Math 155. Homework 4. Sections 1.10-2.2

Do the following problems from the Adler text:

1.10: 14, 16, 28, 42

Also do the following problems.

1. Let V_t represent the voltage of the AV node in the Heart Model.

$$V_{t+1} = \begin{cases} e^{-\alpha\tau} V_t + u & \text{if} \quad V_t \leq e^{\alpha\tau} V_c \\ e^{-\alpha\tau} V_t & \text{if} \quad V_t > e^{\alpha\tau} V_c \end{cases}$$

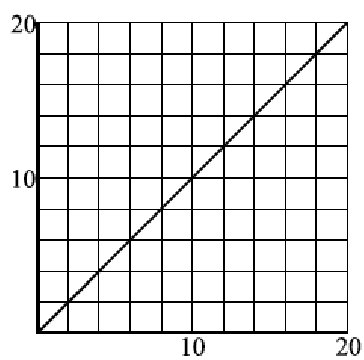
Let $e^{-\alpha\tau} = \frac{1}{5}$, $u = 10$, and $V_c = 2$.

(a) If $V_0 = 6$, calculate V_1. Will the heart beat? Why or why not?

(b) Does this system have an equilibrium? If so, find it algebraically; if not, explain why not.

(c) Graph the updating function and cobweb from an initial value of $V_0 = 6$ to determine if this heart is

i) healthy, ii) has a 2:1 block, or iii) has the Wenckebach phenomenon.

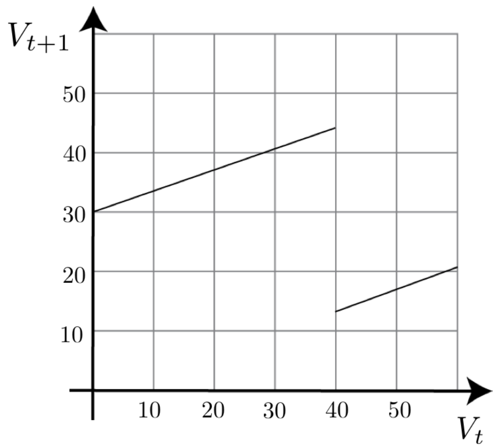


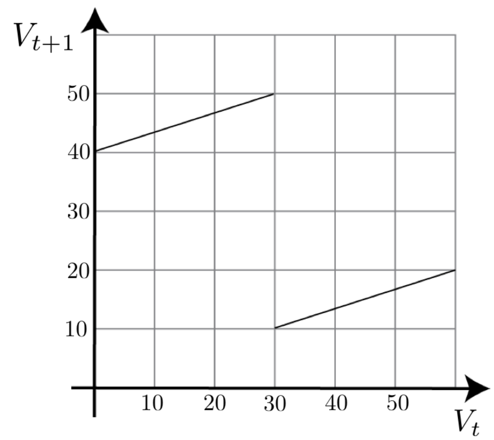
2. Let V_t represent the voltage at the AV node in the heart model

$$V_{t+1} = \begin{cases} e^{-\alpha\tau}V_t + u, & \text{if } V_t \le e^{\alpha\tau}V_c \\ e^{-\alpha\tau}V_t, & \text{if } V_t > e^{\alpha\tau}V_c \end{cases}$$

(a) For each of the following two graphs of the updating function, cobweb starting from an initial value of $V_0 = 10$, and determine if the heart

i) is healthy, ii) has a 2:1 block, or iii) has the Wenckebach phenomenon.





(b) Now let $e^{-\alpha\tau} = 0.25$, $u = 10$, and $V_c = 14$.

i) Does the system have an equilibrium? Justify your answer, and find the equilibrium if there is one.

ii) Recall that $e^{-\alpha\tau} = 0.25$ determines the decay in voltage at the AV node between signals from the SA node. If $\alpha = 0.75$, calculate the time τ between signals. Round to three decimal places.

3. Suppose that the population $f(t)$ of a fungus as a function of time is given by $f(t) = 3t^2 + 1$.
(a) Find a formula for the slope of the secant line to $f(t)$ that passes through $(2, 13)$ and $(2 + \Delta t, f(2 + \Delta t))$.

(b) Find a formula for the average rate of change of the fungus population between times 2 and $2 + \Delta t$ as a function of Δt.

(c) What is the average rate of change in the fungus population between times $t = 2$ and $t = 2.2$?

(d) Find the limit as $\Delta t \to 0$ of your formula in (b). What is the instantaneous rate of change of the fungus population at time $t = 2$?

(e) Graph $f(t)$ and indicate a graphical interpretation of the instantaneous rate of change at time $t = 2$.