

Exam #2

M590

Fall 2007

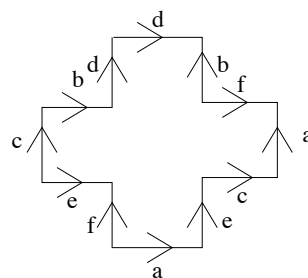
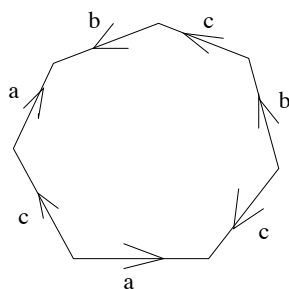
Name: _____ Score: _____

Read the problems. Read them again. Make sure to express your answers in a **clear and logical** way. **Don't rush!**

1. TRUE OR FALSE: (no explanation needed)

- (a) If X is homeomorphic to Y , then X is homotopy equivalent to Y .
- (b) If X is homotopy equivalent to Y , then X is homeomorphic to Y .
- (c) Let S be a closed compact surface. If the Euler characteristic of S is even, then S is orientable.
- (d) Let S be a closed compact surface. If S is orientable, then the Euler characteristic of S is even.

2. Decide whether the pictures below represent a surface or not. In case of a positive answer, recognize the surface.



3. Show that the two maps below are homotopic. You are NOT allowed to use the fact that the fundamental group of the circle is $\mathbb{Z}$ (unless you prove it!)

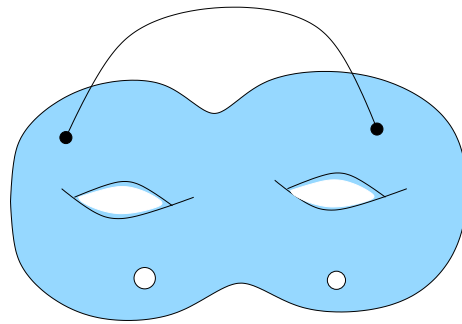
$$f: S^1 \rightarrow S^1$$

$$\theta \mapsto \theta$$

$$g: S^1 \rightarrow S^1$$

$$\theta \mapsto \begin{cases} -3\theta & 0 \leq \theta \leq 1/3 \\ 3\theta & 1/3 \leq \theta \leq 1 \end{cases}$$

4. The figure shown below (a two-torus with two little discs removed and a segment attached at two points) is homotopy equivalent to a bouquet of circles. How many circles? (show a sequence of pictures to substantiate your answer)



5. Assuming the fact:

$$f \sim g \Rightarrow \pi_1(f) = \pi_1(g) \quad ,$$

prove that the fundamental group is a homotopy invariant.

6. Proof that $\mathbb{R}^3$ is not homeomorphic to $\mathbb{R}^3 \setminus \{line\}$. (here you are allowed, if you wish, to use the fact that the fundamental group of a circle is $\mathbb{Z}$).