

Exam #1

M235

Fall 2018

Name: _____ Score: _____

Read** the problems. Read them again. Make sure to express your answers in a **clear and logical** way. **Don't rush!

1. (20 pts.) Prove by induction the following identity. For all natural numbers n ,

$$\sum_{i=1}^n (2i + 1) = n^2 + 2n.$$

Careful! A proof by induction is not just a string of equations. Please supply the proper amount of connecting sentences that makes the proof readable and understandable by your non-mathematician cousin.

2. (20 pts.) Let $\mathcal{U}$ denote the set of all animals. Consider the following definitions:

Definition 1: A set $X \subseteq \mathcal{U}$ is **flick** if there exists one element of X that can fly.

Definition 2: A set $X \subseteq \mathcal{U}$ is **fluck** if every element of X can fly.

Give an example of:

- (a) A set which is flick and fluck.
- (b) A set which is flick but not fluck.
- (c) A set which is fluck but not flick.
- (d) A set which is neither flick nor fluck.

3. (20 pts.) TRUE-FALSE: no explanation is needed for this problem:

(a) If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

(b) If $A \subseteq C$ and $B \subseteq C$, then $A \cup B \subseteq C$.

(c) If $A \setminus B = \phi$, then $B \setminus A = \phi$.

(d) $A \subseteq B$ if and only if $B^c \subseteq A^c$.

(e) If $A \subseteq C$ and $B \subseteq C$, then $A \cap B \subseteq C$.

(f) If $A \cap B \subseteq C$, then $A \subseteq C$ and $B \subseteq C$.

4. (20 pts.) Match each of the statements below with the set theoretical statement it corresponds to:

- (a) If I don't put sunscreen, I burn.
- (b) If $2 + 2 = 5$, then moooo!
- (c) If it rains, it pours.
- (d) I neither ski nor skate.
- (e) I ski and I don't skate.

1:

$$(A \cup B)^c$$

2:

$$A^c \subseteq B$$

3:

$$A \subseteq B$$

4:

$$\phi \subseteq A$$

5:

$$A \cap B^c$$

5. (10 pts.) For any natural number $n \geq 3$, consider the expression

$$(x_1 + x_2 + x_3 + \dots + x_n)^3.$$

Prove that the largest coefficient of a monomial in the expansion of the above expression is 6.

Hint: recall the groupwork proof of Newton's theorem.

6. (10 pts.) Prove that the set of irrational numbers (real numbers which are not rational) is uncountable.