Hmwk #6

Problem 4.15

STEP 1: graph data and choose M (the limiting value or carrying capacity).

STEP 2:

Use model $\Delta p_n = k p_n (M - p_n)$ and find k.

$$E = \sum (\Delta p_n - k p_n (M - p_n))^2$$

$$\frac{\partial E}{\partial k} = \sum 2(\Delta p_n - k p_n (M - p_n))(-p_n (M - p_n)) = 0$$

$$k = \frac{\sum (\Delta p_n) p_n (M - p_n)}{\sum p_n^2 (M - p_n)^2}$$

STEP 3: Iterate (using modified pop2.m)

$p_{n+1} = p_n + k p_n (M - p_n)$ using k from step 2 and M from step 1. for enough points that you can see some long-term behavior.

PROBLEM 4.17

Let $T_1 = 50$, $M = 425$

STEP 1: Use modification of pop2.m to generate 400 pts of $T_{n+1} = T_n + 0.01(M - T_n)^{1.25}$

STEP 2: Use the first 200 points from step 1 to estimate k in $T_{n+1} = T_n + k(M - T_n)$

$$\Delta T_n = k(M - T_n)$$

$$E = \sum (\Delta T_n - k(M - T_n))^2$$

$$\frac{\partial E}{\partial k} = \sum 2(\Delta T_n - k(M-T_n))(-(M-T_n)) = 0$$

$$\sum (\Delta T_n)(M-T_n) = k \sum (M-T_n)^2$$

$$k = \frac{\sum (\Delta T_n)(M-T_n)}{\sum (M-T_n)^2}$$

STEP 3: Use the k from step 2 to iterate $T_{n+1} = T_n + k(M - T_n)$ using modification of pop2.m. with $M = 425$, $T_1 = 50$ (generate 400 pts).

STEP 4: Compute error between pts 200–400 of data from Step 1 and step 3.

$$E = \sum_{n=201}^{400} \left(T_n^{step1} - T_n^{step3}\right)^2$$

Problem 4.18

$$\Delta a_n = c_1 a_n + d_1 b_n$$
$$\Delta b_n = c_2 a_n + d_2 b_n$$
$$E = \sum (\Delta a_n - c_1 a_n - d_1 b_n)^2 + \sum (\Delta b_n - c_2 a_n - d_2 b_n)^2$$

$$(i)\quad \frac{\partial E}{\partial c_1} = \sum 2(\Delta a_n - c_1 a_n - d_1 b_n)(-a_n)$$
$$(ii)\quad \frac{\partial E}{\partial d_1} = \sum 2(\Delta a_n - c_1 a_n - d_1 b_n)(-b_n)$$
$$(iii)\quad \frac{\partial E}{\partial c_2} = \sum 2(\Delta b_n - c_2 a_n - d_2 b_n)(-a_n)$$
$$(iv)\quad \frac{\partial E}{\partial d_2} = \sum 2(\Delta b_n - c_2 a_n - d_2 b_n)(-b_n)$$

$$(\hat{i})\quad \sum \Delta a_n a_n - c_1 \sum a_n^2 - d_1 \sum a_n b_n = 0$$
$$(\hat{ii})\quad \sum \Delta a_n b_n - c_1 \sum a_n b_n - d_1 \sum b_n^2 = 0$$
$$\underbrace{\Rightarrow \begin{bmatrix} \sum a_n^2 & \sum a_n b_n \\ \sum a_n b_n & \sum b_n^2 \end{bmatrix} \begin{bmatrix} c_1 \\ d_1 \end{bmatrix} = \begin{bmatrix} \sum \Delta a_n a_n \\ \sum \Delta a_n b_n \end{bmatrix}}$$

$$(\hat{iii})\quad \sum \Delta b_n a_n - c_2 \sum a_n^2 - d_2 \sum a_n b_n = 0$$
$$(\hat{iv})\quad \sum \Delta b_n b_n - c_2 \sum a_n b_n - d_2 \sum b_n^2 = 0$$
$$\underbrace{\Rightarrow \begin{bmatrix} \sum a_n^2 & \sum a_n b_n \\ \sum a_n b_n & \sum b_n^2 \end{bmatrix} \begin{bmatrix} c_2 \\ d_2 \end{bmatrix} = \begin{bmatrix} \sum \Delta b_n a_n \\ \sum \Delta b_n b_n \end{bmatrix}}$$

CODE IN MATLAB BY MODIFYING PREDATOR-PREY CODE.
AND USING DATA FROM TABLE 2.

$$c_1 = -.8, \quad d_1 = -.4$$
$$c_2 = -.2, \quad d_2 = -.6$$

PROBLEM 4.19

$$E = \sum \left(\Delta f_n + g_1 f_n - c_1 f_n r_n + d_1 f_n^2 \right)^2 + \sum \left(\Delta r_n - g_2 r_n + c_2 f_n r_n + d_2 r_n^2 \right)^2$$

$$
\begin{aligned}
&(i) && \frac{\partial E}{\partial g_1} = 0 = \sum 2 \left(\Delta f_n + g_1 f_n - c_1 f_n r_n + d_1 f_n^2 \right)(f_n) \\
&(ii) && \frac{\partial E}{\partial c_1} = 0 = \sum 2 \left(\Delta f_n + g_1 f_n - c_1 f_n r_n + d_1 f_n^2 \right)(-f_n r_n) \\
&(iii) && \frac{\partial E}{\partial d_1} = 0 = \sum 2 \left(\qquad \text{''} \qquad \right)(f_n^2) \\
&(iv) && \frac{\partial E}{\partial g_2} = 0 = \sum 2 \left(\Delta r_n - g_2 r_n + c_2 f_n r_n + d_2 r_n^2 \right)(-r_n) \\
&(v) && \frac{\partial E}{\partial c_2} = 0 = \sum 2 \left(\qquad \text{''} \qquad \right)(f_n r_n) \\
&(vi) && \frac{\partial E}{\partial d_2} = 0 = \sum 2 \left(\qquad \text{''} \qquad \right)(r_n^2)
\end{aligned}
$$

$$
\begin{aligned}
&(\hat{i}) && +\sum (\Delta f_n) f_n + g_1 \sum f_n^2 - c_1 \sum f_n^2 r_n + d_1 \sum f_n^3 = 0 \\
&(\hat{ii}) && -\sum (\Delta f_n) f_n r_n - g_1 \sum f_n^2 r_n + c_1 \sum f_n^2 r_n^2 - d_1 \sum f_n^3 r_n = 0 \\
&(\hat{iii}) && \sum (\Delta f_n) f_n^2 + g_1 \sum f_n^3 - c_1 \sum f_n^3 r_n + d_1 \sum f_n^4 = 0
\end{aligned}
$$

$$\Longrightarrow \begin{bmatrix} -\sum f_n^2 & \sum f_n^2 r_n & -\sum f_n^3 \\ -\sum f_n^2 r_n & \sum f_n^2 r_n^2 & -\sum f_n^3 r_n \\ -\sum f_n^3 & \sum f_n^3 r_n & -\sum f_n^4 \end{bmatrix} \begin{bmatrix} g_1 \\ c_1 \\ d_1 \end{bmatrix} = \begin{bmatrix} \sum (\Delta f_n) f_n \\ \sum (\Delta f_n) f_n r_n \\ \sum (\Delta f_n) f_n^2 \end{bmatrix}$$

$$
\begin{aligned}
&(\hat{iv}) && -\sum \Delta r_n r_n + g_2 \sum r_n^2 - c_2 \sum f_n r_n^2 - d_2 \sum r_n^3 = 0 \\
&(\hat{v}) && \sum \Delta r_n f_n r_n - g_2 \sum f_n r_n^2 + c_2 \sum f_n^2 r_n^2 + d_2 \sum f_n r_n^3 = 0 \\
&(\hat{vi}) && \sum \Delta r_n r_n^2 - g_2 \sum r_n^3 + c_2 \sum f_n r_n^3 + d_2 \sum r_n^4 = 0
\end{aligned}
$$

$$\Longrightarrow \begin{bmatrix} \sum r_n^2 & -\sum f_n r_n^2 & -\sum r_n^3 \\ \sum f_n r_n^2 & -\sum f_n^2 r_n^2 & -\sum f_n r_n^3 \\ \sum r_n^3 & -\sum f_n r_n^3 & -\sum r_n^4 \end{bmatrix} \begin{bmatrix} g_2 \\ c_2 \\ d_2 \end{bmatrix} = \begin{bmatrix} \sum (\Delta r_n) r_n \\ \sum (\Delta r_n) f_n r_n \\ \sum (\Delta r_n) r_n^2 \end{bmatrix}$$