HW 4
Math 261, F19

Please see the course syllabus for details on how to turn in your homework assignments. This one is due at the beginning of your class on **Friday, October 4**.

1. (5 pts.) **True or False**
   
   (a) Let \( x = 3, y = 4, z = 5t \), and \( f(x, y, z) \) be a function from \( \mathbb{R}^3 \) to \( \mathbb{R} \). Then \( \frac{\partial f}{\partial t} = \frac{\partial f}{\partial z} \).
   
   (b) Let \( x = 3, y = 4, z = 5t \), and \( f(x, y, z) \) be a function from \( \mathbb{R}^3 \) to \( \mathbb{R} \). Then \( \frac{\partial f}{\partial t} = 5 \frac{\partial f}{\partial z} \).
   
   (c) Let \( x = 3, y = 4, z = 5t \), and \( f(x, y, z) \) be a function from \( \mathbb{R}^3 \) to \( \mathbb{R} \). Then \( \frac{\partial f}{\partial t} = \frac{1}{5} \frac{\partial f}{\partial z} \).
   
   (d) Let \( h(x, y) \) be a function from \( \mathbb{R}^2 \) to \( \mathbb{R} \) and \( P \) a point in the domain of \( h \). It is possible that the directional derivative of \( h \) at \( P \) is equal to 1 for every direction vector.
   
   (e) Let \( h(x, y) \) be a function from \( \mathbb{R}^2 \) to \( \mathbb{R} \) and \( P \) a point in the domain of \( h \). It is possible that the directional derivative of \( h \) at \( P \) is equal to 0 for every direction vector.

2. (3 pts.) Suppose function \( f(x, y) \) depends on variables \( x \) and \( y \), which are themselves functions of variables \( \alpha, \beta, \) and \( \gamma \) (i.e., \( x = x(\alpha, \beta, \gamma) \) and \( y = y(\alpha, \beta, \gamma) \)). Fill in the blanks for the chain rule to compute \( \frac{\partial f}{\partial \beta} \):

\[
\frac{\partial f}{\partial \beta} = \frac{\partial}{\partial \beta} \left( \frac{\partial f}{\partial x} \frac{\partial x}{\partial \beta} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial \beta} \right)
\]

3. (3 pts.) Let

\[
g(u, v) = u^2 + v^3, \\
u(t) = \cos(t), \\
v(t) = \ln(t).
\]

Compute \( \frac{dg}{dt} \). (Please use only the variable \( t \) in your response, but do not bother multiplying everything out.)

4. (3 pts.) Suppose \( z \) is a function of \( x \) and \( y \) and that \( x^2z^2 + y\sin(z) = 1 \). Find \( \frac{\partial z}{\partial x} \).
5. (3 pts.) Find the derivative of \( f(x,y) = xy - y^2 \) at point \((1,2)\) in the direction of \( \mathbf{v} = \langle 3, 4 \rangle \). Please simplify your answer to a number. (Notice that \( \mathbf{v} \) is not a unit vector!)

6. (3 pts.) Find a direction vector \( \mathbf{v} \) such that the derivative of \( f(x,y) = xy - y^2 \) at point \((1,2)\) in the direction of \( \mathbf{v} \) is equal to 0.