Math 281

Homework
Oct. 24th

1. If $g, h \in G$ (note: the operation is not necessarily commutative) are group elements, what is the inverse of the element gh in terms of g^{-1} and h^{-1}?

2. For each of the following functions, prove or disprove that it is a group homomorphism:

   (a)
   $$\begin{array}{rccc} f: & \mathbb{Z}/3\mathbb{Z} & \to & \mathbb{Z}/6\mathbb{Z} \\ & [x] & \mapsto & [4x] \end{array}$$

   (b)
   $$\begin{array}{rccc} f: & \mathbb{Z}/3\mathbb{Z} & \to & \mathbb{Z}/6\mathbb{Z} \\ & [x] & \mapsto & [3x] \end{array}$$

   (c)
   $$\begin{array}{rccc} f: & S_2 & \to & S_3 \\ & e & \mapsto & e \\ & (12) & \mapsto & (23) \end{array}$$

   (d)
   $$\begin{array}{rccc} f: & S_2 & \to & S_3 \\ & e & \mapsto & e \\ & (12) & \mapsto & (123) \end{array}$$

   (e)
   $$\begin{array}{rccc} f: & \mathbb{Z} & \to & S_5 \\ & n & \mapsto & (1245)^n \end{array}$$

3. For any of the above that are homomorphisms, write down the kernel and the image.

4. Let $\phi : G \to H$ be a group homomorphism. If $g \in G$ is an element of order k, what can you say about the order of $\phi(g)$? (Remember to look at the various examples of group homomorphisms and functions that are not group homomorphisms in order to understand what is going on!)

5. Let $A = (12)(34)$, $B = (13)(24)$, $C = (14)(23)$ be elements in S_4. Prove that the function below is a group homomorphism. Have you seen it before?

$$
\begin{array}{rccl}
\phi: & S_4 & \to & S_3 \\
 & g & \mapsto & \left\{ \begin{array}{ccc} A & \mapsto & g^{-1}Ag \\ B & \mapsto & g^{-1}Bg \\ C & \mapsto & g^{-1}Cg \end{array} \right\}
\end{array}
$$