Math 281

Homework
Dec. 5th

1. Let $f : \mathbb{R}^5 \to \mathbb{R}$ be a linear function. What is the smallest possible dimension of $ker(f)$? If the kernel is larger dimensional, what can you say about f?

2. Let $f : \mathbb{R}^6 \to \mathbb{R}^3$ be a linear function. Can f be injective? Why?

3. Let $f : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear function. Can f be surjective? Why?

4. Let $f : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear function. Can f be injective but not surjective? How about surjective but not injective?

5. Let X be the vector space of polynomials in one variable of degree less than or equal to 3. What is the dimension of X? Exhibit a basis for it.

6. Let X be the vector space of polynomials in two variables of degree less than or equal to 2. What is the dimension of X? Exhibit a basis for it.

7. Let X be the vector space of 2×2 matrices with real coefficients. What is the dimension of X? Exhibit a basis for it.

8. Let $X \subseteq \mathbb{R}[x]$ be the vector subspace generated by $x^2, x^3+5, 4x^2+8x^4$. What is the dimension of X? Why? Does the polynomial x^4 belong to X?

9. Let $X \subseteq \mathcal{C}[0,1]$ be the vector subspace generated by the functions $1, \sin^2(x), \cos^2(x)$. What is the dimension of X (careful!!)? Explain.