

HW 5
Math 261

Please see the course syllabus for details on how to turn in your homework assignments.

1. (3 pts.) TRUE OR FALSE:
 - (a) If $f(x, y, z)$ is a differentiable function and P is a point of maximum in the interior of the domain of f , then the gradient of f at P is the $\mathbf{0}$ vector.
 - (b) If $f(x, y, z)$ is a differentiable function and the gradient of f at P is the $\mathbf{0}$ vector, then P is a point of maximum.
 - (c) If $f(x, y, z) = 3x + 4y + 5z$ and R is a plane region bounded by a polygon, then the maximum of f on R is obtained at a vertex of the polygon.
2. (3 pts.) Find the derivative of $h(x, y, z) = x + 2y^2 + 3z^3$ at the point $(2, 0, \sqrt{2})$ in the direction of the vector $\mathbf{v} = \langle 1, 1, 0 \rangle$.
3. (3 pts.) Find the equation for the tangent plane to the surface $x^2 - xy - y^2 - z = 0$ at the point $(1, 1, -1)$. Please give your answer in the form $Ax + By + Cz = D$.
4. (3 pts.) Give the best possible upper bound (using the technique from class, i.e. using a linear approximation) for the error in approximating $f(x, y) = x^2 + 3xy - 2y^2$ at the point $(1, 1)$, over the rectangle $|x - 1| \leq 0.1$, $|y - 1| \leq 0.3$. It is OK to leave your answer as a numerical expression (i.e., not simplified down to a number).
5. (3 pts.) Let f be some function of the plane, such that $(1, 1)$ and $(1, -1)$ are critical points. Suppose $f_{xx} = x + 2$, $f_{xy} = x + y - 2$, and $f_{yy} = y + 1$. Classify (min/max/SP) the critical points $(1, 1)$ and $(1, -1)$, clearly indicating any computed values you used to make your decision.
6. (5 pts.)

Consider the sphere S defined by the equation $x^2 + y^2 + (z - 4)^2 = 1$.

 - (a) What are the center and the radius of the sphere? (Possibly help yourself by drawing a sketch of the sphere).
 - (b) Based on your answer to (a), make a guess as to what is the point on S which is closest to the origin $(0, 0, 0) \in \mathbb{R}^3$.
 - (c) Write the system of equations you would need to solve to use Lagrange multipliers to find the closest point on S to the origin.
 - (d) Check whether the point you guessed in (b) satisfies the system of equations you wrote in (c) (if it doesn't, go back to (b) and revise your answer).
 - (e) Is there any other point on S that satisfies the system of equations you wrote in (c)? Please don't just answer yes or no: support your answer with either a computation or a verbal argument.