

Math 472: Introduction to Topology

(A quick and dirty description of the course)

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MWF 1 – 1:50 pm

ENGRG E 106

Textbook:

Basic Topology (M.A. Armstrong)

Springer



A PHILOSOPHICAL DISCUSSION

Topology lies at the heart of **ANY** branch of modern geometry. So let me take a slight detour and chat a little bit of history first.

I like to think that modern geometry is born in 1872, when Felix Klein wrote the **Erlangen Programme**, a mathematical manifesto that addresses a philosophical question: what is geometry?

The answer Klein proposes is the following: a geometry is the study of properties of shapes and spaces - properties that are invariant under a group of transformations. What this means is that, before we make any statement, we must decide two things:

- what kind of objects we consider "shapes and spaces".
- what is the group of transformations, i.e. what is our notion of "being equal".

This also means that there is not a unique geometry, and so yes, it is correct to say that two parallel lines never meet if you are doing euclidean geometry, but it is not if

you are doing projective geometry. And if you are working projectively over the complex numbers, then lines are really spheres, but two of them still meet transversally at one point (huh?).

As you see, geometry is escaping the boundaries of our "common sense intuition", and it will more and more. So it is our job of mathematicians to try and really understand what is going on - i.e. to abstract our intuitive geometric notions to a more general context.

Topology is a fun branch of geometry to study in order to accomplish such a goal. In fact, in topology, Klein's group of transformations is simply huge! Two geometric shapes are considered equal if there are two bijective, continuous functions inverse to each other between them. This means, for example, that a triangle, a square and a circle are all the same, because they can be stretched one into another. But a circle can never be stretched into an infinite straight line, for example. Some people like to call topology "rubber geometry", in the sense that it studies geometric properties of objects that are made of an extremely stretchable and bendable rubber. Even searching for properties to study is a chore to begin with! (lengths, areas, angles, number of sides...all goes topsy turvey if a triangle is the same as a circle). Properties that remain the same under such a huge flexibility are called **topological invariants**.

Now, if this made little sense, don't get scared, but please **read on**.

A MORE DOWN TO EARTH DESCRIPTION OF THE COURSE

Over the course of the semester, I would like to present some basic concepts in both point set and algebraic topology.

We will start of course, by **re-defining** the notion of continuous functions. Of course this new notion will coincide with the usual concept you all know from calculus, but it will be a lot more general. In particular, it will not depend on any notion of distance on our spaces (so away with epsilon and deltas, by golly!), but only on the notion of **open sets** (whatever they might be).

Once we understand continuity, we will be studying continuous functions between various spaces, both familiar and unfamiliar. Two spaces will be considered the same (the technical term will be **homeomorphic**) if you can find a pair of continuous inverses between them.

And now comes the crucial question. If somebody hands you two spaces, can you tell if they are homeomorphic or not? This can be a very difficult problem, because space that "look" very different might actually be homeomorphic and vice-versa. So we have to be clever, and hunt for properties of spaces that **DO NOT** change when two spaces are homeomorphic. These properties are called **topological invariants**. Some such

properties are very down to earth (is the space made up of only one piece or more pieces, can you “walk” from any point in the space to any other point in the space). But the most useful ones are quite sophisticated, and it will take us the rest of the semester to get familiar with a few of them.

Now, if somebody has already vaguely heard about topology, the topics that I would like to cover are:

1. Continuity and Homeomorphisms: what is a topological space?
2. The first topological invariants: connectedness and path connectedness.
3. A substantially more sophisticated invariant: compactness.
4. The topology of compact surfaces, and some more fun invariants.
5. A bite of algebraic topology: the fundamental group and (time permitting) a little theory of covering spaces.