

Homework

M472

Fall 2012

Exercise 1. *Prove that the following two loops into \mathbb{R}^2 are homotopy equivalent relative to the base point:*

$$\gamma_1(s) = (\cos(2\pi s), \sin(2\pi s)),$$

$$\gamma_2(s) = (2\cos(2\pi s) - 1, 2\sin(2\pi s))$$

Exercise 2. *If you consider the two loops in the previous exercise as loops into $\mathbb{R}^2 \setminus \{(0, 0)\}$, are they still homotopy equivalent? How about if you consider them as loops into $\mathbb{R}^2 \setminus \{(-2, 0)\}$? (A heuristic explanation for this last point is sufficient).*

Exercise 3. *Let f be the identity function in \mathbb{R}^n and $g : \mathbb{R}^n \rightarrow \mathbb{R}^n$ be the constant function $g(x) = \mathbf{0}$. Prove that f and g are homotopy equivalent functions.*

Exercise 4. *Use the previous exercise to prove that \mathbb{R}^n is homotopy equivalent to a point.*

Exercise 5. *Prove that a loop parameterizing the equator of a sphere is homotopy equivalent to a constant loop.*