

Final Exam

M472

Fall 2008

Name: _____ Score: _____

Read the problems. Read them again. Make sure to express your answers in a **clear and logical** way. **Don't rush!**

1. TRUE OR FALSE: (no explanation needed)

- (a) If X is homeomorphic to Y (and X is path connected), then $\pi_1(X)$ is isomorphic to $\pi_1(Y)$.
- (b) If $\pi_1(X)$ is isomorphic to $\pi_1(Y)$, then X and Y are homeomorphic.
- (c) If X is homeomorphic to Y (and X is NOT necessarily path connected), then $\pi_1(X, x_0)$ is isomorphic to $\pi_1(Y, y_0)$.
- (d) If $f : X \rightarrow Y$ is a homeomorphism, (X is NOT necessarily path connected), then $\pi_1(X, x_0)$ is isomorphic to $\pi_1(Y, f(x_0))$.

2. Let X be a topological space, and x a point of X such that any closed set that does NOT contain x is finite. Prove that X is compact.

3. Use the theorem of classification of compact surfaces to prove which of the following three surfaces are homeomorphic:
- S_1 : the connected sum of 4 tori.
 - S_2 : the connected sum of 3 tori and 2 projective planes.
 - S_3 : the connected sum of 2 tori and 4 projective planes.

4. Consider the group $\mathbb{Z}/2\mathbb{Z}$ acting on the unit disc, in such a way the non-trivial group element reflects points about the y axis. Pick a point x_0 in the quotient space and let y_0 be a preimage of x_0 via the quotient map. Exhibit a loop based at x_0 in the quotient space that admits two liftings starting at y_0 but with different endpoints.

5. Consider the torus as the product space $S^1 \times S^1$, and consider the second projection function:

$$\begin{aligned} f_2 : \quad T &\rightarrow S^1 \\ (\theta_1, \theta_2) &\mapsto \theta_2 \end{aligned}$$

Describe $\pi_1(f_2)$.

6. Prove the **Brauer Fixed Point Theorem**. Here D^2 denotes the closed two dimensional disc.

Theorem 1. *Any continuous map $f : D^2 \rightarrow D^2$ has a fixed point, i.e. a point P such that $f(P) = P$.*