HW 7
Math 261, F19

Please see the course syllabus for details on how to turn in your homework assignments. This one is due at the beginning of your class on **Friday, November 1**.

1. (5 pts.) TRUE OR FALSE:

   (a) Let R denote a region in the xy-plane, given polar coordinates (r, θ). The expression $\int_R dr d\theta$ computes the area of R.

   (b) Let R denote a region in the xy-plane, given polar coordinates (r, θ). The expression $\int_R r dr d\theta$ computes the area of R.

   (c) Let R denote the plane region bounded by a circle of radius one centered at the origin. Then $\int_R x dx dy = 0$.

   (d) Let R denote the plane region bounded by a circle of radius one centered at the origin and (r, θ) polar coordinates. Then $\int_R r dr d\theta = 0$.

   (e) If (x, y) are cartesian coordinates and (r, θ) polar coordinates for the plane, and R some plane region, the expression

   $$\int_R f(x, y) dx dy = \int_R g(r, \theta) dr d\theta$$

   is true when

   $$g(r, \theta) = r f(r \cos(\theta), r \sin(\theta)).$$

2. (3 pts.) Set up but do **NOT** evaluate a double integral to compute the integral of $f(x, y) = \cos(xy)$ over the part of the unit disk (the region inside the circle of radius 1 centered at the origin) in the first quadrant (where $x > 0$, $y > 0$).

3. (3 pts.) Convert the following double integral to an equivalent polar form but do **NOT** evaluate:

   $$\int_0^1 \int_y^{\sqrt{4-y^2}} x^2 + y^2 dx dy$$

4. (3 pts.) Set up but do **NOT** evaluate a triple integral to compute the volume of the tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 1)$. The top plane of the tetrahedron is given by $2x + y + 2z = 2$. **USE** the order $dz\ dy\ dx$.

5. (3 pts.) Consider the tetrahedron T with vertices $(1, 0, 0)$, $(1, -1, 1)$, $(1, 1, 1)$, and $(0, 0, 1)$. How many regions must T be split into in order to integrate some function over T with the following variable orders (each worth 1 point)? (Each answer is just 1 number!)

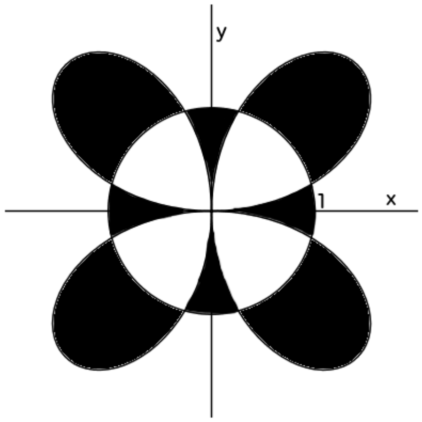
   (a) $dx\ dy\ dz$

   (b) $dx\ dz\ dy$

   (c) $dy\ dz\ dx$

   (It would be good practice to try setting these integrals up, but that's not required for the problem.)

6. (3 pts.) Let R be the region colored in black in the figure below. The two curves bounding R are the circle $x^2 + y^2 = 1$ and the curve described in polar coordinates by the equation $r = 2\sin(2\theta)$. Set up but do **NOT** evaluate a (sum of) double integral(s) in *polar* coordinates to find the area of R.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1
x