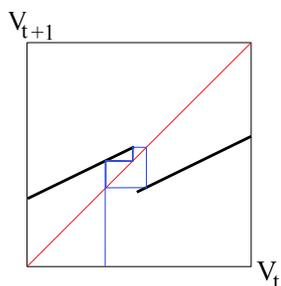
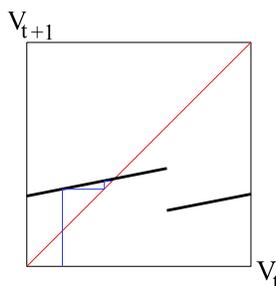
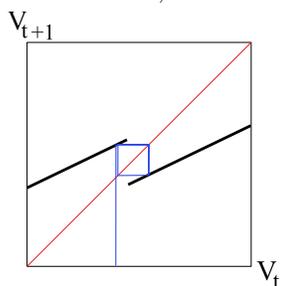
- i. Identify the equilibria on the graph and classify them as stable or unstable.
- ii. What is the long term behavior if  $x_0 < 10$ ?

5. (12 points) Let  $V_{t+1}$  represent the voltage of the AV node in the Heart Model.

$$V_{t+1} = \begin{cases} e^{-\alpha\tau}V_t + u & \text{if } e^{-\alpha\tau}V_t \leq V_c \\ e^{-\alpha\tau}V_t & \text{if } e^{-\alpha\tau}V_t > V_c \end{cases}$$

(a) Let  $e^{-\alpha\tau} = .5$ ,  $u = 10$  and  $V_c = 22$ . If  $V_0 = 24$ , will the heart beat? Why?

(b) Classify each cobweb diagram as representing a healthy heart, a heart with Wenkebach, 2:1 Block, 3:1 Block, or neither.



6. (16 points) Suppose the distance that Sonic the Hedgehog has rolled is given by the function  $y(t) = 4t^2 - 3t + 1$  where  $y$  is in meters and  $t$  is in seconds.

(a) Compute Sonic's average speed (average rate of change) between time  $t$  and time  $t + \Delta t$ .

(b) Using the result of Part 1, compute Sonic's velocity (instantaneous rate of change) as a function of  $t$ .

(c) What is Sonic's velocity at time  $t=2$ ?