

APPLICATION OF THE SINGULAR VALUE DECOMPOSITION TO IMAGE REPRESENTATION

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ABSTRACT. This is a sample report for M435. We discuss latex and matlab and in particular how to get your matlab results into latex. Of course, we will talk about writing abstracts in class. All reports need one! It will be a very brief summary of the work.

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1. INTRODUCTION

General introduction to the problem. Why is it interesting? Instead, a few comments on Latex to get us prepared to write reports. Good news: it is really quite easy and I am prepared to help when you get stuck. One writes an equation quite simply by typing in

```
$$X = U \Sigma V^T $$
```

and this produces

$$X = U\Sigma V^T$$

If I want to be able to reference the equation then we need a little more. Now,

```
\begin{equation}
\label{SVD-eq}
X = U \Sigma V^T
\end{equation}
```

Notice the dollar signs are now missing. This code produces

(1.1) $X = U\Sigma V^T$

and we can now draw attention to Equation (1.1) using

...now draw attention to Equation (\ref{SVD-eq})...

2. MATHEMATICAL THEORY AND MODELING APPROACH

Now the SVD is all about truncating the representation. We can re-express Equation (1.1) as

$$X = \sum_{i=1}^k \sigma_i u_i v_i^T$$

A k -term approximation will be exact when k is equal to the rank of X . In matlab we can examine rank k approximations very easily!!

$$X_k = U(:, 1 : k) * \Sigma(1 : k, 1 : k) * V(:, 1 : k)^T$$

3. DATA

The Saturn image for this problem may be downloaded from the class web page. The format of the data is "uint8", which stands for unsigned integer, 8 bits. Before you apply the SVD, change it to "double" format:

```
A=imread('saturn.tif')
A=double(A);
```

To display the image use

```
imagesc(A);
colormap(gray);
```

Use the singular value decomposition to construct a low-rank approximation to Saturn.

```
[U,D,V] = svd(X);
```

To see the results incorporated into our document examine Figure 1.

4. RESULTS

Use Figures embedded in the text and cited in the text. Here are a few tasks for this class.

- embed the picture of the eigenvectors into the text as done above.
- compute a 10 term reconstruction of Saturn and embed in text.
- compute the column mean and subtract this from all the columns of the original Saturn and reproduce the eigenvectors. What do you notice about the eigenvectors and eigenvalues (the latter is not trivial).

4.1. **Analysis of Results.** Predictions, added value.

4.2. **Weaknesses of the approach.**

5. CONCLUSIONS AND SUMMARY

Answer the questions: what did you learn about this question that you could not have discovered without your mathematics? " if you had 3 months to work on this problem what would you do?

Matlab Code Background information or theory

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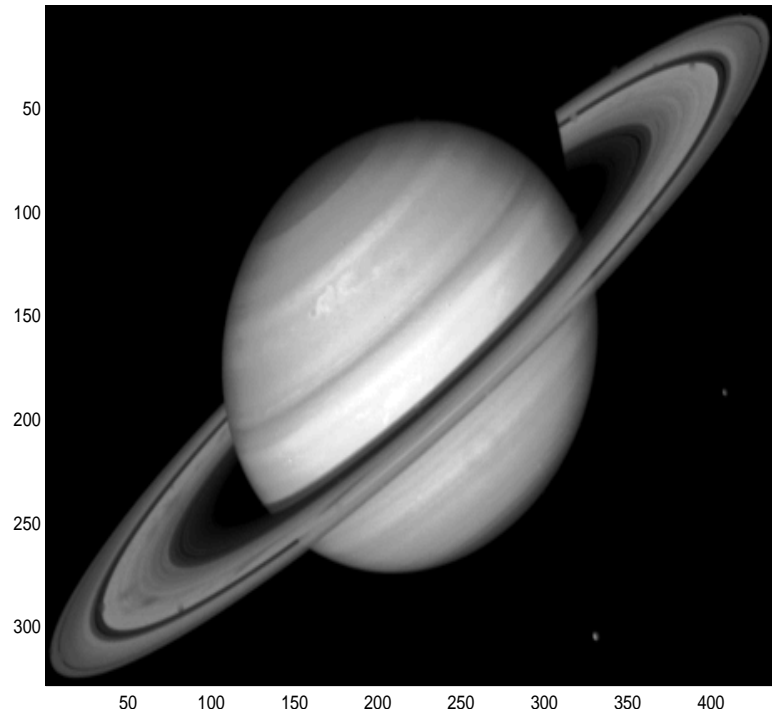


FIGURE 1. The matlab picture of Saturn.