M561 Problem Set Number 10

1. Find the global minimizers of

$$f(x,y) = e^{x-y} + e^{y-x}$$

2. Determine all the global and local minimizers of

$$f(x,y) = x^3 - 12xy + 8y^3$$

3. Consider

$$f(x,y) = 4x^2 - 4xy + 2y^2$$

Compute the steepest descent iterations $x^{(1)}, x^{(2)}, x^{(3)}$ taking $x^{(0)} = (2,3)^T$.

4. Show that if A is a symmetric positive definite matrix then the quadratic form

$$Q_A(x) = \frac{1}{2}x^T Ax - x^T b$$

has a strict global minimum for $x = x^*$ where $Ax^* = b$. Suggest an iterative approach for solving $Ax = b$.

5. The surface in n-dimensional space defined by the equation

$$x^T Ax = 1$$

where A is a real symmetric matrix of order n is known as a *central quadric*. Let $n = 3$ and classify the five types of possible surfaces. Hint: use the appropriate coordinates and Maple.