

Exam #2

M472

Fall 2008

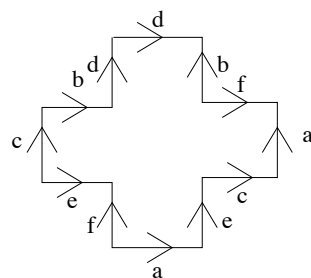
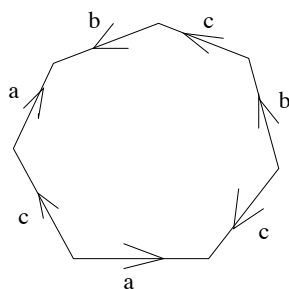
Name: _____ Score: _____

Read the problems. Read them again. Make sure to express your answers in a **clear and logical** way. **Don't rush!**

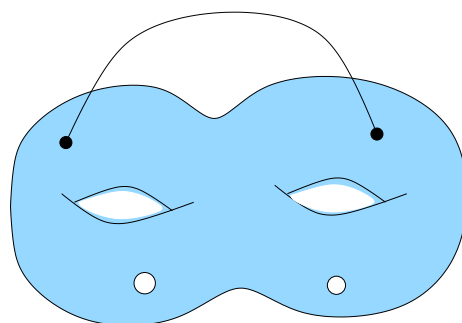
1. TRUE OR FALSE: (no explanation needed)

- (a) If X is homeomorphic to Y , then X is homotopy equivalent to Y .
- (b) If X is homotopy equivalent to Y , then X is homeomorphic to Y .
- (c) Let S be a closed compact surface. If the Euler characteristic of S is even, then S is orientable.
- (d) Let S be a closed compact surface. If S is orientable, then the Euler characteristic of S is even.

2. Decide whether the pictures below represent a surface or not. In case of a positive answer, recognize the surface.



3. The figure shown below (a two-torus with two little discs removed and a segment attached at two points) is homotopy equivalent to a bouquet of circles. How many circles? (show a sequence of pictures to substantiate your answer)



4. Assuming the fact:

$$f \sim g \Rightarrow \pi_1(f) = \pi_1(g) \quad ,$$

prove that the fundamental group is a homotopy invariant.

5. Proof that $\mathbb{R}^3$ is not homeomorphic to $\mathbb{R}^3 \setminus \{line\}$. (here you are allowed to use the fact that the fundamental group of a circle is $\mathbb{Z}$).

6. Prove the **Brauer Fixed Point Theorem**, following the outline below. Here D^2 denotes the closed two dimensional disc.

Theorem 1. *Any continuous map $f : D^2 \rightarrow D^2$ has a fixed point, i.e. a point P such that $f(P) = P$.*

- (a) Prove this by contradiction. Assume f is a continuous map with no fixed points.
- (b) Geometrically construct a continuous function g from the whole disc to the boundary circle: to a point Q in the disc, associate a point on the circle found using Q and $f(Q)$ (Caution! There are two natural ways to do this, and only one works in the end!).
- (c) Observe. What is g restricted just to the circle?
- (d) Now consider the inclusion of the circle into D^2 followed by g .

$$S^1 \xrightarrow{i} D^2 \xrightarrow{g} S^1$$

Apply π_1 to everything and use functoriality to deduce a contradiction.