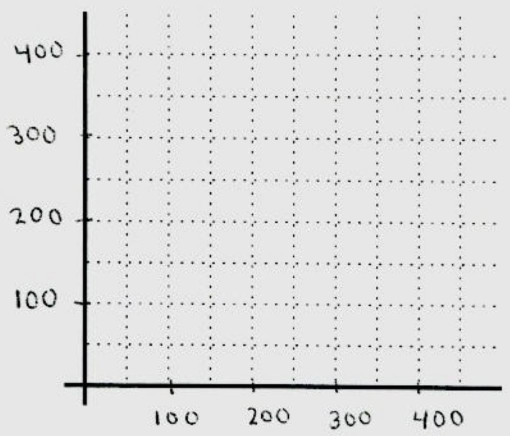
M155 Exam 1 Spring 2006

1. (16 pts) The women's Olympic record for the 1500 m race in 1972 was 4 min. 1.4 s and in 1988 was 3 min. 53.9 s.

a) Convert these times to seconds and find the equation of the line in slope-intercept form connecting these points.

b) Graph the line.



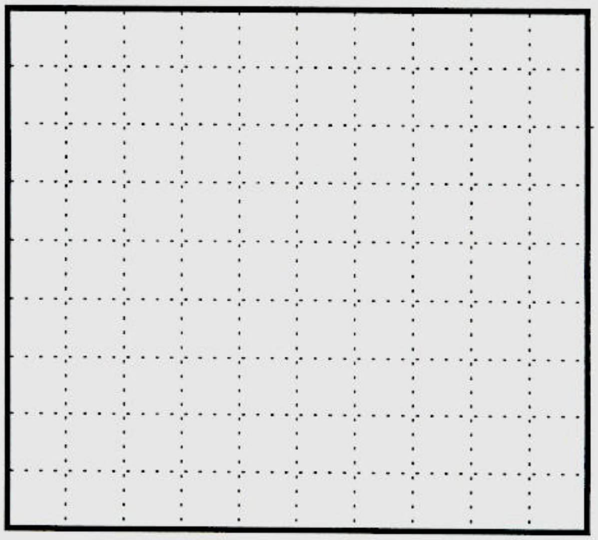
c) If things continue at this rate, when would the women finish in exactly no time?

2. (12 pts) A population of bacteria doubles every hour, but 1.0×10^6 individuals are removed after reproduction to be converted into valuable biological by-products.

a) Write a discrete-time dynamical system to describe the population.

b) Find the equilibrium solution.

c) Graph the updating function and cobweb from an initial population of $b_0 = 2.0 \times 10^6$.



3. (10 pts) Suppose the size of a population of bacteria at time t is given by $b(t) = 1.5t + 6$ where t is measured in hours.

a) Find the average rate of change of the population between times t and $t + \Delta t$. Justify your answer.

b) Suppose $t = 2$ and $\Delta t = 0.1$. Find the equation of the secant line in slope-intercept form between times t and $t + \Delta t$.

4. (12 pts) The amount of C^{14} left t years after the death of an organism is given by

$$Q(t) = Q_0 e^{-0.000122t}$$

where Q_0 is the amount left at the time of death.
a) When will one tenth of the original amount of C^{14} be left in the remains?

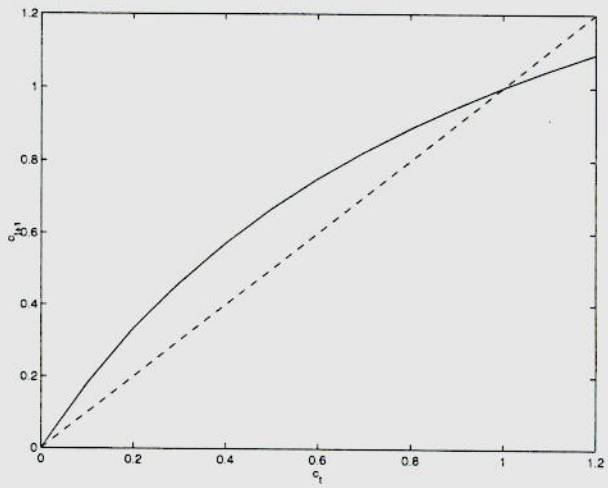
b) If 2.5×10^{-8} grams are present after 10,000 hours, how much C^{14} was originally present in the organism?

5. (14 pts) An organism is breathing a chemical that modifies the depth of its breaths. Suppose the discrete-time dynamical system for the concentration c_t of the chemical in the lung is

$$c_{t+1} = c_t - \frac{c_t^2}{c_t + 1} + \frac{c_t}{c_t + 1}$$

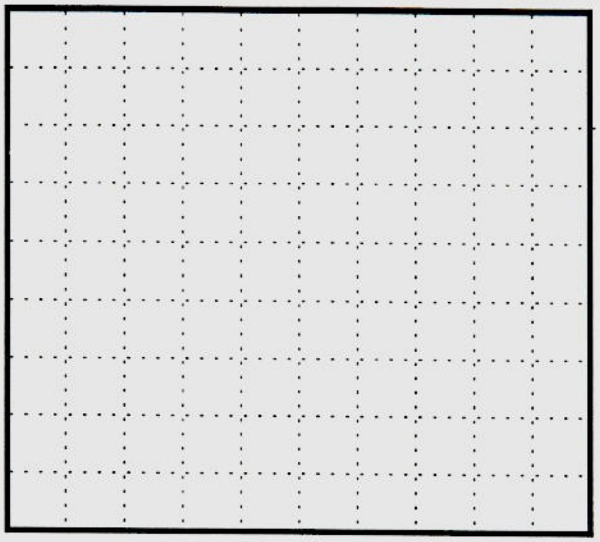
a) Find the equilibria. Show all of your work to receive credit. No points will be given for answers only.

b) Draw a cobweb diagram starting from the initial concentration $c_0 = .4$. What is the long-term concentration of the chemical in the lung?



6. (12pts) Accurately graph the updating function

$$V_{t+1} = \begin{cases} \frac{1}{4}V_t + 2, & \text{if} \quad V_t \leq 4 \\ V_t - 1, & \text{if} \quad V_t > 4 \end{cases}$$



b) Is the function continuous? Why or why not?

c) Find $\lim_{V_t \to 3} V_{t+1}$, if it exists.

7. (14 pts) a) Find the average rate of change of the function $f(x) = 6x^2 + 3$ as a function of Δx.

b) Find the limit in your answer to part b as $\Delta x \to 0$.

c) What is the instantaneous rate of change of this function at $x = 1$?

8. (10 pts) Find the following limits, if they exist. Show all of your work and justify your answers.

a) $\lim_{x \to 0^+} \sqrt{x} + 12$

b) $\lim_{\Delta x \to 0} \dfrac{2(x^2 + 2x\Delta x + \Delta x^2) - 2x^2}{\Delta x}$

c) $\lim_{x \to 2} \dfrac{1}{(x-2)^2}$