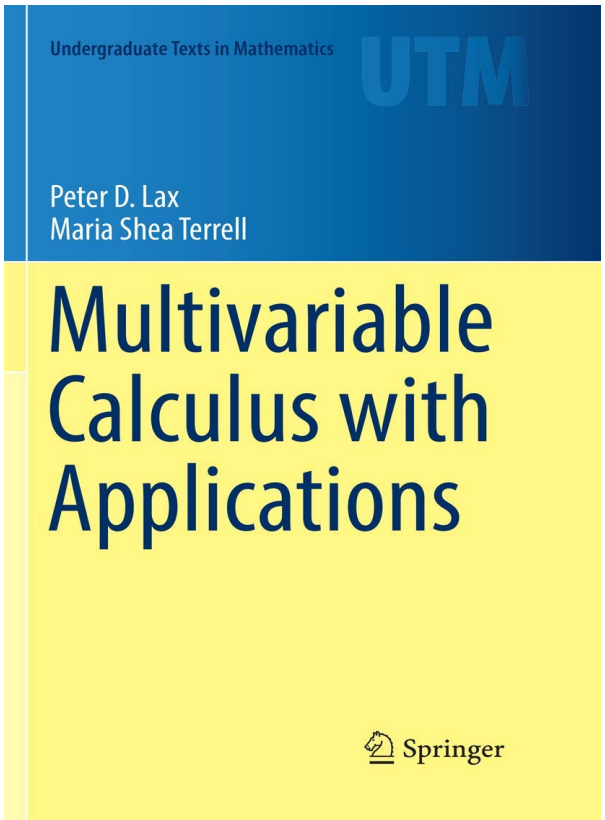
MATH 417 "Advanced Calculus" of Multivariables @ ColoState

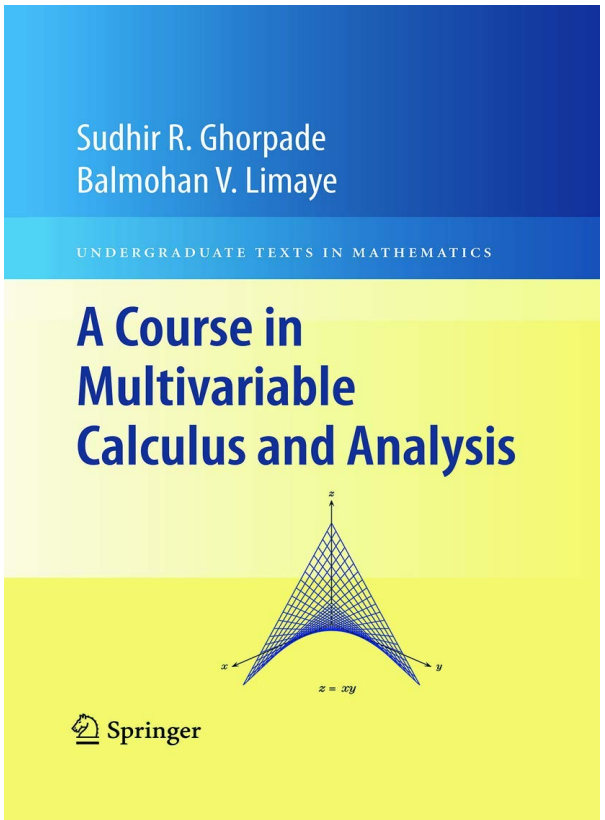
Fall 2024: Instructor Dr. James Liu

Classroom: Engrg. E104 (Not affected by Clark Building renovation)

Textbook? (still in search)



(2017)



(2010)

MATH 317 vs. MATH 417

MATH 317 "Advanced Calculus of One Variable"

– Denseness of $\mathbb{Q}$ in $\mathbb{R}$; Cauchy criterion;

– Uniform continuity; Mean value theorems;

– Uniform convergence; FTC: $\int_{[a,b]} F'(x) = [F(x)]_a^b$

MATH 417 "Advanced Calculus" of Multivariables

- Implicit function theorem;
- Taylor expansion; Hessian matrix for local min./max.
- Gradient, Divergence, Curl: theory and applications

  Gradient flow, Incompressible fluid flow, Electromagnetic fields;
- New: Differential forms on manifolds: $\int_\Omega d\omega = \int_{\partial\Omega} \omega$

Exploration to renovate MATH 417 for students in the new era.

Taylor Expansion

MATH 317:

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

MATH 417: For a scalar-valued multivar. fxn. $f(\mathbf{x}) = f(x_1, \cdots, x_n)$,

$$f(\mathbf{x}) = f(\mathbf{a}) + \mathcal{D}f(\mathbf{a})\,(\mathbf{x}-\mathbf{a}) + \frac{1}{2}(\mathbf{x}-\mathbf{a})^T \mathcal{H}f(\mathbf{a})(\mathbf{x}-\mathbf{a}) + \cdots$$

– 1st order derivatives form the gradient vector;

– 2nd order derivatives form symmetric Hessian matrix $\left[\frac{\partial^2 f}{\partial x_i \partial x_j}\right]$;

"Eigenvalues all positive" implies local minimum;

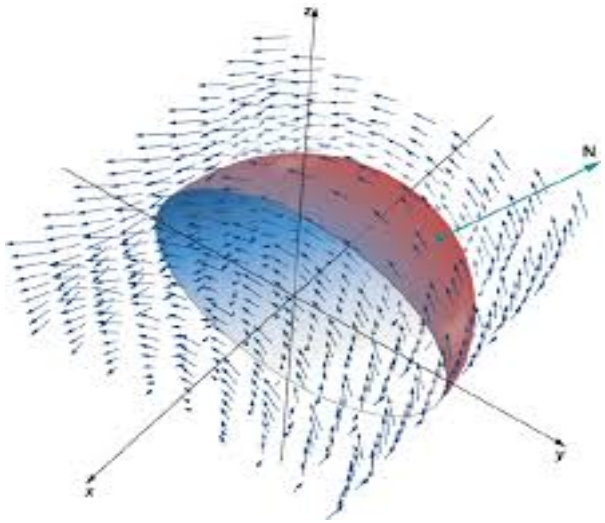
– How about higher order terms?

– How about a vector-valued function $\mathbf{f} : \mathbb{R}^n \to \mathbb{R}^m$?

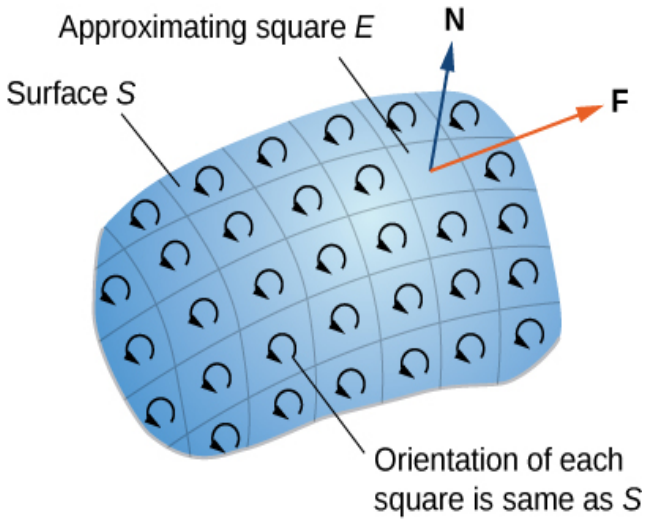
Stokes' Theorem

Curl – Circulation

$$\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S} = \int_{\partial S} \mathbf{F} \cdot d\mathbf{r}$$



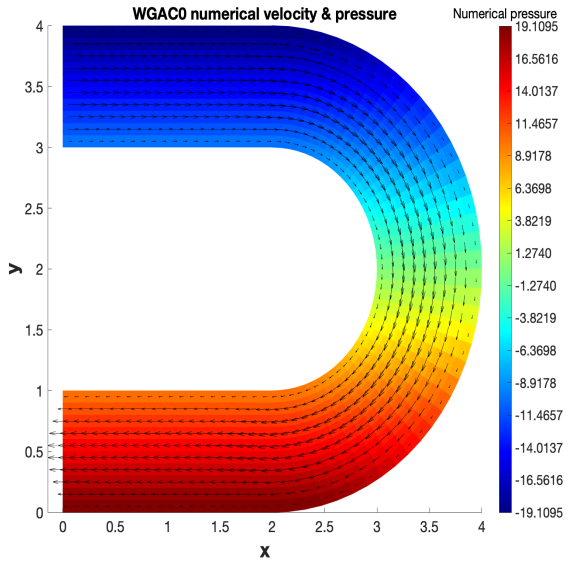
(a) Verifying theorem on a hemisphere



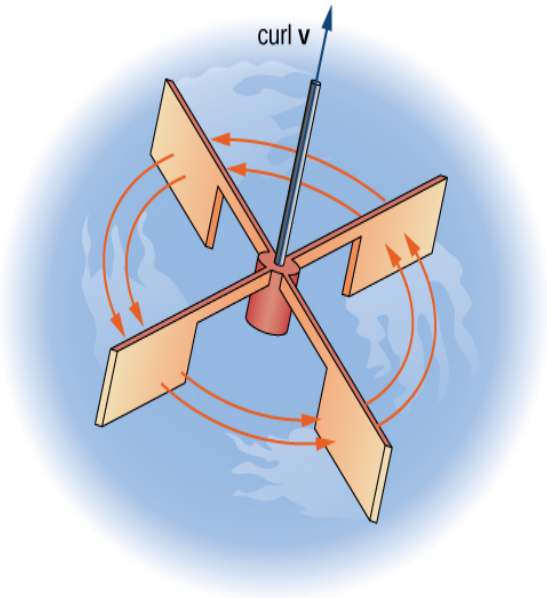
(b) Intuition for proof

Figure: Sources: (a)(b) `math.libretexts.org`

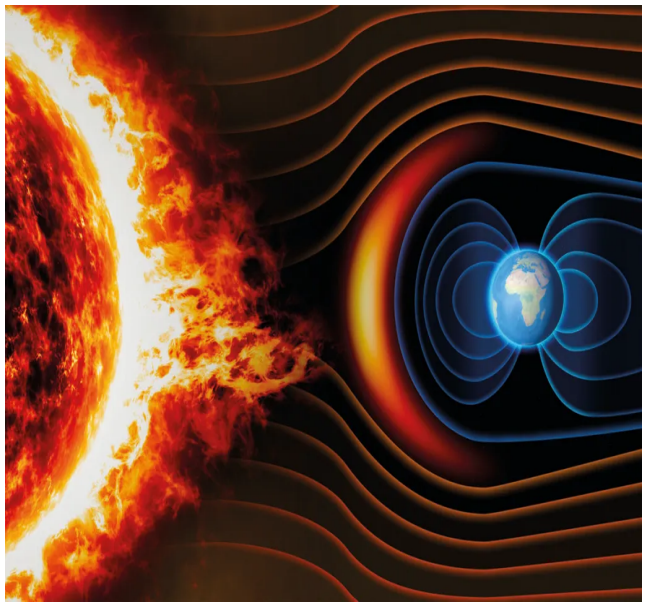
Vector Fields in applications



(a) Div-free flow thru a duct



(b) Curl (paddle-wheel)



(c) Shield by magnetosphere

Figure: Sources: (b) `math.libretexts.org`; (c) BBC `skyatnightmagazine`