HW 3
Math 261, F19

Please see the course syllabus for details on how to turn in your homework assignments. This one is due at the beginning of your class on Friday, September 27.

1. (5 pts.) True or False
   (a) Let \( h(x, y) \) be a continuous function. Then for any point \((x_0, y_0)\) in the domain of \( h \), the limit of \( h \) as \((x, y)\) approach the point \((x_0, y_0)\) exists.
   (b) Let \( h(x, y) = x/y \). The limit of \( h \) as \((x, y)\) approaches the point \((1, 1)\) exists.
   (c) Let \( h(x, y) = x/y \). The limit of \( h \) as \((x, y)\) approaches the point \((0, 0)\) exists.
   (d) Let \( h(x, y) = \begin{cases} 3 & (x, y) = (0, 0) \\ 1 & (x, y) \neq (0, 0) \end{cases} \). The limit of \( h \) as \((x, y)\) approach the point \((0, 0)\) doesn’t exist.
   (e) Let \( h(x, y) = \begin{cases} 3 & (x, y) = (0, 0) \\ 1 & (x, y) \neq (0, 0) \end{cases} \). The limit of \( h \) as \((x, y)\) approach the point \((0, 0)\) is equal to 3.

2. (3 pts.) If \( f(x, y, z) = \sqrt{x^3 + \sin(y) - y \ln(z)} \), find \( f(2, \frac{\pi}{2}, 1) \). Perform elementary simplifications.

3. (3 pts.) Sketch the domain of \( g(x, y) = \ln(1 - 2x - 2y) \).

4. (3 pts.) Let \( h(x, y, z) = 3x^2z + z \cos(\pi y - \pi x) + 3e^z \). Determine \( \lim_{(x,y,z)\to(1,2,0)} h(x, y, z) \).

5. (3 pts.) The function \( k(x, y) = \frac{7x^8 y}{-2x^9 + 9y^9} \) has no limit as \((x, y) \to (0, 0)\).
   
   Show this by computing the limit of the function along the two following paths:
   (a) \( t \mapsto (t, 0) \). This notation indicates the path \((x(t), y(t)) = (t, 0)\), or equivalently, the path given by \( y = 0 \).
   (b) \( t \mapsto (t, t) \). This notation indicates the path \((x(t), y(t)) = (t, t)\), or equivalently, the path given by \( y = x \).

   Note (and hint): the nice thing about the parametric notation for the paths \( t \mapsto (f(t), g(t)) \) is that it suggests what you should do to compute the limit along the path: plug in the function \( f(t) \) for \( x \), the function \( g(t) \) for \( y \), and then take the limit as \( t \to 0 \).

6. (3 pts.) Compute \( \frac{\partial h}{\partial x} \) for the function in #4.