

Equivalence Relations

1. Give an example of an equivalence relation.
2. Give an example of a surjective function.
3. Give an example of a partition of a set.
4. Let X be the set of students of our class. Does the rule
 $x \sim y$ if x and y are born in the same year
define an equivalence relation?
5. Let $X = \{1, 2, 3, 4, 5, 6, 7\}$. For each of (a), (b) and (c) do the subsets:

$$(a) \quad Y_1 = \{1, 7\}, Y_2 = \{2, 3, 4\}, Y_3 = \{6\}, Y_4 = \{5\}$$

$$(b) \quad Z_1 = \{1, 7\}, Z_2 = \{2, 3, 4\}, Z_3 = \{6\}$$

$$(c) \quad W_1 = \{1, 2, 5, 7\}, W_2 = \{2, 3, 4, 5, 6\}$$

define an equivalence relation on X ? How many elements does the quotient set have? Is $[2] = [3]$?

6. Let $X = \{Cat, Dog, Mouse, Sheep, Ram, Horse, Kangaroo\}$. Define an equivalence relation on X as follows: two elements of X are equivalent if they have the same number of letters.

Comments:

You don't have to prove that this rule is an equivalence relation. You can assume I gave you a good one! Also, if you want to save some time in writing you can just use the first letter to indicate each animal.

- (a) How many elements does the quotient set X/\sim have?
 - (b) Write down the equivalence classes for this equivalence relation.
 - (c) Write down the partition that this equivalence relation determines for the set X .
 - (d) Write down explicitly the projection function $p : X \rightarrow X/\sim$.
7. Let $X = \mathbb{Z}$ and $Y = \{0, 1, 2, 3, 4\}$ and define a surjective function:

$$rem : X \rightarrow Y$$

by $rem(x) =$ the remainder of division of x by 5. Describe the equivalence relation induced by rem on the set X . Describe the natural bijection between Y and the quotient set X/\sim .