Homework

M472 Fall 2012

Exercise 1. *Prove that any finite subset of a topological space is compact.*

Exercise 2. *Give an example (in any topological space other than a set with the stupid topology) of a set which is compact but not closed.*

Exercise 3. *Is* $\mathbb{R}$ *with the finite complement topology a compact topological space? What subsets of it are compact?*

Exercise 4. *Is the closed interval* $[0,1]\cap\mathbb{Q}$ *a compact subsect of* $\mathbb{Q}$ *(with subspace topology induced by the euclidean topology on* $\mathbb{R}$*)?*

Exercise 5. *Define the diameter of a subset of* $\mathbb{R}^n$ *to be the sup of the distances of all pairs of points of the subset:*

$$diam(A) = sup_{x,y\in A} d(x,y)$$

Let C_n *be a sequence of closed sets in* $\mathbb{R}^n$ *such that:*

1. *For every* n, $C_{n+1} \subseteq C_n$.
2. $diam(C_n) \to 0$ *as* $n \to \infty$.

Prove that the intersection of all C_n*'s consists of exactly one point.*

$$\bigcap_{i=1}^{\infty} C_n = \{pt.\}$$