Homework

M472 Fall 2012

Important note: in this homework I use several times the words "smallest", "largest"... anytime I do not specify which ordering relation I refer to I am implicitely assuming I am talking about inclusion!

Exercise 1. *If X is a set with exactly three points, describe all possible topologies you can put on X.*

Exercise 2. *Let X be a topological space, let $A \subseteq X$.*

1. *Prove that the set of interior points of A is the largest open set contained in A. We call such set the* ***interior*** *of A.*

2. *Prove that the union of A with set of limit points of A is the smallest closed set containing A. We call such set the* ***closure*** *of A.*

3. *Prove that the complement of the interior of A equals the closure of the complement of A.*

Exercise 3. *Let X have the stupid topology, and $x \in X$. For what subsets of X is x:*

- *an interior point?*
- *a limit point?*

Exercise 4. *Let X be any set, and make it into a topological space by giving it the discrete topology. What is the smallest basis you can produce for this topology?*

Exercise 5. *Consider $X = Y = \mathbb{R}$ and the function $f : X \to Y$ given by $y = f(x) := [x]$ (y is the integer part of x, e.g. $f(2.4) = 2$) Recall that $\mathbb{R}$ can be given a topology τ_1 by declaring open sets to be open half lines of the form $(-\infty, a)$, or it can be given a topology τ_2 by declaring open sets to be open half lines of the form (a, ∞) (and remember that in both cases you have to toss in the empty set and all of $\mathbb{R}$ as well). Consider all for possibile combinations of assigning a topology to X and to Y from τ_1 and τ_2, and tell me when f is continuous and when it is not.*