

Exam #1

M235

Fall 2020

Name: _____ Score: _____

Read** the problems. Read them again. Make sure to express your answers in a **clear and logical** way. **Don't rush!

1. (20 pts.) Prove the following identity by induction. For all natural numbers n ,

$$\sum_{i=1}^n (2i + 1) = n^2 + 2n.$$

Careful! A proof by induction is not just a string of equations. Please supply the proper amount of connecting sentences that makes the proof readable and understandable by your non-mathematician cousin.

2. (20 pts.) Let $\mathcal{U}$ denote the set of all animals. Consider the following definitions:

Definition 1: A set $X \subseteq \mathcal{U}$ is **flick** if there exists one element of X that can fly.

Definition 2: A set $X \subseteq \mathcal{U}$ is **fluck** if every element of X can fly.

Give an example of:

- (a) A set which is flick and fluck.
- (b) A set which is flick but not fluck.
- (c) A set which is fluck but not flick. *Careful here...this is almost a trick question.*
- (d) A set which is neither flick nor fluck.

3. (20 pts.) TRUE-FALSE: no explanation is needed for this problem:

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 (a) If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

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 (b) If $A \subseteq C$ and $B \subseteq C$, then $A \cup B \subseteq C$.

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 (c) If $A \setminus B = \phi$, then $B \setminus A = \phi$.

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 (d) $A \subseteq B$ if and only if $B^c \subseteq A^c$.

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 (e) If $A \subseteq C$ and $B \subseteq C$, then $A \cap B \subseteq C$.

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 (f) If $A \cap B \subseteq C$, then $A \subseteq C$ and $B \subseteq C$.

4. (20 pts.)

- (a) Write the definition of **countable set**.
- (b) Prove that if X is a countable set and Y is a finite set, then the cartesian product $X \times Y$ is a countable set.

5. (20 pts.)

- (a) Write the definition of $\binom{n}{k}$.
- (b) If X is a set with n elements, what type of subsets of X does the binomial coefficient $\binom{n}{k}$ count?
- (c) Write down Newton's binomial theorem.
- (d) Use (b) and (c) to prove that if X is a set with n elements, the total number of subsets of X is 2^n .