

**HW 8**  
**Math 261**

Please see the course syllabus for details on how to turn in your homework assignments.

1. (5 pts.) TRUE OR FALSE:

- (a) Let  $R$  denote a plane region, and  $(u, v) = (u(x, y), v(x, y))$  be a different set of coordinates for the Cartesian plane. Then for any function  $F(u, v)$

$$\int_R F(u, v) du dv = \int_R F(u(x, y), v(x, y)) dx dy.$$

- (b) Let  $R$  denote a plane region, and  $(u, v) = (u(x, y), v(x, y))$  be a different set of coordinates for the Cartesian plane. Then

$$\int_R du dv = \int_R |u_x v_y - u_y v_x| dx dy.$$

- (c) Let  $R$  denote a square of sidelength 2 defined by the inequalities  $|x| \leq 1, |y| \leq 1$ , and  $(u, v) = (3y, 2x)$ . Then the area of  $R$  is computed as

$$\int_{-2}^2 \int_{-3}^3 du dv.$$

- (d) Let  $R$  denote a square of sidelength 2 defined by the inequalities  $|x| \leq 1, |y| \leq 1$ , and  $(u, v) = (3y, 2x)$ . Then the area of  $R$  is computed as

$$\int_{-2}^2 \int_{-3}^3 (1/6) du dv.$$

- (e) Let  $R$  denote a square of sidelength 2 defined by the inequalities  $|x| \leq 1, |y| \leq 1$ , and  $(u, v) = (3y, 2x)$ . Then the area of  $R$  is computed as

$$\int_{-1}^1 \int_{-1}^1 (1/6) du dv.$$

2. (3 pts.) Using *cylindrical* coordinates, set up the integral to find the volume of the region enclosed by the vertical cylinder  $x^2 + y^2 = 4$  and the planes  $z = 0$  and  $y + z = 4$ . Do **NOT** evaluate the integral; just set it up.
3. (3 pts.) Using *spherical* coordinates, set up the integral to find the volume of the region enclosed by the vertical cylinder  $x^2 + y^2 = 4$  and the planes  $z = 0$  and  $z = 2$ . Do **NOT** evaluate the integral; just set it up.
4. (3 pts.) Use *cylindrical* coordinates and set up the integral to find the volume of the region enclosed by a circular cone of base radius 1 and height 2. Do **NOT** evaluate the integral; just set it up. Your set-up should include a description of how the cone is positioned in  $\mathbb{R}^3$ .

5. (3 pts.) Consider using the substitution  $\begin{cases} x = u - v, \\ y = 2u + v \end{cases}$  for the integral of  $x + y^2 - 2$ .

What is the *integrand* in terms of  $u$  and  $v$ ? (Don't bother with the integral signs, the bounds, or the  $du dv$ .)

6. (3 pts.) Using the same substitution as in the previous problem, suppose the  $(x, y)$  region over which we wish to integrate includes the boundary line  $2x - y = 3$ . Convert this line into a  $(u, v)$  boundary line.