We will not answer questions about this page during the exam.

f is a real-valued function and $\mathbf{F}(x,y,z) = \langle M, N, P\rangle$ is vector-valued. (If in $\mathbb{R}^2$, $\mathbf{F} = \langle M, N\rangle$.)
$\mathbf{T}$ is an appropriate unit tangent vector and $\mathbf{n}$ is an appropriate unit normal vector.
$\mathbf{r}(t) = \langle f(t), g(t), h(t)\rangle$ is a parameterization of a curve in $\mathbb{R}^3$ ($\mathbf{r}(t) = \langle f(t), g(t)\rangle$ in $\mathbb{R}^2$);
$\mathbf{r}(u,v) = \langle f(u,v), g(u,v), h(u,v)\rangle$ is a parameterization of a surface, with $\mathbf{r}_u = \frac{\partial \mathbf{r}}{\partial u}$ and $\mathbf{r}_v = \frac{\partial \mathbf{r}}{\partial v}$.

Line Integral along a curve C : $\displaystyle\int_C f(x,y,z)\, ds = \int_a^b f(\mathbf{r}(t))|\mathbf{v}(t)|\, dt$, where $\mathbf{v}(t) = \mathbf{r}'(t)$.

Work/Circ/Flow along a curve C :

in $\mathbb{R}^2$: Work/Circ/Flow $\displaystyle = \int_C \mathbf{F}\cdot d\mathbf{r} = \int_C M dx + N dy$. Also see Green's Theorem.

in $\mathbb{R}^3$: Work/Circ/Flow $\displaystyle = \int_C \mathbf{F}\cdot d\mathbf{r} = \int_C M dx + N dy + P dz$. Also see Stokes' Theorem.

Flux of vector field $\mathbf{F}$:

across curve $C \subset \mathbb{R}^2$: $\displaystyle\int_C \mathbf{F}\cdot\mathbf{n}\, ds = \int_C M dy - N dx$. Also see Green's Theorem.

through surface $S \subset \mathbb{R}^3$: $\displaystyle\iint_S \mathbf{F}\cdot\mathbf{n}\; d\sigma = \iint_S \mathbf{F}\cdot(\mathbf{r}_u \times \mathbf{r}_v)\, du dv$. Also see Divergence Theorem.

Component Test: $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}, \dfrac{\partial M}{\partial z} = \dfrac{\partial P}{\partial x}, \dfrac{\partial N}{\partial z} = \dfrac{\partial P}{\partial y}$. Equivalently, $curl(\mathbf{F}) = \nabla \times \mathbf{F} = \mathbf{0}$.

Fundamental Theorem for Line Integrals: If $\mathbf{F} = \nabla f$ and curve C goes from A to B, then

$$\int_C \mathbf{F}\cdot d\mathbf{r} = f(B) - f(A).$$

Green's Theorem: Region $R \subset \mathbb{R}^2$ has closed boundary curve C.

$$\text{Work/Circ/Flow } = \oint_C \mathbf{F}\cdot d\mathbf{r} = \int_C M dx + N dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\, dx\, dy = \iint_R \nabla\times\mathbf{F}\cdot\mathbf{k}\, dx\, dy,$$

$$\text{Flux } = \oint_C \mathbf{F}\cdot\mathbf{n}\, ds = \int_C M dy - N dx = \iint_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right)\, dx\, dy = \iint_R \nabla\cdot\mathbf{F}\, dx\, dy$$

Surface Integral of g over the surface S ($g(x,y,z) = 1$ for surface area):

$$\iint_S g(x,y,z)\, d\sigma = \iint_R g(\mathbf{r}(u,v))|\mathbf{r}_u \times \mathbf{r}_v|\, du\, dv, \text{ with parameters } u, v \text{ in } R.$$

Stokes' Theorem: Surface S with closed boundary curve C (where C has counterclockwise orientation with respect to the normal direction of S).

$$\text{Work/Circ/Flow } = \oint_C \mathbf{F}\cdot d\mathbf{r} = \int_C M dx + N dy + P dz = \iint_S \nabla\times\mathbf{F}\cdot\mathbf{n}\, d\sigma = \iint_S \nabla\times\mathbf{F}(\mathbf{r}(u,v))\cdot(\mathbf{r_u}\times\mathbf{r_v})\, du\, dv.$$

Divergence Theorem : Solid D with boundary surface S.

$$\text{Flux} = \iint_S \mathbf{F}\cdot\mathbf{n}\; d\sigma = \iint_S \mathbf{F}(\mathbf{r}(u,v))\cdot(\mathbf{r}_u\times\mathbf{r}_v)\, du dv = \iiint_D \nabla\cdot\mathbf{F}\, dV = \iiint_D \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z} dV.$$