

HOMEWORK: Sep.5th

Math 472

1 Life with a metric

Exercise 1. Which of these functions define a metric on $\mathbb{R}^2$?

1. $\max_{i=1,2} |x_i - y_i|$
2. $\max |x_1 - y_1|$ (NOT)

Exercise 2. Draw open balls for the above metrics.

Exercise 3. What are limit points for sets in the discrete metric?

Exercise 4. Prove that a set is open/closed in the euclidean metric iff it is open/closed in the taxi-cab metric.

Exercise 5. What are open and closed sets in the discrete metric?

2 Aug 29

Definition of a topological space and a topology (X, τ) . Show two ways. Can define topology by opens and define complements of opens to be closed or viceversa.

Example 1. • stupid (trivial) topology. Discrete topology.

- topology induced by a metric.
- topologies on a set with two points.
- the topology that loves one point.
- $[a, b]$ on $\mathbb{R}$ (NOT)
- induced topology on a subset.

Remark: Notice that the induced topology can be tricky to digest. $[0, a)$ is not open in $\mathbb{R}$, but it is in $[0, 1]$ with the induced topology!

Exercise 6. Which of these are topologies on $\mathbb{R}$?

- $A = (-\infty, a)$
- $B = (a, +\infty)$
- $A \cup B$
- $(a, b]$

2.1 basic concepts

1. neighborhood.
2. interior point/interior of a set.
3. limit points/closure of a set.

Can give two equivalent definitions for interior and closure.

Exercise 7. Let X be a topological space with the stupid topology, and $x \in X$ a point of X . For what subsets of X is x :

- an interior point?
- a limit point?

What is the smallest neighborhood of x ?

Define concept of fineness of a topology.

Exercise 8. What are the coarsest and the finest topology you can define on a set X ?

Exercise 9. Partial order the following topologies on $\mathbb{R}$ by fineness:

- stupid topology
- discrete topology
- euclidean topology
- finite complement topology
- $(-\infty, a)$ topology

If there is time, can introduce the notion of a base for a topology.

3 Sep 3

Continuity and homeomorphisms

4 Sep 5

Philosophy: how to induce topologies on sets.

5 Sep 8
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7 Sep 12
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