HW 10
Math 261, F19

This is the last homework set of the semester! This one is due at the beginning of your class on **Friday, December 6**.

1. (5 pts.) True or False:
   (a) Let $\mathbf{F} = \langle M(x), N(y) \rangle$ be a vector field on the plane and C a closed curve that bounds a region R. The circulation of $\mathbf{F}$ along C is equal to 0.
   (b) Let $\mathbf{F} = \langle M(y), N(x) \rangle$ be a vector field on the plane and C a closed curve that bounds a region R. The circulation of $\mathbf{F}$ along C is equal to 0.
   (c) Let $\mathbf{F} = \langle M(x), N(y) \rangle$ be a vector field on the plane and C a closed curve that bounds a region R. The flux of $\mathbf{F}$ through C is equal to 0.
   (d) Let $\mathbf{F} = \langle M(y), N(x) \rangle$ be a vector field on the plane and C a closed curve that bounds a region R. The flux of $\mathbf{F}$ through C is equal to 0.
   (e) Let $\mathbf{F} = \langle a, b \rangle$ be a constant vector field on the plane and C a closed curve that bounds a region R. The circulation of $\mathbf{F}$ along C and the flux of $\mathbf{F}$ through C are both equal to 0.
2. (3 pts.) Let C denote the unit circle, oriented clockwise. Evaluate the line integral $\displaystyle\int_C y\,dx - x\,dy$ in two different ways: first by parameterizing the curve and using the definition of line integral; then, use Green's theorem.
3. (3 pts.) Set up AND EVALUATE the integral(s) to compute the outward flux of the vector field $\mathbf{F} = \langle xy, y^2 \rangle$ for the region enclosed by the curves $y = x^2$ and $y = x$ in the first quadrant, <u>without</u> parameterizing the curves.
4. (3 pts.) Use Green's Theorem (either version) to set up AND EVALUATE $\displaystyle\int_C y^2 dx + x^2 dy$ where C is the triangle bounded by $x = 0$, $x + y = 1$, and $y = 0$ (with counter-clockwise orientation).
5. (3 pts.) Parameterize the portion of the plane $2x - 3y + z = 5$ over the rectangle $0 \le x \le 1$, $2 \le y \le 5$. In particular, please provide $\mathbf{r}(x, y)$ in the parameters x and y.
6. (3 pts.) Suppose some surface S is parameterized by $\mathbf{r}(u, v) = \langle u^2, uv, v^2 \rangle$, for $-1 \le u \le 1$, $0 \le v \le 2$. Set up but DO NOT EVALUATE an integral to find the surface area of S.

There will not be time to cover the divergence theorem before this final homework set is due, so here's one to try on your own time before the final exam (<u>not worth any points</u>):

Let $\mathbf{F} = \langle 3x^2, -2xy, -3xz \rangle$ and let D be the solid cut from the first octant by the plane $x + 2y + z = 2$. Write down an integral to find the outward flux of $\mathbf{F}$ across the boundary of D using the Divergence Theorem, i.e., set up (BUT DO NOT EVALUATE) the triple integral in the Divergence Theorem. Use the variable order $dz\ dy\ dx$.