Math 155 **Final Exam** **Spring 2009**

NAME: ______________________________

SECTION: _______________ TIME: _______________

INSTRUCTOR: ______________________________

Instructions: The exam is closed book and closed notes. You may use an approved calculator, but be sure to show your work on each problem for full credit. Work that is crossed out or erased will not be graded. Turn in any scratch paper that you use during the exam. You will have two hours to work on the exam.

Problem	Points	Score
1	16	
2	10	
3	10	
4	16	
5	12	
6	14	
7	12	
8	12	
Total	100	

CONFIDENTIALITY PLEDGE

I agree that I will not share any information, either specific or general, about the problems on this examination with any other person.

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(Signature)

1. (16 pts) Find the following limits. Show all your work. If you use L'Hospital's rule, justify why it can be applied each time you use it. If you use knowledge of leading behaviors, justify your work.

a) $\lim_{x\to 0} \frac{2-2e^{-2x}}{x^2+3x}$ form $\frac{0}{0}$ L'Hôpital's

$$\lim_{x\to 0} \frac{4e^{-2x}}{2x+3} = \boxed{\frac{4}{3}}$$

b) $\lim_{x\to\infty} \frac{10x-1}{10\ln x}$ form $\frac{\infty}{\infty}$ L'Hôpital's

$$\lim_{x\to\infty} \frac{10}{10\left(\frac{1}{x}\right)} = \lim_{x\to\infty} x = \boxed{\infty}$$

c) $\lim_{x\to 1} \frac{\sin(\pi/2)}{\cos \pi x} = \frac{\sin(\pi/2)}{-1} = \boxed{-1}$

d) Find $f_\infty(x)$, the leading behavior of $f(x)$ as $x \to \infty$ for

$$f(x) = \frac{\overset{\nearrow\infty}{6x^2} + \overset{\nearrow\infty}{\ln x} + \overset{\nearrow\infty}{2x} + \overset{\nearrow 100}{100}}{\underset{\searrow 0}{e^{-x}} + \underset{\searrow\infty}{2x} + \underset{\searrow 12}{12}}$$

$top_\infty(x) = 6x^2$
$bot_\infty(x) = 2x$

$$f_\infty(x) = \frac{6x^2}{2x} = 3x$$

e) Find $f_0(x)$, the leading behavior of $f(x)$ as $x \to 0$, for

$$f(x) = \frac{\overset{\nearrow 1}{e^{x^2}} + \overset{\nearrow 0}{12x} + \overset{\nearrow\infty}{x^{-1}}}{\underset{\searrow 0}{x^2} + \underset{\searrow 0}{x} + \underset{\searrow}{7}}$$

$top_0(x) = x^{-1}$
$bot_0(x) = 7$

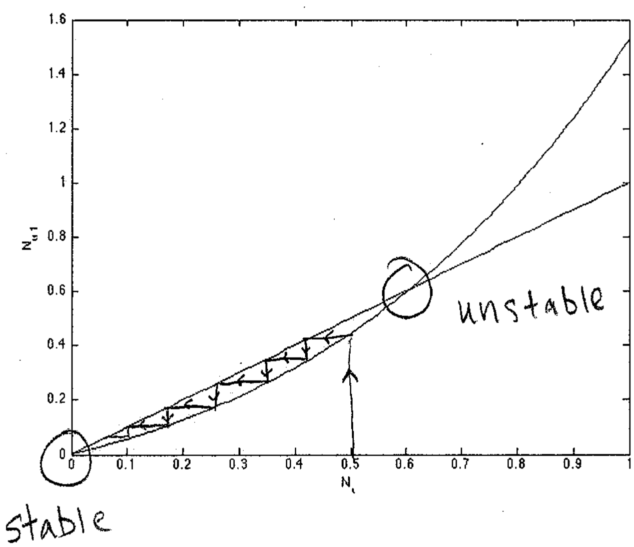
$$f_0(x) = \frac{x^{-1}}{7} = \frac{1}{7x}$$

2. (10 pts) The following updating function describes the number of fish N_t of a certain fish population.

$$N_{t+1} = 0.6N_t e^{N_t} - hN_t$$

The term hN_t is the harvest and h is the harvesting effort.

a) The graph below corresponds to $h = 0.1$. Draw a cobweb diagram on the graph, starting at $N_t = 0.5$. Circle all equilibria on the graph. Determine the stability of each equilibrium from your cobweb diagram.



b) Use the stability criterion to check your answer to part a for the nonzero equilibrium $N^* = \ln(1.833)$. (Here $h = 0.1$, as in part a).

$f(N) = .6Ne^{N} - .1N$

$f'(N) = .6Ne^{N} + e^{N}(.6) - .1$

$f'(\ln(1.833)) = f(.6060)$

$= .6(.606)e^{.606} + e^{.606}(.6) - .1$

$= |1.6664| > 1$

unstable

3. (10 pts) a) Find the equation of the tangent line at $t = 1$ to the function $f(t) = e^{2-t^3}$. Write your answer in the form $y = mt + b$.

$f'(t) = e^{2-t^3}(-3t^2)$

$f'(1) = e^{2-1}(-3) \approx -8.1548$

$f(1) = e^{2-1} = e^1 \approx 2.7183$

$2.7183 = -8.1548(1) + b$

$10.8731 = b$

$\boxed{y = -.81548t + 10.8731}$

b) Use Newton's method to approximate the value of the zero of the function $g(x) = x^2 - \sin(2x) - 3$. Take an initial guess of $x_0 = 2$. Do two iterations (find x_2). The formula for Newton's method is

$$x_{n+1} = x_n - \frac{g(x_n)}{g'(x_n)}$$

$g'(x) = 2x - 2\cos(2x)$

$X_0 = 2$

$X_1 = 2 - \frac{4 - \sin(4) - 3}{4 - 2\cos(4)} \approx 1.6690$

$X_2 = 1.669 - \frac{(1.669)^2 - \sin(2(1.669)) - 3}{2(1.669) - 2\cos(2(1.669))}$

$\boxed{X_2 \approx 1.6726}$

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~~3~~ (16 pts) Evaluate the following definite and indefinite integrals. If necessary, use substitution. Show all of your work to receive full credit.

a) $\int (x^{-1} - \pi - \cos x)\ dx$ $= \boxed{\ln(|x|) - \pi x - \sin(x) + c}$

b) $\int_1^2 x^2\sqrt{x^3 - 1}\ dx$

$w = x^3 - 1$

$\frac{dw}{dx} = 3x^2$

$\frac{dw}{3x^2} = dx$

$\int x^2 \sqrt{w}\ \frac{dw}{3x^2}$

$= \frac{1}{3}\int \sqrt{w}\ dw$

$= \frac{1}{3}\left(\frac{2}{3}\right) w^{3/2} = \frac{2}{9}(x^3-1)^{3/2}\Big|_1^2 = \frac{2}{9}(8-1)^{3/2} - \frac{2}{9}(0)$

$= \boxed{\frac{2}{9}(7)^{3/2} \approx 4.1156}$

c) $\int_1^\infty (e^{-3x} - x^{-3/2})\ dx$

$\lim_{b\to\infty} \int_1^b (e^{-3x} - x^{-3/2})\,dx$

$= \lim_{b\to\infty} \left(-\frac{1}{3}e^{-3x} + 2x^{-1/2}\right)\Big|_1^b = \lim_{b\to\infty}\left[-\frac{1}{3}e^{-3b} + 2b^{-1/2} - \left(-\frac{1}{3}e^{-3} + 2\right)\right]$

$= 0 + \frac{1}{3}e^{-3} - 2 \approx \boxed{-1.9834}$

d) $\int (\sec^2(2t) - e^t - t)\ dt$ $= \int \sec^2(2t)\,dt - \int e^t\,dt - \int t\,dt$

$w = 2t$

$\frac{dw}{dt} = 2$

$\frac{dw}{2} = dt$

$\int \sec^2(w)\left(\frac{dw}{2}\right) - e^t - \frac{1}{2}t^2$

$= \frac{1}{2}\int \sec^2(w)\,dw - e^t - \frac{1}{2}t^2$

$= \boxed{\frac{1}{2}\tan(2t) - e^t - \frac{1}{2}t^2 + c}$

5 ~~4~~ (14 pts) Suppose the rate at which a chemical product is formed in a reaction is

$$\frac{dP}{dt} = 42.4e^{-0.3t} \quad \text{moles/s}$$

a) If there is no product at time $t = 0$ (so $P(0) = 0$), how much is present at time t? (Solve the differential equation.)

$$\int 42.4e^{-0.3t}\,dt = \frac{42.4e^{-0.3t}}{-0.3} + C \approx -141.33e^{-0.3t} + C$$

$$P(0) = -141.33e^{0} + C = 0$$

$$C = 141.33$$

$$\boxed{P(t) = -141.33e^{-0.3t} + 141.33}$$

b) Use a definite integral to represent and find the net change in product between times $t = 10$ and $t = 20$.

$$\int_{10}^{20} 42.4e^{-0.3t}\,dt = -141.33e^{-0.3t}\Big|_{10}^{20}$$

$$\approx \boxed{6.6861 \text{ moles}}$$

c) What is the limiting value of the rate $\frac{dP}{dt}$ as $t \to \infty$? (Take a limit.)

$$\lim_{t\to\infty} 42.4e^{-0.3t} = \boxed{0 \text{ moles/s}}$$

d) What is the limiting value of the product $P(t)$ as $t \to \infty$? (Take a limit.)

$$\lim_{t\to\infty} (-141.33e^{-0.3t} + 141.33) = \boxed{141.33 \text{ moles}}$$

6 ~~8~~ a) (8 pts) The following differential equation is a model for the rate of change of the volume of blood in liters in the liver with respect to time t in seconds. Solve the initial value problem for the volume of blood $V(t)$.

$$\frac{dV}{dt} = 0.8 + \sin(2\pi t - \pi), \quad V(0) = 0.75$$

$$\int .8 + \sin(2\pi t - \pi)\,dt = .8t + \int \sin(2\pi t - \pi)\,dt$$

$w = 2\pi t - \pi$

$\frac{dw}{dt} = 2\pi$

$\frac{dw}{2\pi} = dt$

$.8t + \int \sin(w)\left(\frac{dw}{2\pi}\right)$

$= .8t + \frac{1}{2\pi}\int \sin(w)\,dw$

$= .8t + \frac{1}{2\pi}(-\cos(2\pi t - \pi)) + c$

$$V(0) = .8(0) - \frac{1}{2\pi}\cos(2\pi(0) - \pi) + c = 0.75$$

$$-\frac{1}{2\pi}(-1) + c = 0.75 \quad \text{so } c \approx .5908$$

$$\boxed{V(t) = .8t - \frac{1}{2\pi}\cos(2\pi t - \pi) + .5908}$$

b) (6 pts) Find the total change in a population between times $t = 10$ and $t = 15$ hours if the population $P(t)$ is governed by the differential equation

$$\frac{dP}{dt} = (3t+1)^2$$

$$\int_{10}^{15} (3t+1)^2\,dt$$

$w = 3t + 1$

$\frac{dw}{dt} = 3$

$\frac{dw}{3} = dt$

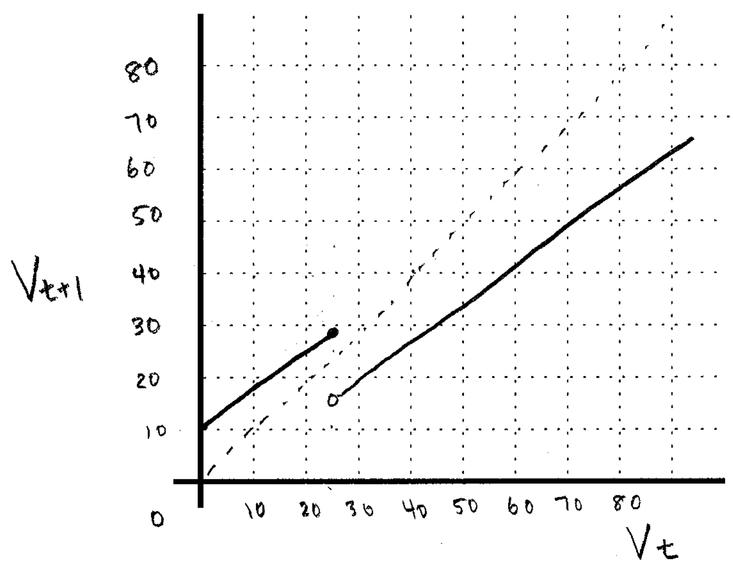
$\int w^2\left(\frac{dw}{3}\right)$

$= \frac{1}{3}\left(\frac{1}{3}w^3\right) = \frac{1}{9}(3t+1)^3\Big|_{10}^{15}$

$= \boxed{7505}$

~~8~~ 7. a) (6 pts) Sketch the updating function for V_t below. Label your axes. Does the updating function have an equilibrium for positive values of V_t? Why or why not?

$$V_{t+1} = \begin{cases} 0.75V_t + 10 & \text{if } V_t \leq 25 \\ 0.75V_t & \text{if } V_t > 25 \end{cases}$$



No, there is no equilibrium because it does not intersect the diagonal.

b) (6 pts) Use integration by parts to evaluate the indefinite integral.

$$\int 3xe^{2x}\, dx$$

$u = 3x$ $\qquad dv = e^{2x}$

$du = 3$ $\qquad V = \frac{1}{2}e^{2x}$

$$3x\left(\tfrac{1}{2}e^{2x}\right) - \int 3\left(\tfrac{1}{2}e^{2x}\right)dx$$

$$= \frac{3}{2}xe^{2x} - \frac{3}{2}\left(\frac{1}{2}e^{2x}\right) + c$$

$$= \boxed{\frac{3}{2}xe^{2x} - \frac{3}{4}e^{2x} + c}$$

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~~7~~. (12 pts) Suppose the total food collected by a bee is given by

$$F(t) = \frac{t}{2+t}$$

and the rate at which nectar is collected is given by

$$R(t) = \frac{F(t)}{t+1} = \frac{t}{(2+t)(t+1)} = \frac{t}{2t+2+t^2+t} = \frac{t}{t^2+3t+2}$$

Find the time t that will maximize the rate at which nectar is collected on the time interval $[0, 3]$. Verify that you have found a maximum by using either the first or second derivative test and determine if it is a global maximum on the interval.

$$R'(t) = \frac{(t^2+3t+2)(1) - t(2t+3)}{(t^2+3t+2)^2} = \frac{t^2+3t+2-2t^2-3t}{(t^2+3t+2)^2}$$

$$= \frac{-t^2+2}{(t^2+3t+2)^2} \overset{\text{set}}{=} 0$$

$$t^2 = 2 \qquad \boxed{t = \sqrt{2}}$$

First Derivative Test:

$$\overset{+}{\longleftarrow}\ \underset{\sqrt{2}}{|}\ \overset{-}{\longrightarrow}$$

$R'(0) = 2 > 0$
$R'(2) < 0$

so we have a local min

$R(0) = 0$

$R(\sqrt{2}) = \frac{\sqrt{2}}{\sqrt{2}^2+3\sqrt{2}+2} \approx .1716$

$R(3) = \frac{3}{9+9+2} = .15$

so we have a global max at $t = \sqrt{2}$