

Homework

M472

Fall 2012

Exercise 1. *Prove that the following two definitions of homeomorphism are equivalent:*

1. *X and Y are homeomorphic if there exist two continuous functions $f : X \rightarrow Y$ and $g : Y \rightarrow X$ such that $f \circ g = 1_Y$ and $g \circ f = 1_X$.*
2. *X and Y are homeomorphic if there exist a set bijection $f : X \rightarrow Y$ that induces a bijection between τ_X and τ_Y .*

Exercise 2. *Prove that \mathbb{R} with the finite complement topology is NOT homeomorphic to \mathbb{R} with the euclidean topology.*

Exercise 3. *Prove that the open disc minus the origin is homeomorphic to an open cylinder (aka a cylinder without its boundaries).*

Exercise 4. *Suppose that X is a topological space, Y only a measley set, and that there exists a set bijection f between X and Y . Prove that you can endow Y with a topology in such a way that f becomes a homeomorphism of topological spaces.*