manuscripta mathematica manuscript No.
(will be inserted by the editor)

Darren Glass[1] · Rachel Pries[2]

Corrigendum: Hyperelliptic curves with prescribed p-torsion

Manuscripta Math., volume 117, 2005, 3:299-317

The statement of [4, Prop. 2] is correct, but a case is missing from the proof. Here is the material necessary to complete the proof.

Consider the moduli space $\mathcal{H}_g$ of smooth hyperelliptic curves of genus g and its closure $\overline{\mathcal{H}}_g$ in $\overline{\mathcal{M}}_g$. Let $V_{g,f}$ denote the sublocus of $\overline{\mathcal{M}}_g$ whose points correspond to curves of genus g with p-rank at most f.

Proposition 1. *[4, Prop. 2] The locus $V_{g,f} \cap \overline{\mathcal{H}}_g$ is pure of codimension $g-f$ in $\overline{\mathcal{H}}_g$.*

The proof uses the intersection of $V_{g,f}$ with $\Delta_0 := \Delta_0[\overline{\mathcal{H}}_g]$, the sublocus of $\overline{\mathcal{H}}_g$ whose points correspond to stable hyperelliptic curves which are not of compact type, i.e., whose Picard variety is not represented by an abelian scheme. The text of page 3 contains the incorrect statement: "The boundary $\overline{\mathcal{H}}_g - \mathcal{H}_g$ consists of components Δ_0 and Δ_i for integers $1 \leq i \leq g/2$." In fact, Δ_0 is not irreducible for $g \geq 3$. By [3], [6], it is the union of components $\Xi_i := \Xi_i[\overline{\mathcal{H}}_g]$ for $0 \leq i \leq g-2$, where each Ξ_i is an irreducible divisor in $\overline{\mathcal{H}}_g$ and where Ξ_i and Ξ_{g-i-1} denote the same substack of $\overline{\mathcal{H}}_g$.

The following description of Ξ_0 and Ξ_i for $i \geq 1$ can be found in [1, Section 2.3]. If η is the generic point of Ξ_0, then the curve Y_η is an irreducible hyperelliptic curve self-intersecting in an ordinary double point P. The normalization of Y_η is a smooth hyperelliptic curve Y_1 of genus $g-1$ and the inverse image of P in the normalization consists of an orbit under the hyperelliptic involution. For $1 \leq i \leq g-2$, if η is the generic point of Ξ_i, then the curve Y_η has two components Y_1 and Y_2, which are smooth irreducible hyperelliptic curves, of genera $g_1 = i$ and $g_2 = g-1-i$, intersecting in two ordinary double points P and Q. The hyperelliptic involution ι stabilizes each of Y_1 and Y_2. The points P and Q form an orbit of the restriction of ι to each of Y_1 and Y_2.

Let C_0 be a component of $V_{g,f} \cap \overline{\mathcal{H}}_g$. The first paragraph of [4, proof of Prop. 2] shows that $\dim(C_0) \geq g-1+f$ and that C_0 intersects Δ_0. The second and third paragraphs of [4, proof of Prop. 2] prove the result in the case that C_0 intersects Ξ_0.

The second author is partially supported by NSF grant DMS 07-01303.

D. Glass: Gettysburg College, Gettysburg, PA 17325 e-mail: glass@math.columbia.edu

R. Pries: Colorado State University, Fort Collins, CO 80523-1874 e-mail: pries@math.colostate.edu

Proof. (Material needed to complete the proof of [4, Prop. 2].) To complete the proof, it suffices to consider the case that C_0 intersects Ξ_i for some $i \geq 1$. A point of $C_0 \cap \Xi_i$ is the moduli point of a curve Y with two components Y_1 and Y_2 as described above. Let f_1 (resp. f_2) be the p-rank of Y_1 (resp. Y_2). Since the toric part of $\mathrm{Jac}(Y)[p]$ contains a copy of the group scheme μ_p, [2, Ex. 9.2.8] implies that $f_1 + f_2 \leq f - 1$. For $\ell = 1, 2$, the curve Y_ℓ corresponds to a point of $V_{g_\ell, f_\ell} \cap \overline{\mathcal{H}}_{g_\ell}$. The hyperelliptic orbit $\{P, Q\}$ is determined by the choice of hyperelliptic orbits $\{P_1, Q_1\}$ on Y_1 and $\{P_2, Q_2\}$ on Y_2.

The clutching morphism

$$\lambda_{g_1,g_2} : \overline{\mathcal{H}}_{g_1;1} \times \overline{\mathcal{H}}_{g_2;1} \to \overline{\mathcal{H}}_{g_1+g_2+1}.$$

is a finite, unramified morphism between moduli spaces of labeled curves [5, Cor. 3.9], see also [1, Section 2.3]. Now $C_0 \cap \Xi_i$ is in the image of the restriction of λ_{g_1,g_2} to

$$(V_{g_1,f_1} \cap \overline{\mathcal{H}}_{g_1;1}) \times (V_{g_2,f_2} \cap \overline{\mathcal{H}}_{g_2;1}).$$

Since $g_1, g_2 < g$, one can apply an inductive approach. Applying the inductive hypothesis gives $\dim(V_{g_1,f_1} \cap \overline{\mathcal{H}}_{g_1;1}) = g_i + f_i$; (note that the marked orbit increases the dimension by 1). Thus $\dim(C_0 \cap \Xi_i) \leq (g_1 + f_1) + (g_2 + f_2) = g + f - 2$. Since Ξ_i is an irreducible divisor in $\overline{\mathcal{H}}_g$, this yields $\dim(C_0) \leq g + f - 1$, and thus $\mathrm{codim}(C_0, \overline{\mathcal{H}}_g) = g - f$.

References

[1] Jeffrey D. Achter and Rachel Pries. The p-rank strata of the moduli space of hyperelliptic curves. arXiv:0902.4637.

[2] Siegfried Bosch, Werner Lütkebohmert, and Michel Raynaud. *Néron models*, volume 21 of *Ergebnisse der Mathematik und ihrer Grenzgebiete (3) [Results in Mathematics and Related Areas (3)]*. Springer-Verlag, Berlin, 1990.

[3] Maurizio Cornalba and Joe Harris. Divisor classes associated to families of stable varieties, with applications to the moduli space of curves. *Ann. Sci. École Norm. Sup. (4)*, 21(3):455–475, 1988.

[4] Darren Glass and Rachel Pries. Hyperelliptic curves with prescribed p-torsion. *Manuscripta Math.*, 117(3):299–317, 2005.

[5] Finn F. Knudsen. The projectivity of the moduli space of stable curves. II. The stacks $M_{g,n}$. *Math. Scand.*, 52(2):161–199, 1983.

[6] Kazuhiko Yamaki. Cornalba-Harris equality for semistable hyperelliptic curves in positive characteristic. *Asian J. Math.*, 8(3):409–426, 2004.