1. (16 pts) Evaluate the first derivative of the following functions. You do **NOT** need to simplify your answers.

a) $s(x) = x^9 + 4x^3 - 5x^2 + \frac{1}{6}x^{-1/2} - 10^{5/8} + \pi$

b) $g(x) = e^{9x} \sin\left(\alpha^2 x^7\right)$ where α is a constant.

c) $p(x) = \dfrac{\ln\left(2x^3\right)}{\tan(3x)}$

d) $g(x) = \left(2x^5 + 17\right)\left(3x^{-2} - 9\right)$

2. (12 pts) Consider a population of fish governed by the logistic dynamical system with harvesting rate h where $0 \leq h \leq 1$

$$x_{t+1} = 2x_t(1 - x_t) - hx_t.$$
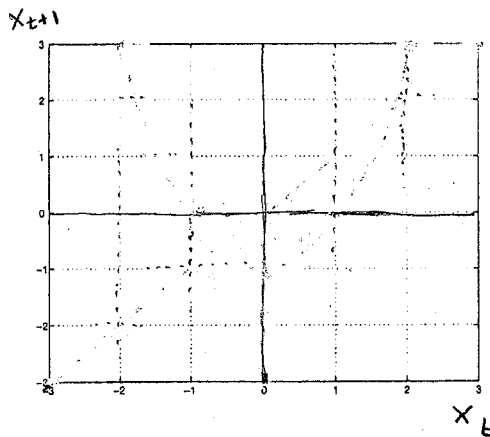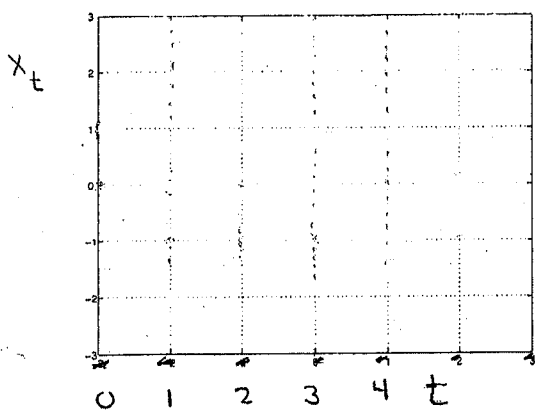
a) Find all equilibria.

b) Use the stability criterion to determine all values of h for which the <u>nonzero</u> equilibrium is stable. State or show clearly how you are using the criterion to receive full credit.

3. (14 pts) Given the updating function

$$x_{t+1} = x_t^2 - 1$$

a) Determine the long-term behavior of the system by finding x_1, x_2, x_3, x_4 and x_5. Begin at $x_0 = 1$. What is the long-term behavior of the system?

b) Draw your iterates $x_0, x_1, \ldots, x_5$ on the graph on the left (a graph of x_t versus t). Draw a cobweb diagram on the graph to the right starting this time from $x_0 = 0$. In each case, label your axes.

c) Find any equilibria. Show all of your work. Based on the results of parts (a) and (b), do you expect each of them to be stable or unstable?

4. (16 pts) Let $f(x) = \frac{1}{3}x^3 - \frac{1}{6}x^6$ on $(-\infty, \infty)$.

a) Find the critical points, showing all your work and using calculus to justify your answer.

b) List the intervals where the function is increasing and decreasing, showing all your work and using calculus to justify your answer.

c) Find $f''(x)$

d) List intervals where the function is concave up and concave down, showing all your work and using calculus to justify your answer.

e) Sketch a curve of $f(x)$ using your previous work.

5. (16 pts) Evaluate the following limits. Show all of your work. If you use L'Hopital's Rule, justify why it can be applied each time you use it. If you use knowledge of leading behavior, justify your work by explaining all of your steps.

a) $\lim\limits_{x \to \infty} \dfrac{\ln x + x}{e^x + x^2}$

b) $\lim\limits_{x \to 0} \dfrac{x \cos x}{\tan x}$

c) $\lim\limits_{x \to 0} \dfrac{3x^2 + x^3}{x^4 - x}$

d) $\lim\limits_{x \to 1} \dfrac{e^{(x-1)}}{\sqrt{x} \cos(x - 1)}$

6. (12 pts) For the following functions $f(x)$, find $f_\infty(x)$, the leading behavior of $f(x)$ as $x \to \infty$, and $f_0(x)$, the leading behavior of $f(x)$ as $x \to 0$

a) $f(x) = e^{-4x} + \frac{10}{x^2} + \frac{3}{x^5} + 12e^{-2x}$

b) $f(x) = \frac{4x+2}{e^x - x}$

c) $f(x) = \frac{6x}{1+x}$

d) For the function in part (c), use the method of matched leading behaviors to sketch a graph of $f(x)$ on the interval $[0, \infty)$.

7. (14 pts) Suppose the size of a bird population in hundreds is governed by $h(x) = \dfrac{3 + x^2}{x}$ for x in $[1, 10]$ where x represents the number (in thousands) of a species of tree borer that serves as its food supply.

a) Find the critical points and classify them as local minima or maxima using the *second derivative*.

b) Find the values of x for which h attains a global minimum and for which h attains a global maximum. Justify your answers.

c) What is the meaning of your answers to part b) in terms of the bird population and its food supply?