Exam #1

M590 Fall 2007

Name:_____ Score: _____

Read *the problems. Read them again. Make sure to express your answers in a* ***clear and logical*** *way.* ***Don't rush****!*

1. True or False: (no explanation needed)

   (a) If a set S in a topological space X is not open, then it must be closed.

   (b) The identity function
   $$Id : X \longrightarrow X$$
   is always continuous, no matter what topologies you put on X (you may put different topologies on domain and range!)

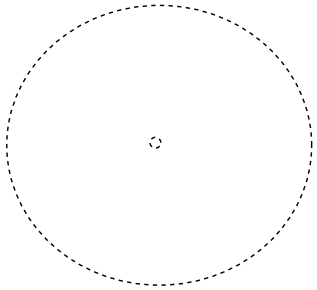
   (c) If
   $$f : X \longrightarrow Y$$
   is a homeomorphism, then there must exist a homeomorphism
   $$g : Y \longrightarrow X.$$

   (d) any subset of a space with the finite complement topology is compact.

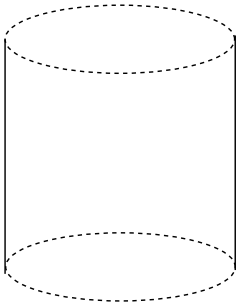
2. X, τ is a topological space. Y is a dense subset of X and it is a closed subset of X. What is Y? (explain)

3. Construct a homeomorphism between the punctured open disc and the open cylinder. Assume both spaces have the topology induced from the euclidean topology in $\mathbb{R}^2$ and $\mathbb{R}^3$ respectively. You can construct the homeomorphism both geometrically or algebraically. Even better if you do it in two different ways!

   Show also that open cilinder and punctured open disc are homeomorphic to a product space.



punctured open disc



open cylinder

4. Let X be a Hausdorff topological space. Prove that if a point $\{x\}$ is an open set, then it is a connected component of X.

5. If X and Y are **Hausdorff** topological spaces, consider the product space and the projections:

$$\pi_X : X \times Y \longrightarrow X,$$

$$\pi_Y : X \times Y \longrightarrow Y,$$

   (a) Prove that the projections are open functions (i.e. the image of any open set is open).

   (b) Are the projections closed functions in general?

   (c) Prove that, if X and Y are compact, then the projections are closed functions (i.e. the image of any closed set is closed).

   Hint: use some psychology in approaching this problem. Question 3 should suggest what the answer to question 2 should be...so try to construct a counterexample to 2! Get yourself the most familiar noncompact space (e.g. $\mathbb{R}$) and see if you can come up with a closed set in the plane that does not project to a closed set to one of the axes. Keep in mind that in $\mathbb{R}^n$ bounded closed sets are compact, so by point 3 they won't work to build a counterexample!!