

Homework

M472

Fall 2012

Exercise 1. *Let X and Y be topological spaces and let $X \times Y$ be the product space (i.e. the cartesian product endowed with the product topology). Prove that $X \times Y$ is path connected if and only if X and Y are path connected.*

Exercise 2. *Assume that a topological space X has a finite number of connected components. Prove that each component is an open and closed subset. In fact prove that in this case the open and closed subsets of X are precisely the connected components.*

Exercise 3. *Let X be a topological space, and β_X a base for the topology on X . Given a subset $A \subseteq X$, prove that*

$$\beta_A := \{B \cap A \text{ such that } B \in \beta_X\}$$

is a base for the subspace topology on A .