M 645 Fall 2004

The course will roughly follow Chapters 5, 6 and 7 of "***Partial Differential Equations***" by L. C. Evans but will supplement this text with various notes that will be available on the web site. The goal of the course is to develop the functional analysis tools needed to treat the questions of existence, uniqueness and regularity for linear partial differential equations of elliptic, parabolic and hyperbolic type. These tools are fundamental in continuing on to the study of nonlinear PDE's which is the subject of the next course, M646.

Course grades will be based on problems which will be assigned throughout the semester.

Introduction

Notation
Linear problems
Well posed problems

Building a Function Space

$C(U)$ *and* $C(\bar{U})$
$L^p(U)$ *and* $L^p_{loc}(U)$
    T_F *on* L^1 *and* L^2
    Weak derivatives on $L^p_{loc}(U)$
Definition of $W^{m,p}(U)$ *and* $H^m(U)$; completeness
$W^{m,p}_o(U)$ *and* $H^m_o(U)$
Mollifiers
    approximation by smooth functions in $L^p(U)$ *and* $L^p_{loc}(U)$
    applications of mollifiers
Hilbert-Sobolev spaces
Properties of $H^m(U)$
    Poincare inequality
    a closed subspace of $H^m(U)$; equivalent norms

Properties of a General Hilbert Space

Subspaces
    Projection theorem
Linear functionals and bilinear forms
    Riesz theorem
    Equivalence of problems lemma
Applications of the lemma
    proof of projection theorem
    orthogonal complement of $H^1_0(U)$
Lax-Milgram Lemma
Convergence

bounded sequences contain weakly convergent subsequences

More Advanced Properties of the Hilbert-Sobolev Spaces

T_F and the spaces H^s

$H^S \approxeq H^m(R^n)$, s=m

$P(D) : H^s \rightarrow H^{s-m}$

Sobolev embedding theorem

Rellich's embedding lemma

Trace theorem on R^n

Local charts and flattening the boundary

trace theorems for $H^m(U)$

extension from the boundary to the interior

Sobolev embedding

Rellich lemma

Properties of the Banach-Sobolev Spaces

Approximation

Extension

Trace theorem

Sobolev Inequalities

Embeddings: continuous, compact

Distributions

Test functions

Linear functionals on D(U)

linear space D'(U)

$L^1_{loc}(U) \subset D'(U)$

Derivatives

Convergence in D'(U)

Fourier transform: S and S'

The dual of a Hilbert space: pivot space, H^{-s} and $H^{-1}(U)$

Second Order Elliptic PDE's

Notation and terminology

Existence of weak solutions

Galerkin existence proof

L-M existence proof

Uniqueness of weak solutions

Regularity

Linear Evolution Equations

- Notation and terminology
- Existence of weak solutions
  - Galerkin existence proof for parabolic problems
  - Galerkin existence proof for hyperbolic problems
- Uniqueness of weak solutions
- Regularity
- Semigroup Approach to Linear Evolution Equations
  - Semigroup existence proof for parabolic problems
  - Semigroup existence proof for hyperbolic problems