Homework

M472 Fall 2012

Exercise 1. *Let X and Y be topological spaces and let $X \times Y$ be the product space (i.e. the cartesian product endowed with the product topology). Prove that for any $x \in X$ the subspace $\{x\} \times Y$ (endowed with the subspace topology) is homeomorphic to Y.*

Exercise 2. *Is $\mathbb{R}$ with the finite complement topology connected?*

Exercise 3. *Assuming that you know that $\mathbb{R}$ (with euclidean topology) is connected, prove that the circle (with euclidean topology) is also connected. Can you somehow "adapt" this argument to prove that the circle with the finite complement topology is connected?*

Exercise 4. *Let $X = \mathbb{R} \setminus \{0\}$ (with euclidean topology), and $Y = \{\pm 1\}$ a SET with two elements. Consider the following functions f and $g : X \to Y$.*

$$f(x) = \begin{cases} 1 & \text{if } x < 0 \\ -1 & \text{if } x > 0 \end{cases}$$

$$g(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \\ -1 & \text{if } x \notin \mathbb{Q} \end{cases}$$

What is the finest topology on Y that makes f is continuous? What is the finest topology on Y that makes g is continuous?

Exercise 5. *Let (X, τ_X) be a connected topological space and (Y, τ_{disc}) be a topological space with the discrete topology. What are the continuous functions from X to Y?*