

Pries: 460 Information and Coding Theory:
Sample Quiz 2, 2015.

1. How many source words do there need to be to force every UD binary encoding to have average word length at least 4? Explain why.
2. Find two vectors \vec{x} and \vec{y} and a radius ρ such that the spheres $S_2^5(\vec{x}, \rho)$ and $S_2^5(\vec{y}, \rho)$ do not overlap and the union of the spheres is $(\mathbb{Z}/2)^5$.
3. This is the generator matrix for the Hamming (7, 4) code:

$$H_{7,4} = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

A codeword \vec{c} is sent, one error occurs, and you receive $(1, 1, 1, 1, 1, 0, 0)$. What is \vec{c} ?

4. Let $C = \{c_1 = 000, c_2 = 111\}$. Let $P(c_1) = \frac{1}{3}$, $P(c_2) = \frac{2}{3}$.
Let $P(0 | 1) = P(1 | 0) = \frac{1}{4}$ and $P(1 | 1) = P(0 | 0) = \frac{3}{4}$.
 - (a) What is the probability that c_1 is sent and you receive a vector of weight 2?
 - (b) If you receive 110, what is the probability that c_1 was sent? what is the probability that c_2 was sent?

5. Use the same code and probabilities as the previous problem.
A codeword is sent and you use nearest neighbor decoding to interpret it.
Show that the probability that you decode incorrectly is $\frac{5}{25}$.

6. The code is $C = \{c_1 = 00000, c_2 = 11111\}$ and the channel has cross-over probabilities:

$$P(1 \text{ received} | 0 \text{ sent}) = \frac{1}{5}, \quad P(0 \text{ received} | 1 \text{ sent}) = \frac{2}{5}.$$

You receive the message 00011. Which codeword was most likely sent?

7. Let C be the code of all vectors in $(\mathbb{Z}/2)^n$ with even weight.
 - (a) Find the dimension and a basis of C . What is the information rate?
 - (b) Find the minimum distance of C . How many errors can this code correct? How many errors can it detect?
 - (c) Explain whether or not C achieves the Singleton bound.
8. Let C be an r -ary code of length n , dimension k , and minimum distance d .
Let $T : C \rightarrow (\mathbb{Z}/r)^{n-d+1}$ be the truncation map $T((x_1, \dots, x_n) = (x_d, \dots, x_n)$.
Prove that T is 1-to-1, in other words, if $\vec{x} \neq \vec{y}$, then $T(\vec{x}) \neq T(\vec{y})$.
Use this to prove the Singleton bound.