

Exam #2

M235

Spring 2018

Name: _____ Score: _____

Read the problems. Read them again. Make sure to express your answers in a *clear and logical* way. *Don't rush!*

1. (20 pts.) TRUE-FALSE: no explanation is needed for this problem.

Let $f : X \rightarrow Y$ be a function.

- (a) If $x_1 = x_2$, then $f(x_1) = f(x_2)$.
- (b) If $f(x_1) = f(x_2)$, then $x_1 = x_2$.
- (c) If f is injective and $f(x_1) = f(x_2)$, then $x_1 = x_2$.
- (d) If f is surjective and $f(x_1) = f(x_2)$, then $x_1 = x_2$.
- (e) If $x \in f^{-1}(\{y\})$, then $f(x) = y$.
- (f) If $x_1 \in f^{-1}(\{y\})$ and $x_2 \in f^{-1}(\{y\})$, then $f(x_1) = f(x_2)$.
- (g) If $x_1 \in f^{-1}(\{y\})$ and $x_2 \in f^{-1}(\{y\})$, then $x_1 = x_2$.
- (h) If f is injective, $x_1 \in f^{-1}(\{y\})$ and $x_2 \in f^{-1}(\{y\})$, then $x_1 = x_2$.
- (i) If f is injective, then $f(X) = Y$.
- (j) If f is surjective, then $f(X) = Y$.

2. (20 pts.) Let $X = \{N, E, S, W\}$ and $Y = \{0, \pi/2, \pi, 3\pi/2\}$.

- (a) Give an example of an invertible function $f : X \rightarrow Y$.
- (b) Write down the inverse function f^{-1} .
- (c) Give an example of an invertible function $g : Y \rightarrow X$ which is not the inverse of f .
- (d) Compute the compositions $g \circ f$ and $f \circ g$.

3. (20 pts.) Let X be the set of all possible orderings (or permutations) of the natural numbers $1, 2, \dots, 7$, and Y be the set of all possible orderings of the natural numbers $1, 2, \dots, 6$. Define the function $p : X \rightarrow Y$ as follows: given an ordering of the first 7 integers, obtain an ordering of the first 6 integers simply by deleting the number 7.

- (a) What is $p(1, 3, 5, 7, 2, 4, 6)$?
- (b) Prove that p is a surjective function.
- (c) Write all elements of $p^{-1}(1, 2, 4, 6, 5, 3)$.

Define an equivalence relation by setting two orderings of $1, 2, \dots, 7$ to be equivalent if and only if they take the same value under the function p .

(If you prefer symbols, for $\sigma_1, \sigma_2 \in X$: $\sigma_1 \sim \sigma_2 \iff p(\sigma_1) = p(\sigma_2)$).

- (d) How many elements are in each equivalence class?
- (e) How many elements does the quotient set $X / \sim$ have?

4. (20 pts.) Recall that a function $f : \mathbb{R} \rightarrow \mathbb{R}$ is called **continuous** if it is *continuous at every point* $x_0 \in \mathbb{R}$.
- (a) Write the definition of $f : \mathbb{R} \rightarrow \mathbb{R}$ being continuous at a point $x_0 \in \mathbb{R}$.
 - (b) Prove that if $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ are continuous functions, then the composition $g \circ f : \mathbb{R} \rightarrow \mathbb{R}$ is a continuous function.

5. (20 pts.) Let X and Y be two finite sets with the same number of elements ($|X| = |Y|$), and $f : X \rightarrow Y$ a function.
- (a) Prove that if f is injective, then f is surjective.
 - (b) If X and Y are countable sets, does the statement in (a) still hold? (explain why it holds or give a counterexample if it doesn't).