HMWK #8 Nov 6, 2002 M331

AVG: 11.43/15
MEDIAN: 11.5/15

NOTES:

- The uniform approximation is <u>not</u> the same as solving a least squares problem.

5.1 (2pts).

- There were alot of good answers for this such as things to sell at a bake sale, what kind of cars to make, the best way to get drunk, ... my example:

I want to maximize my profit selling quilts in a month
A baby quilt takes 5 yrds of fabric and
a queen size quilt needs 16 yards of fabric.
I have available 100 yards of fabric to use.
A baby quilt takes 5 hours to assemble and
it takes 11 hours to assemble a queen size quilt.
I have 10 hours a week (40 hrs/month) to work on
the quilts. What is my profit and how many of
each quilt should I sell if I get $25 for each
baby quilt and $75 for each queen size quilt.
more decision variables: twin quilt, king quilt, lap quilt
more constraints: print/solid material, thread, machine/hand quilting.

5.2 (3 pts)

OBJECTIVE FXN:

max $p = x_1 + x_2 \longrightarrow x_2 = p - x_1$

CONSTRAINTS:

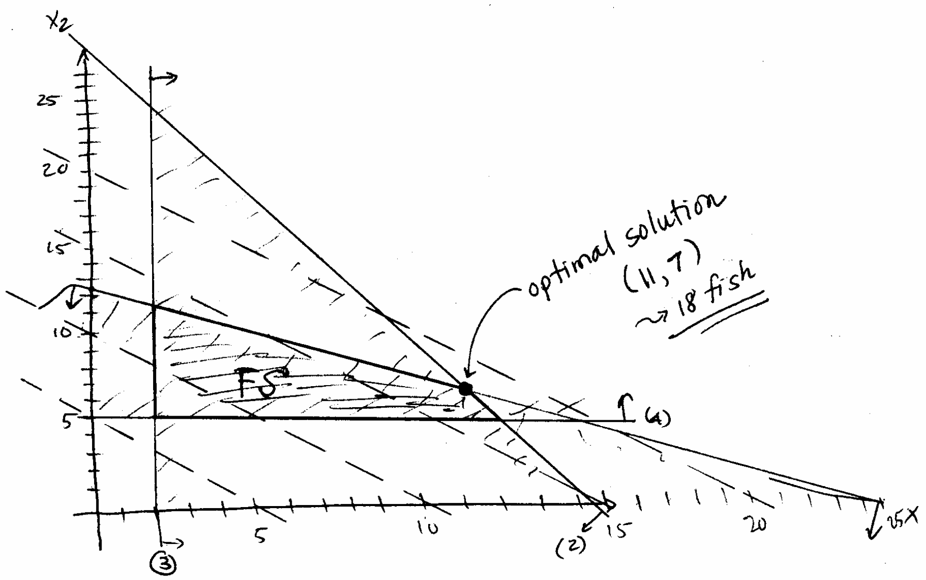
$2x_1 + 4x_2 \le 50 \longrightarrow 4x_2 \le 50 - 2x_1$ (1)

$x_2 \le \frac{25}{2} - \frac{1}{2}x_1$

$2x_1 + x_2 \le 29 \longrightarrow x_2 \le 29 - 2x_1$ (2)

$x_1 \ge 2$ (3)

$x_2 \ge 5$ (4)



11 G. Fish

7 D. Fish

Total of 18 fish

5.3 (4 pts)

OBJECTIVE FXN:

max $p = 45B + 25W$ $\rightsquigarrow$ $25W = p - 45B$

$W = \frac{p}{25} - \frac{45}{25}B \rightsquigarrow W = \frac{p}{25} - \frac{9}{5}B$

CONSTRAINTS:

① $\frac{1}{3}B + \frac{1}{4}W \le 100 \rightsquigarrow 4B + 3W \le 1200$

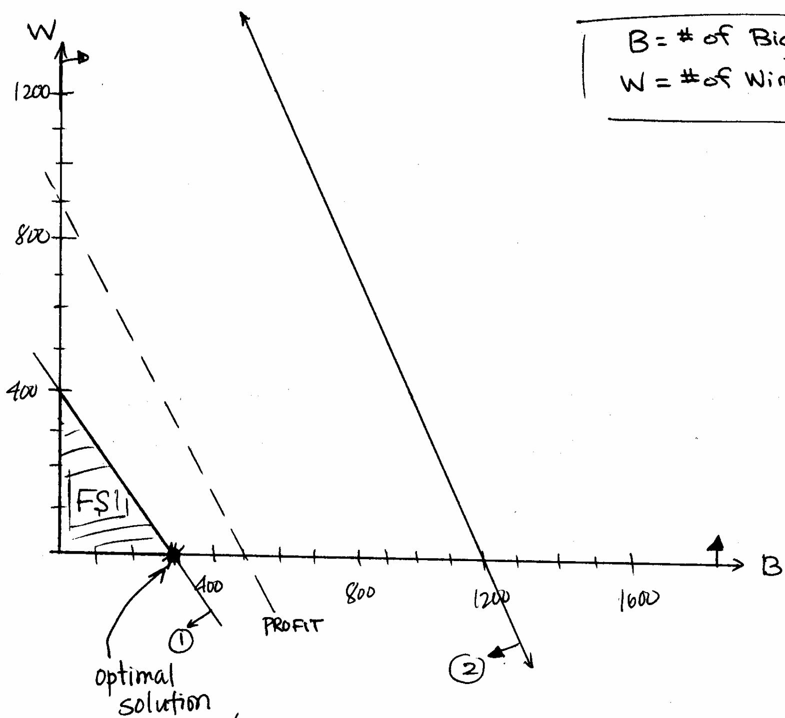
$3W \le 1200 - 4B$

$W \le 400 - \frac{4}{3}B$ ①

② $2B + W \le 2400 \rightsquigarrow W \le 2400 - 2B$ ②

$B \ge 0,\ W \ge 0$

B = # of Big Whoopies
W = # of Wimpy Whoopies



(300,0) ie no wimpy burgers and 300 big burgers)
Profit of \$135.00

b)

$p = 45B + \alpha W$

slope $\frac{-45}{\alpha}$ < slope of constraint ①

$\frac{-45}{\alpha} < \frac{-4}{3}$

$\frac{45}{\alpha} > \frac{4}{3}$

$\frac{45 \cdot 3}{4} > \alpha$

$\alpha < 33.75$

$\alpha \le 33$ cents

ie: price of whimpys $\le$ 33 cents to keep this optimal soln.

c) $p = \beta B + 25W$

TO MAKE ONLY BIG BURGERS:

$\frac{-\beta}{25}$ < slope constraint ①

$\frac{-\beta}{25} < \frac{-4}{3}$

$\beta > \frac{4 \cdot 25}{3}$

$\beta > 33.33$

$\beta \geq 34$ cents

cost of big burger $\geq$ 34 cents

TO MAKE ONLY WIMPY BURGERS:

$\frac{-\beta}{25}$ > slope constraint ①

$\frac{-\beta}{25} > -\frac{4}{3}$

$\beta < \frac{4 \cdot 25}{3}$

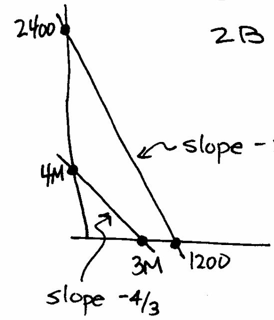
$\beta < 33.33$

$\beta \leq 33$ cents

cost of big burger $\leq$ 33 cents

d) Constraints: $\frac{1}{3}B + \frac{1}{4}W \leq M \rightsquigarrow W \leq 4M - 4/3 B$

$2B + W \leq 2400 \rightsquigarrow W \leq 2400 - 2B$



since slope of objective function is between the slopes of the two constraints:

1) if $3M < 1200$ then optimum is $(3M, 0)$
2) if $2400 < 4M$ then optimum is $(0, 2400)$
3) otherwise the optimum is at the intersection of the two constraints.

In this problem we have case 1, thus $(B, W) = (3M, 0)$

So in objective function, $p = 45B + 25W$

$= 45(3M) + 0$

$= 135M$

So if we increase meat by 1 lb, $M \to M+1$ then profit $p = 135M \to 135M + 135$. So by adding 1 lb of meat we increase our profit by \$1.35. Thus to justify buying more meat, the cost should be less than \$1.35/lb.

5.4
(4 pts)

$y = ax$

x_i	8	16	32
y_i	10	14	31

objective function

$\underline{\max\ \varepsilon_{max}}$, $\varepsilon_{max} = \max_i |\varepsilon_i|$, $\varepsilon_i = y_i - ax_i$

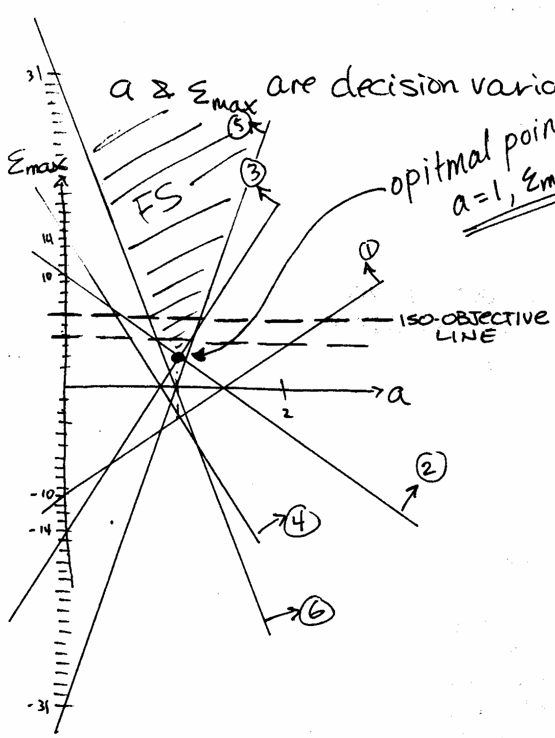
constraints:

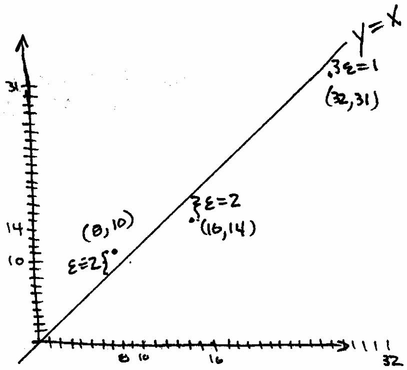
$|y_i - ax_i| \le \varepsilon_{max} \Rightarrow -\varepsilon_{max} \le y_i - ax_i \le \varepsilon_{max}$

or

$$\left.\begin{array}{l} -\varepsilon_{max} - y_i + ax_i \le 0 \\ -\varepsilon_{max} + y_i - ax_i \le 0 \end{array}\right\} \xrightarrow{\text{data}} \begin{array}{ll} -\varepsilon_{max} + a\cdot 8 \le 10 & (1) \\ -\varepsilon_{max} - a\cdot 8 \le -10 & (2) \\ -\varepsilon_{max} + a\cdot 16 \le 14 & (3) \\ -\varepsilon_{max} - a\cdot 16 \le -14 & (4) \\ -\varepsilon_{max} + a\cdot 32 \le 31 & (5) \\ -\varepsilon_{max} - a\cdot 32 \le -31 & (6) \end{array}$$

a & ε_{max} are decision variables.





here we can see there are actually two points which have the error of ε_{max}.

5.6

(2 pts) $y = ax^2 + bx + c$

x_i	1	2	3	4
y_i	2.3	5.3	8.7	16.1

~~max~~ $\min\ \varepsilon_{max}$

s.t. $|y_i - ax_i^2 - bx_i - c| \le \varepsilon_{max}$

$\Rightarrow \min\ \varepsilon_{max}$

s.t. $y_i - ax_i^2 - bx_i - c - \varepsilon_{max} \le 0$

$-y_i + ax_i^2 + bx_i + c - \varepsilon_{max} \le 0$

$\Rightarrow \min\ \varepsilon_{max}$

s.t. $-ax_i^2 - bx_i - c - \varepsilon_{max} \le -y_i$

$ax_i^2 + bx_i + c - \varepsilon_{max} \le y_i$

$\Rightarrow \min\ \varepsilon_{max}$

decision variables

s.t.
$$\begin{bmatrix} -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & -1 \\ -4 & -2 & -1 & -1 \\ 4 & 2 & 1 & -1 \\ -9 & -3 & -1 & -1 \\ 9 & 3 & 1 & -1 \\ -16 & -4 & -1 & -1 \\ 16 & 4 & 1 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ \varepsilon_{max} \end{bmatrix} \le \begin{bmatrix} -2.3 \\ 2.3 \\ -5.3 \\ 5.3 \\ -8.7 \\ 8.7 \\ -16.1 \\ 16.1 \end{bmatrix}$$

If you solve you find $\underline{y = 1.1x^2 - 1.2x + 2.85,\ \varepsilon_{max} = .45}$