Linear Algebra for Data Science

Math/DSCI 569: Course Outlines A–D

Colorado State University

July 29, 2026

Course A	Matrices and Vector Spaces (a primer)
Course B	Geometric Techniques for Data Reduction
Course C	Matrix Factorizations and Transformations
Course D	Theoretical Foundations

Contents

Course A Matrices and Vector Spaces (a primer)

Module 1 — Terminology and Basic Operations

Reading Assignment

LADS 1.1, 1.2

Problem Set

Solve Exercises: 1.1-1.20, 1.62-1.69.

- Lecture 1: Data, Vectors, and Matrices

  Learning Outcome: Students will be able to define vectors and matrices and describe the notation used to represent their entries.

- Lecture 2: Vector Operations

  Learning Outcome: Students will be able to perform vector addition and scalar multiplication.

- Lecture 3: Geometry of vectors

  Learning Outcome: Students will be able to compute the length of a vector and the distance between two vectors.

- Lecture 4: Angles

  Learning Outcome: Students will be able to compute the dot product of two vectors and use it to determine the angle between them.

- Lecture 5: Classification

  Learning Outcome: Students will be able to apply the dot product to classify the geometric relationship between vectors.

Module 2 — Matrix Operations

Reading Assignment

LADS 1.3, 1.4

Problem Set

Solve Exercises: 1.21, 1.22, 1.23, 1.25, 1.28, 1.29, 1.32, 1.37, 1.39, 1.44, 1.45.

- Lecture 1: Basic Matrix Operations

  Learning Outcome: Students will be able to perform matrix addition, subtraction, and scalar multiplication.

- Lecture 2: Multiplication of a vector by a matrix

  Learning Outcome: Students will be able to compute the product of a matrix and a vector.

- Lecture 3: Matrix–Matrix Multiplication

  Learning Outcome: Students will be able to compute the product of two matrices.

- Lecture 4: The Matrix Transpose

  Learning Outcome: Students will be able to compute the transpose of a matrix and verify its basic properties.

Module 3 — Linear Systems

Reading Assignment

LADS 2.1-2.4

Problem Set

Solve Exercises: 2.2, 2.4, 2.11, 2.12, 2.17, 2.18, 2.24, 2.25, 2.28, 2.31

- Lecture 1: An Illustrative Example

  Learning Outcome: Students will be able to set up and solve a linear system using a motivating example.

- Lecture 2: Elementary Row Operations via Elementary Matrices

  Learning Outcome: Students will be able to represent elementary row operations as multiplication by elementary matrices.

- Lecture 3: Echelon Form and Pivots

  Learning Outcome: Students will be able to identify pivot positions and reduce a matrix to echelon form.

- Lecture 4: Gaussian Elimination

  Learning Outcome: Students will be able to solve a linear system using Gaussian elimination.

- Lecture 5: Rank and Nullity

  Learning Outcome: Students will be able to compute the rank and nullity of a matrix.

Module 4 — The Geometry of Solutions

Reading Assignment

LADS 2.4

Problem Set

Solve Exercises: 2.40, 2.41, 2.43, 2.45, 2.46, 2.47, 2.48, 2.49.

- Lecture 1: Geometry of Solutions

  Learning Outcome: Students will be able to describe the geometric interpretation of the solution set of a linear system.

- Lecture 2: No Solution

  Learning Outcome: Students will be able to identify when a linear system has no solution.

- Lecture 3: Non-unique solutions

  Learning Outcome: Students will be able to identify when a linear system has infinitely many solutions and describe the solution set.

- Lecture 4: Underdetermined System $m < n$

  Learning Outcome: Students will be able to analyze the solution set of an underdetermined linear system.

- Lecture 5: Overdetermined systems $m > n$

  Learning Outcome: Students will be able to analyze the solution set of an overdetermined linear system.

Module 5 — The Matrix Inverse

Reading Assignment

LADS 2.5

Problem Set

Solve Exercises: 2.50, 2.51, 2.52, 2.53, 2.54, 2.55, 2.56, 2.57, 2.59.

- Lecture 1: The Matrix Inverse

  Learning Outcome: Students will be able to define the inverse of a matrix.

- Lecture 2: Basic Properties of Inverses

  Learning Outcome: Students will be able to prove basic properties of matrix inverses.

- Lecture 3: Elementary Matrices and the Inverse

  Learning Outcome: Students will be able to compute a matrix inverse using elementary matrices.

- Lecture 4: Zero Solution Existence Criteria

  Learning Outcome: Students will be able to determine the conditions under which a homogeneous system has only the zero solution.

- Lecture 5: Matrix Inversion Identities

  Learning Outcome: Students will be able to prove and apply identities involving matrix inverses.

Module 6 — Vector spaces and subspaces

Reading Assignment

LADS 3.1, 3.2

Problem Set

Solve Exercises: 3.1, 3.2, 3.5, 3.6, 3.8, 3.9, 3.10, 3.11, 3.14

- Lecture 1: What Is a Set?

  Learning Outcome: Students will be able to describe the basic set-theoretic language used to define a vector space.

- Lecture 2: What Is a Vector Space?

  Learning Outcome: Students will be able to verify whether a given set together with defined operations forms a vector space.

- Lecture 3: Subspace of a Vector Space

  Learning Outcome: Students will be able to verify whether a subset of a vector space is a subspace.

- Lecture 4: The Null Space is a Subspace

  Learning Outcome: Students will be able to prove that the null space of a matrix is a subspace.

- Lecture 5: More Subspace Examples

  Learning Outcome: Students will be able to identify and verify subspaces in a variety of examples.

Module 7 — The Direct Sum Subspaces

Reading Assignment

LADS 3.3

Problem Set

Solve Exercises: 3.16, 3.17 , 3.18, 3.19, 3.21, 3.22, 3.23, 3.27, 3.28

- Lecture 1: Affine Subspaces

  Learning Outcome: Students will be able to define an affine subspace and distinguish it from a linear subspace.

- Lecture 2: Sum of Subspaces

  Learning Outcome: Students will be able to compute the sum of two subspaces.

- Lecture 3: The Direct Sum

  Learning Outcome: Students will be able to determine when a sum of subspaces is a direct sum.

- Lecture 4: Uniqueness of the Direct Sum and Applications

  Learning Outcome: Students will be able to prove that every vector in a direct sum has a unique decomposition.

Module 8 — The Column Space

Reading Assignment

LADS 4.1

Problem Set

Solve Exercises: 4.1-4.10

- Lecture 1: Linear Combinations

  Learning Outcome: Students will be able to form and identify linear combinations of vectors.

- Lecture 2: The span is a subspace

  Learning Outcome: Students will be able to prove that the span of a set of vectors is a subspace.

- Lecture 3: The Column Space/Range of a Matrix

  Learning Outcome: Students will be able to compute the column space of a matrix.

- Lecture 4: Linear Combinations in Data

  Learning Outcome: Students will be able to interpret linear combinations of data vectors.

Module 9 — The Null Space and Independence

Reading Assignment

LADS 4.2

Problem Set

Solve Exercises: 4.12, 4.13, 4.14, 4.15, 4.16, 4.18, 4.20, 4.23

- Lecture 1: Dependence and Independence by Example

  Learning Outcome: Students will be able to determine whether a set of vectors is linearly independent using illustrative examples.

- Lecture 2: The Null Space of a Matrix

  Learning Outcome: Students will be able to compute the null space of a matrix.

- Lecture 3: $k+1$ Vectors in $\mathbb{R}^k$ Are Always Dependent

  Learning Outcome: Students will be able to prove that any set of $k+1$ vectors in $\mathbb{R}^k$ is linearly dependent.

- Lecture 4: Matrix Invertibility and Linear Independence

  Learning Outcome: Students will be able to relate the invertibility of a matrix to the linear independence of its columns.

- Lecture 5: Two Propositions on Independence

  Learning Outcome: Students will be able to prove two fundamental propositions concerning linear independence.

Module 10 — The Row Space and Left Null Space

Reading Assignment

LADS 4.3, 4.4

Problem Set

Solve Exercises: 4.32, 4.33, 4.34, 4.35, 4.36, 4.39, 4.41

- Lecture 1: The Row Space

  Learning Outcome: Students will be able to define the row space of a matrix.

- Lecture 2: Finding the Row Space

  Learning Outcome: Students will be able to compute a basis for the row space of a matrix.

- Lecture 3: Row Operations Preserve the Row Space

  Learning Outcome: Students will be able to prove that elementary row operations preserve the row space of a matrix.

- Lecture 4: The Left Null Space

  Learning Outcome: Students will be able to compute the left null space of a matrix.

- Lecture 5: The Four Fundamental Subspaces of a Matrix

  Learning Outcome: Students will be able to identify and compute the four fundamental subspaces of a matrix.

Module 11 — The Basis of a Vector Space

Reading Assignment

LADS 5.1-5.4

Problem Set

Solve Exercises: 5.1, 5.2, 5.3, 5.4, 5.7, 5.8, 5.13

- Lecture 1: The Basis

  Learning Outcome: Students will be able to determine whether a set of vectors forms a basis for a vector space.

- Lecture 2: Independent Sets are never bigger than Spanning Sets

  Learning Outcome: Students will be able to prove that an independent set can never be larger than a spanning set.

- Lecture 3: The Dimension of a Vector Space

  Learning Outcome: Students will be able to compute the dimension of a vector space.

- Lecture 4: The Goldilocks Algorithm

  Learning Outcome: Students will be able to apply the Goldilocks Algorithm to construct a basis from a spanning set.

Module 12 — Bases for the Four Fundamental Subspaces

Reading Assignment

LADS 5.5

Problem Set

Solve Exercises: 5.16, 5.17, 5.19, 5.20, 5.21, 5.25, 5.27, 5.34

- Lecture 1: Bases for the Row and Column Spaces

  Learning Outcome: Students will be able to compute bases for the row and column spaces of a matrix.

- Lecture 2: Basis for the Null Space

  Learning Outcome: Students will be able to compute a basis for the null space of a matrix.

- Lecture 3: The Rank-Nullity Theorem

  Learning Outcome: Students will be able to state and prove the Rank-Nullity Theorem.

- Lecture 4: Row Rank Equals Column Rank

  Learning Outcome: Students will be able to prove that the row rank of a matrix equals its column rank.

Module 13 — Change of Basis

Reading Assignment

LADS 5.7

Problem Set

Solve Exercises: 5.35, 5.36, 5.37, 5.38, 5.39, 5.40, 5.44, 5.48

- Lecture 1: Why Change Basis?

  Learning Outcome: Students will be able to explain the motivation for representing a vector with respect to a different basis.

- Lecture 2: Coordinates with Respect to a Basis

  Learning Outcome: Students will be able to compute the coordinates of a vector with respect to a given basis.

- Lecture 3: Standard Basis to a New Basis

  Learning Outcome: Students will be able to convert the coordinates of a vector from the standard basis to a new basis.

- Lecture 4: Similarity Transformations

  Learning Outcome: Students will be able to construct and apply a similarity transformation to change a matrix representation.

Module 14 — Orthogonality

Reading Assignment

LADS 6.1, 6.2

Problem Set

Solve Exercises: 6.1, 6.2, 6.3, 6.6, 6.9, 6.11, 6.13, 6.14, 6.15, 6.17.

- Lecture 1: The Angle Between Two Vectors (Lax's Derivation)

  Learning Outcome: Students will be able to derive the formula for the angle between two vectors following Lax's approach.

- Lecture 2: Orthogonal Vectors, Matrices and Subspaces

  Learning Outcome: Students will be able to determine whether given vectors, matrices, or subspaces are orthogonal.

- Lecture 3: The Orthogonal Direct Sum

  Learning Outcome: Students will be able to prove that a subspace and its orthogonal complement form a direct sum.

- Lecture 4: The Four Fundamental Subspaces of a Matrix

  Learning Outcome: Students will be able to describe the orthogonality relationships among the four fundamental subspaces of a matrix.

- Lecture 5: The Orthogonal Complement

  Learning Outcome: Students will be able to compute the orthogonal complement of a subspace.

Module 15 — Projections

Reading Assignment

LADS 6.3

Problem Set

Solve Exercises: 6.19, 6.20, 6.22, 6.23, 6.26, 6.28, 6.29, 6.30, 6.32, 6.36.

- Lecture 1: What Is a Projection?

  Learning Outcome: Students will be able to define a projection operator on a vector space.

- Lecture 2: Range and Null Space of a Projection

  Learning Outcome: Students will be able to determine the range and null space of a projection.

- Lecture 3: Projections and Direct Sums

  Learning Outcome: Students will be able to relate a projection to the direct sum decomposition it defines.

- Lecture 4: Orthogonal Projections

  Learning Outcome: Students will be able to determine whether a given projection is orthogonal.

- Lecture 5: Constructing Projections using the Outer Product Formula

  Learning Outcome: Students will be able to construct a projection matrix using the outer product formula.

Course B Geometric Techniques for Data Reduction

Module 1 — Orthogonalization and QR Factorization

Reading Assignment

LADS 6.4

Problem Set

Solve Exercises: 6.37, 6.38, 6.39

- Lecture 1: The Gram–Schmidt Procedure

  Learning Outcome: Students will be able to apply the Gram-Schmidt procedure to construct an orthonormal set of vectors.

- Lecture 2: Gram–Schmidt in $\mathbb{R}^4$

  Learning Outcome: Students will be able to carry out the Gram-Schmidt procedure on a concrete example in $\mathbb{R}^4$.

- Lecture 3: Gram-Schmidt Step k

  Learning Outcome: Students will be able to derive the general step k of the Gram-Schmidt procedure.

- Lecture 5: Volume via QR

  Learning Outcome: Students will be able to compute the volume of a parallelepiped using the QR factorization.

Module 2 — Fitting Models to Data

Reading Assignment

LADS 7.1

Problem Set

Solve Exercises: 7.1, 7.3, 7.4, 7.9, 7.14

- Lecture 1: Data Fitting

  Learning Outcome: Students will be able to formulate a data-fitting problem as a linear system.

- Lecture 2: Approximate Solution of the Overdetermined Problem

  Learning Outcome: Students will be able to derive the least-squares solution to an overdetermined system.

- Lecture 3: A Rank Deficient Example

  Learning Outcome: Students will be able to identify and analyze a rank-deficient least-squares problem.

- Lecture 4: Examples of Solving Overdetermined Systems

  Learning Outcome: Students will be able to solve overdetermined systems using the least-squares method.

Module 3 — Classification

Reading Assignment

LADS 7.2

Problem Set

Solve Exercises:

- Lecture 1: The Classification Problem

  Learning Outcome: Students will be able to formulate a classification problem in terms of linear algebra.

- Lecture 2: Hyperplanes, Half-Spaces, and Convexity

  Learning Outcome: Students will be able to define hyperplanes and half-spaces and determine whether a given set is convex.

- Lecture 3: Fisher Discriminant Analysis

  Learning Outcome: Students will be able to evaluate the quality of a projection used for classification.

- Lecture 4: Derivation

  Learning Outcome: Students will be able to derive the optimal classification rule.

- Lecture 5: Simplification of Solution, Example

  Learning Outcome: Students will be able to simplify the classification solution and apply it to a worked example.

Module 4 — The Determinant

Reading Assignment

LADS 8

Problem Set

Solve Exercises:

- Lecture 1: Axioms for the Determinant

  Learning Outcome: Students will be able to state the axioms that uniquely define the determinant function.

- Lecture 2: Computing the Determinant

  Learning Outcome: Students will be able to compute the determinant of a matrix.

- Lecture 3: Minors and Cofactors

  Learning Outcome: Students will be able to compute the minors and cofactors of a matrix.

- Lecture 4: Properties of Determinants

  Learning Outcome: Students will be able to prove key properties of determinants.

- Lecture 5: Volumes of Parallelepipeds and Applications

  Learning Outcome: Students will be able to compute the volume of a parallelepiped using the determinant.

Module 5 — Eigenvalues and Eigenvectors

Reading Assignment

LADS 9.1

Problem Set

Solve Exercises:

- Lecture 1: Eigenvalues and Eigenvectors

  Learning Outcome: Students will be able to define the eigenvalues and eigenvectors of a matrix.

- Lecture 2: Computing Eigenvalues and Eigenvectors

  Learning Outcome: Students will be able to compute the eigenvalues and eigenvectors of a matrix.

- Lecture 3: Eigenspaces

  Learning Outcome: Students will be able to determine the eigenspace associated with a given eigenvalue.

- Lecture 4: The Characteristic Polynomial

  Learning Outcome: Students will be able to construct the characteristic polynomial of a matrix and use it to find eigenvalues.

Module 6 — Diagonalization

Reading Assignment

LADS 9.2

Problem Set

Solve Exercises:

- Lecture 1: Google's PageRank Algorithm

  Learning Outcome: Students will be able to explain how Google's PageRank algorithm uses eigenvectors to rank webpages.

- Lecture 2: Diagonalization

  Learning Outcome: Students will be able to determine whether a matrix is diagonalizable and construct its diagonalization.

- Lecture 3: Eigenvectors and Change of Basis

  Learning Outcome: Students will be able to use eigenvectors to change the basis used to represent a matrix.

- Lecture 4: Powers of Matrices

  Learning Outcome: Students will be able to compute powers of a matrix using its diagonalization.

- Lecture 5: Spectral Theory Key Results

  Learning Outcome: Students will be able to summarize the key results of spectral theory covered in this module.

Module 7 — The Singular Value Decomposition

Reading Assignment

LADS 10.1, 10.2

Problem Set

Solve Exercises:

- Lecture 1: What is the Singular Value Decomposition?

  Learning Outcome: Students will be able to define the Singular Value Decomposition of a matrix.

- Lecture 2: Applications

  Learning Outcome: Students will be able to describe applications of the Singular Value Decomposition.

- Lecture 3: Construction of the SVD

  Learning Outcome: Students will be able to construct the Singular Value Decomposition of a matrix.

- Lecture 4: Singular Vectors are Eigenvectors

  Learning Outcome: Students will be able to prove the relationship between singular vectors and the eigenvectors of related matrices.

Module 8 — The Geometry Revealing Factorization & Bases for the Fundamental Subspaces

Reading Assignment

LADS 10.3, 10.4

Problem Set

Solve Exercises:

- Lecture 1: The Geometry Revealing Factorization

  Learning Outcome: Students will be able to explain how the SVD reveals the geometric structure of a linear transformation.

- Lecture 2: Geometric Illustration

  Learning Outcome: Students will be able to illustrate the geometric action of a matrix using its SVD.

- Lecture 3: The Four Fundamental Subspaces via SVD

  Learning Outcome: Students will be able to identify the four fundamental subspaces of a matrix from its SVD.

- Lecture 4: Basis for the Left Null Space

  Learning Outcome: Students will be able to compute a basis for the left null space using the SVD.

- Lecture 5: Row Space and Null Space

  Learning Outcome: Students will be able to compute bases for the row space and null space using the SVD.

Module 9 — SVD-LDA

Reading Assignment

LADS 10.7

Problem Set

Solve Exercises:

- Lecture 1: High-Dimensional Fisher Discriminant Analysis

  Learning Outcome: Students will be able to describe the Fisher Discriminant Analysis problem in high dimensions.

- Lecture 2: SVD Dimension Reduction

  Learning Outcome: Students will be able to use the SVD to reduce the dimension of high-dimensional data.

- Lecture 3: FDA in Reduced Coordinates

  Learning Outcome: Students will be able to perform Fisher Discriminant Analysis in reduced coordinates.

- Lecture 4: SVD-FDA Algorithm Summary

  Learning Outcome: Students will be able to summarize the steps of the SVD-FDA algorithm.

- Lecture 5: Projection and Classification

  Learning Outcome: Students will be able to apply the SVD-FDA projection to classify data.

Module 10 — Introduction to Principal Component Analysis

Reading Assignment

LADS 10.8.1

Problem Set

Solve Exercises:

- Lecture 1: Which Direction Fits the Data Best?

  Learning Outcome: Students will be able to formulate the problem of finding the direction that best fits a data set.

- Lecture 2: The Optimal Subspace Problem

  Learning Outcome: Students will be able to formulate the optimal subspace problem for dimension reduction.

- Lecture 3: Mean Squared Projection Revisited

  Learning Outcome: Students will be able to derive the mean squared projection error onto a subspace.

- Lecture 4: Connection to SVD

  Learning Outcome: Students will be able to explain the connection between Principal Component Analysis and the Singular Value Decomposition.

Module 11 — PCA – The Optimal Eigenbasis, Mean Subtraction, and Whitening

Reading Assignment

LADS 10.8.2-10.8.4

Problem Set

Solve Exercises:

- Lecture 1: The Optimal Eigenbasis

  Learning Outcome: Students will be able to derive the optimal eigenbasis for representing a data set.

- Lecture 2: Geometry and Eigenvalues

  Learning Outcome: Students will be able to relate the geometry of a data set to the eigenvalues of its covariance matrix.

- Lecture 3: When Eigenvalues must be zero

  Learning Outcome: Students will be able to determine the conditions under which eigenvalues of the covariance matrix must be zero.

- Lecture 4: Rank drop with mean subtracted data and sign ambiguity

  Learning Outcome: Students will be able to explain the rank drop that occurs with mean-subtracted data and the resulting sign ambiguity in the eigenvectors.

Module 12 — Data Preprocessing and a Sample Study

Reading Assignment

Problem Set

Solve Exercises:

- Lecture 1: Mean Subtraction

  Learning Outcome: Students will be able to compute the mean-subtracted version of a data set.

- Lecture 2: Data Whitening

  Learning Outcome: Students will be able to apply data whitening to a data set.

- Lecture 3: A Sample PCA Study

  Learning Outcome: Students will be able to carry out a complete PCA study on a sample data set.

Module 13 — The Snapshot Method

Reading Assignment

LADS 10.3

Problem Set

Solve Exercises:

- Lecture 1: The Snapshot Method Motivation

  Learning Outcome: Students will be able to explain the motivation for the snapshot method.

- Lecture 2: Deriving the Snapshot Formula

  Learning Outcome: Students will be able to derive the snapshot method formula.

- Lecture 3: Snapshot Method Examples

  Learning Outcome: Students will be able to apply the snapshot method to compute an SVD-based decomposition.

Module 14 — Laplacian Eigenmaps

Reading Assignment

LADS 11.1

Problem Set

Solve Exercises:

- Lecture 1: Graphs

  Learning Outcome: Students will be able to represent data using a graph structure.

- Lecture 2: Graph Drawing

  Learning Outcome: Students will be able to construct a graph drawing from a given graph.

- Lecture 3: The Cost Function

  Learning Outcome: Students will be able to formulate the cost function used in Laplacian eigenmaps.

- Lecture 4: Normalization Example

  Learning Outcome: Students will be able to apply normalization to a Laplacian eigenmap example.

Module 15 — Morse Code Application and Manifold Learning

Reading Assignment

LADS 11.1

Problem Set

Solve Exercises:

- Lecture 1: Morse Code Data

  Learning Outcome: Students will be able to represent Morse code data in a form suitable for manifold learning.

- Lecture 2: Three Nearest Neighbors

  Learning Outcome: Students will be able to construct a nearest-neighbor graph using the three nearest neighbors of each point.

- Lecture 3: Manifold Learning for Dimension Reduction

  Learning Outcome: Students will be able to apply manifold learning techniques to reduce the dimension of a data set.

Course C Matrix Factorizations and Transformations

Module 1 — Graph Clustering

Reading Assignment

LADS 11.1.2

Problem Set

Solve Exercises:

- Lecture 1: The Graph Clustering Approach

  Learning Outcome: Students will be able to formulate a graph clustering problem as a partition of a node set.

- Lecture 2: Graph Cuts via Optimization (Shi and Malik)

  Learning Outcome: Students will be able to state the Shi-Malik graph cut optimization formulation.

- Lecture 3: Graph Clustering

  Learning Outcome: Students will be able to apply spectral methods to perform graph clustering.

Module 2 — The Multidimensional Scaling Algorithm

Reading Assignment

LADS 11.2

Problem Set

Solve Exercises:

- Lecture 1: The Distance Matrix

  Learning Outcome: Students will be able to construct a distance matrix from a set of points.

- Lecture 2: The MDS Algorithm

  Learning Outcome: Students will be able to describe and apply the steps of the MDS algorithm.

- Lecture 3: The Unit Distance Graph with Three Points

  Learning Outcome: Students will be able to construct an MDS embedding for a simple three-point example.

- Lecture 4: Schoenburg's Theorem

  Learning Outcome: Students will be able to state and apply Schoenberg's Theorem to characterize valid distance matrices.

Module 3 — MDS and Geometric Structure

Reading Assignment

LADS 11.2

Problem Set

Solve Exercises:

- Lecture 1: Best approximation theorem

  Learning Outcome: Students will be able to state the Best Approximation Theorem as it applies to multidimensional scaling.

- Lecture 2: MDS of the Circle (sampled)

  Learning Outcome: Students will be able to compute an MDS embedding of points sampled from a circle.

- Lecture 3: MDS embedding of Morse code digits

  Learning Outcome: Students will be able to construct an MDS embedding of Morse code digit data.

- Lecture 4: Pot Pourri

  Learning Outcome: Students will be able to apply MDS techniques to a variety of additional examples.

Module 4 — Angles Between Subspaces

Reading Assignment

LADS 12.1, 12.2

Problem Set

Solve Exercises:

- Lecture 1: Philosophy of Subspace Encoding

  Learning Outcome: Students will be able to explain the motivation for encoding the relationship between two subspaces.

- Lecture 2: The Optimization Problem for Angles Between Subspaces

  Learning Outcome: Students will be able to formulate the optimization problem that defines the principal angles between subspaces.

- Lecture 3: Computing Principal Angles and Principal Vectors

  Learning Outcome: Students will be able to compute the principal angles and principal vectors between two subspaces.

- Lecture 4: Worked Example

  Learning Outcome: Students will be able to apply the principal angle computation to a worked example.

Module 5 — Canonical Correlation Analysis

Reading Assignment

LADS 12.2

Problem Set

Solve Exercises:

- Lecture 1: Canonical Correlation Analysis Formulation

  Learning Outcome: Students will be able to formulate the Canonical Correlation Analysis problem.

- Lecture 2: Reduction via QR Factorization

  Learning Outcome: Students will be able to use QR factorization to reduce the Canonical Correlation Analysis problem.

- Lecture 3: Side-by-Side Derivation

  Learning Outcome: Students will be able to derive the Canonical Correlation Analysis solution using a side-by-side comparison approach.

- Lecture 4: CCA Algorithm and Joint Algorithm

  Learning Outcome: Students will be able to implement the Canonical Correlation Analysis algorithm and its joint formulation.

Module 6 — Subspace Geometry

Reading Assignment

LADS 12.3-12.5

Problem Set

Solve Exercises:

- Lecture 1: Geometric Framework of the Grassmannian

  Learning Outcome: Students will be able to describe the geometric framework of the Grassmannian manifold.

- Lecture 2: Geodesics on the Grassmannian

  Learning Outcome: Students will be able to compute geodesics on the Grassmannian.

- Lecture 3: Averaging Subspaces

  Learning Outcome: Students will be able to compute the average of a collection of subspaces on the Grassmannian.

- Lecture 4: The Flag Mean

  Learning Outcome: Students will be able to compute the flag mean of a set of subspaces.

Module 7 — Generalized Singular Value Decomposition

Reading Assignment

LADS 13.1-13.3

Problem Set

Solve Exercises:

- Lecture 1: Introduction to the GSVD

  Learning Outcome: Students will be able to define the Generalized Singular Value Decomposition.

- Lecture 2: The CS Decomposition Theory Sketch

  Learning Outcome: Students will be able to outline the CS Decomposition and explain its relation to the GSVD.

- Lecture 3: The Cosine-Sine decomposition implementation

  Learning Outcome: Students will be able to implement the cosine-sine decomposition.

- Lecture 4: Geometric interpretation of c_i

  Learning Outcome: Students will be able to interpret the generalized singular values geometrically.

Module 8 — GSVD and Generalized Singular Vectors

Reading Assignment

LADS 13.4, 13.5

Problem Set

Solve Exercises:

- Lecture 1: The GSVD (Full rank case)

  Learning Outcome: Students will be able to compute the GSVD for a full-rank pair of matrices.

- Lecture 2: GSVD Algorithm

  Learning Outcome: Students will be able to describe the steps of the GSVD algorithm.

- Lecture 3: The Generalized Singular Vector Equation

  Learning Outcome: Students will be able to derive the generalized singular vector equation.

- Lecture 4: Separating a Superposed Data Matrix

  Learning Outcome: Students will be able to use the GSVD to separate a superposed data matrix into its component sources.

Module 9 — Applications of the GSVD

Reading Assignment

LADS 13.7, 13.8

Problem Set

Solve Exercises:

- Lecture 1: Application to Hyperspectral Denoising

  Learning Outcome: Students will be able to apply the GSVD to denoise hyperspectral data.

- Lecture 2: Signal Unmixing

  Learning Outcome: Students will be able to formulate the signal unmixing problem using the GSVD.

- Lecture 3: Signal Unmixing Algorithm with Application

  Learning Outcome: Students will be able to implement the GSVD signal unmixing algorithm and apply it to data.

- Lecture 4: Bases and the GSVD

  Learning Outcome: Students will be able to relate the GSVD to bases for its associated subspaces.

Module 10 — Matching Pursuit

Reading Assignment

LADS 14.2

Problem Set

Solve Exercises:

- Lecture 1: Sparse Approximation Problem

  Learning Outcome: Students will be able to formulate the sparse approximation problem.

- Lecture 2: Greedy Selection Rule (w/ Example)

  Learning Outcome: Students will be able to apply the greedy selection rule of matching pursuit to a worked example.

- Lecture 3: Possible Repeated Column Selection

  Learning Outcome: Students will be able to explain why matching pursuit may select the same column more than once.

- Lecture 4: Matching Pursuit Pseudocode

  Learning Outcome: Students will be able to implement the matching pursuit algorithm from its pseudocode.

Module 11 — Orthogonal Matching Pursuit

Reading Assignment

LADS 14.3

Problem Set

Solve Exercises:

- Lecture 1: Overview of the Algorithm

  Learning Outcome: Students will be able to describe the overall structure of the orthogonal matching pursuit algorithm.

- Lecture 2: Worked OMP Example

  Learning Outcome: Students will be able to apply orthogonal matching pursuit to a worked example.

- Lecture 3: A key geometric property

  Learning Outcome: Students will be able to prove a key geometric property underlying orthogonal matching pursuit.

- Lecture 4: OMP Pseudocode

  Learning Outcome: Students will be able to implement orthogonal matching pursuit from its pseudocode.

Module 12 — Dictionary Learning with the K-SVD

Reading Assignment

Problem Set

Solve Exercises:

- Lecture 1: Problem Statement

  Learning Outcome: Students will be able to formulate the dictionary learning problem.

- Lecture 2: Exploiting sparse solutions

  Learning Outcome: Students will be able to explain how sparse solutions are exploited in dictionary learning.

- Lecture 3: Detailed Example

  Learning Outcome: Students will be able to work through a detailed example of dictionary learning.

- Lecture 4: The K-SVD Algorithm Summary and Example

  Learning Outcome: Students will be able to summarize the K-SVD algorithm and apply it to an example.

Module 13 — The Haar Wavelet via Projections

Reading Assignment

LADS 15.1.1-15.1.3

Problem Set

Solve Exercises:

- Lecture 1: Introduction to the Haar Wavelet

  Learning Outcome: Students will be able to define the Haar wavelet and its associated scaling function.

- Lecture 2: First Scaling Subspace Analysis

  Learning Outcome: Students will be able to analyze the first scaling subspace of the Haar wavelet decomposition.

- Lecture 3: Haar Scaling Space Reconstruction (synthesis)

  Learning Outcome: Students will be able to reconstruct a signal from its Haar scaling space representation.

- Lecture 4: Multiresolution Summary

  Learning Outcome: Students will be able to summarize the multiresolution structure of the Haar wavelet decomposition.

Module 14 — Recursive Analysis and Synthesis

Reading Assignment

LADS 15.1.4, 15.1.5

Problem Set

Solve Exercises:

- Lecture 1: Recursive Decomposition (analysis)

  Learning Outcome: Students will be able to perform a recursive wavelet decomposition of a signal.

- Lecture 2: Recursive Reconstruction (synthesis)

  Learning Outcome: Students will be able to reconstruct a signal recursively from its wavelet decomposition.

- Lecture 3: Equivalence of the Projection and Recursive Formulations

  Learning Outcome: Students will be able to prove the equivalence of the projection-based and recursive wavelet formulations.

- Lecture 4: 2D wavelet transform

  Learning Outcome: Students will be able to extend the wavelet transform to two-dimensional signals.

Module 15 — Discrete Global Transforms

Reading Assignment

LADS 15.2

Problem Set

Solve Exercises:

- Lecture 1: The Fourier Matrix

  Learning Outcome: Students will be able to construct the discrete Fourier matrix.

- Lecture 2: Complex inner product space

  Learning Outcome: Students will be able to define the complex inner product space used in the discrete Fourier transform.

- Lecture 3: The Fourier Spectrum

  Learning Outcome: Students will be able to compute the Fourier spectrum of a discrete signal.

- Lecture 4: The Discrete Cosine Transform

  Learning Outcome: Students will be able to construct and apply the Discrete Cosine Transform.

Course D Theoretical Foundations

Module 1 — The Basis of a Vector Space Theory

Reading Assignment

LADS 4.2, 5.3-5.5

Problem Set

Solve Exercises: 4.29, 4.30, 4.39, 5.14, 5.51

- Lecture 1: A Lemma on Independence

  Learning Outcome: Students will be able to prove an important theorem relating span and independence.

- Lecture 2: The Basis

  Learning Outcome: Students will be able to prove that independent sets cannot exceed spanning sets.

- Lecture 3: Adding and Removing Vectors to Form a Basis

  Learning Outcome: Students will be able to prove a series of theorems concerning the properties of a basis.

- Lecture 4: Existence of Complementary Spaces

  Learning Outcome: Students will be able to prove that any subspace of a vector space has a complementary space that can be used to form a direct sum.

Module 2 — Geometric Structure Theory

Reading Assignment

LADS 6.3-6.5

Problem Set

Solve Exercises: 6.46, 6.49, 6.51, 6.54

- Lecture 1: Normed Vector Spaces

  Learning Outcome: Students will be able to verify whether a given function defines a norm on a vector space.

- Lecture 2: Inner Product Spaces

  Learning Outcome: Students will be able to verify whether a given function defines an inner product on a vector space.

- Lecture 3: Norms Induced by Inner Product Spaces

  Learning Outcome: Students will be able to prove that every inner product induces a norm and derive its basic properties.

- Lecture 4: Best Approximation Theorem

  Learning Outcome: Students will be able to prove the Best Approximation Theorem and use it to find the closest vector in a subspace.

Module 3 — The Determinant

Reading Assignment

LADS 8.1-8.4

Problem Set

Solve Exercises: 8.14, 8.15, 8.16, 8.18, 8.19

- Lecture 1: Axioms for the Determinant

  Learning Outcome: Students will be able to state the axioms that uniquely define the determinant function.

- Lecture 2: Properties of Determinants

  Learning Outcome: Students will be able to prove key properties of determinants that follow from the defining axioms.

- Lecture 3: Invertibility and the Determinant

  Learning Outcome: Students will be able to prove that a matrix is invertible if and only if its determinant is nonzero.

- Lecture 4 : Derivation of Laplace's Expansion

  Learning Outcome: Students will be able to derive Laplace's expansion formula and use it to compute determinants.

Module 4 — The Spectral Theorem for Real Symmetric Matrices

Reading Assignment

LADS 9.3

Problem Set

Solve Exercises: 9.16, 9.43, 9.46, 9.49

- Lecture 1: Symmetric Matrices Have Real Eigenvalues

  Learning Outcome: Students will be able to prove that every eigenvalue of a real symmetric matrix is real.

- Lecture 2: Symmetric Matrices Have Orthogonal Eigenvectors

  Learning Outcome: Students will be able to prove that eigenvectors of a symmetric matrix corresponding to distinct eigenvalues are orthogonal.

- Lecture 3: Schur's Theorem

  Learning Outcome: Students will be able to state and prove Schur's Theorem on unitary triangularization of a matrix.

- Lecture 4: Schur's Theorem Example

  Learning Outcome: Students will be able to apply Schur's Theorem to compute a Schur decomposition of a given matrix.

- Lecture 5: The Spectral Theorem

  Learning Outcome: Students will be able to prove the Spectral Theorem for real symmetric matrices.

- Lecture 6: Positive Semi-Definite Matrices

  Learning Outcome: Students will be able to characterize positive semi-definite matrices using the Spectral Theorem.

Module 5 — SVD Theory

Reading Assignment

LADS 10.6

Problem Set

Solve Exercises: 10.22, 10.24, 10.25, 10.261

- Lecture 1: The Eckart-Young Theorem

  Learning Outcome: Students will be able to state the Eckart-Young Theorem and explain its role in low-rank matrix approximation.

- Lecture 2: Orthogonal Matrices Preserve $\|\cdot\|_2$

  Learning Outcome: Students will be able to prove that orthogonal matrices preserve the 2-norm of a vector.

- Lecture 3: Statement of the Theorem

  Learning Outcome: Students will be able to state the Singular Value Decomposition Theorem and identify its component matrices.

- Lecture 4: The Frobenius Norm Eckart Young Theorem

  Learning Outcome: Students will be able to prove the Eckart-Young Theorem with respect to the Frobenius norm.

Module 6 — MDS Theory

Reading Assignment

LADS 11.2.2, 11.2.3

Problem Set

Solve Exercises: 11.20, 11.21, 11.22, 11.23

- Lecture 1: The MDS Algorithm Review and a Lemma

  Learning Outcome: Students will be able to outline the steps of the MDS algorithm and prove a supporting lemma.

- Lecture 2: Schoenberg's Theorem

  Learning Outcome: Students will be able to state and prove Schoenberg's Theorem characterizing Euclidean distance matrices.

- Lecture 3: Best Approximation Theorem

  Learning Outcome: Students will be able to prove the Best Approximation Theorem in the context of multidimensional scaling.

- Lecture 4: MDS Connection to SVD and PCA

  Learning Outcome: Students will be able to explain the mathematical connections between MDS, SVD, and PCA.

Module 7 — Generalized Singular Value Decomposition Theory

Reading Assignment

LADS 13.3-13.8

Problem Set

Solve Exercises: 13.8, 13.18, 13.19

- Lecture 1: The CS Decomposition Theory

  Learning Outcome: Students will be able to state and prove the CS Decomposition Theorem.

- Lecture 2: Orthogonality of the Columns

  Learning Outcome: Students will be able to prove that the columns of the matrices in the CS decomposition are orthogonal.

- Lecture 3: Generalized singular vectors

  Learning Outcome: Students will be able to define and compute generalized singular vectors.

- Lecture 4: Angles between subspaces

  Learning Outcome: Students will be able to compute the principal angles between two subspaces.

Module 8 — Variational Characterization of the Eigenvector Problem

Reading Assignment

LADS 16.1, 16.2

Problem Set

Exercises: 16.1, 16.2, 16.3, 16.4 16.5

- Lecture 1: The Rayleigh Quotient and Critical Points

  Learning Outcome: Students will be able to compute the critical points of the Rayleigh quotient.

- Lecture 2: The Rayleigh-Ritz Theorem

  Learning Outcome: Students will be able to state and prove the Rayleigh-Ritz Theorem.

- Lecture 3: Variational Characterization of Singular Values

  Learning Outcome: Students will be able to derive a variational characterization of singular values.

- Lecture 4: Finding the Second Eigenvector

  Learning Outcome: Students will be able to apply variational methods to find the second eigenvector of a matrix.

Module 9 — Trace Optimization

Reading Assignment

LADS 16.3

Problem Set

Solve Exercises: 16.6, 16.8, 16.11, 16.19

- Lecture 1: The Ky Fan Maximum Principle

  Learning Outcome: Students will be able to state the Ky Fan Maximum Principle.

- Lecture 2: The Lagrange Multiplier Approach

  Learning Outcome: Students will be able to apply the method of Lagrange multipliers to trace optimization problems.

- Lecture 3: Proof of the Ky Fan Max Principle

  Learning Outcome: Students will be able to prove the Ky Fan Maximum Principle.

- Lecture 4: Change of Coordinates (proof conclusion)

  Learning Outcome: Students will be able to complete the proof of the Ky Fan Maximum Principle using a change of coordinates.

- Lecture 5: PCA via the Trace

  Learning Outcome: Students will be able to derive Principal Component Analysis as a trace optimization problem.

Module 10 — The Procrustes Problem

Reading Assignment

LADS 16.3.2

Problem Set

Solve Exercises: 16.15, 16.16, 16.17

- Lecture 1: The Procrustes Problem Statement

  Learning Outcome: Students will be able to state the Procrustes problem.

- Lecture 2: The Procrustes Solution

  Learning Outcome: Students will be able to derive the solution to the Procrustes problem using the SVD.

- Lecture 3: Application to a Procrustes Rotation

  Learning Outcome: Students will be able to apply the Procrustes solution to compute an optimal rotation between two configurations.

Module 11 — The Linear Transformation Framework

Reading Assignment

LADS 17.1, 17.2

Problem Set

Solve Exercises: 17.3, 17.4, 17.8, 17.9

- Lecture 1: The Linear Transformation

  Learning Outcome: Students will be able to define a linear transformation between vector spaces.

- Lecture 2: Linearity

  Learning Outcome: Students will be able to verify whether a given transformation is linear.

- Lecture 3: Properties and Examples

  Learning Outcome: Students will be able to identify key properties of linear transformations through examples.

- Lecture 4: The Coordinate Transformation

  Learning Outcome: Students will be able to compute the coordinate representation of a vector with respect to a given basis.

- Lecture 5: The Range and Null Space of a Linear Transformation

  Learning Outcome: Students will be able to determine the range and null space of a linear transformation.

Module 12 — Injectivity, and Surjectivity, and Inverses

Reading Assignment

LADS 17.3, 17.4

Problem Set

Solve Exercises: 17.16, 17.17, 17.18, 17.19

- Lecture 1: Injections and Surjections

  Learning Outcome: Students will be able to determine whether a given linear transformation is injective or surjective.

- Lecture 2: Differentiation (surjective) and Integration (Injective)

  Learning Outcome: Students will be able to prove that differentiation is surjective and integration is injective on appropriate function spaces.

- Lecture 3: Theorems about Injectivity, Surjectivity and Inverses

  Learning Outcome: Students will be able to prove theorems relating injectivity, surjectivity, and invertibility of linear transformations.

- Lecture 4: Additional Properties of Inverse

  Learning Outcome: Students will be able to prove additional properties of inverse linear transformations.

Module 13 — Isomorphisms and Matrix Representations

Reading Assignment

LADS 17.5

Problem Set

Solve Exercises: 17.22, 23, 30, 32, 34.

- Lecture 1: Isomorphisms

  Learning Outcome: Students will be able to define an isomorphism between vector spaces and determine whether a given transformation is an isomorphism.

- Lecture 2: Isomorphic Spaces Have the Same Dimension

  Learning Outcome: Students will be able to prove that isomorphic vector spaces have the same dimension.

- Lecture 3: Matrix Representations

  Learning Outcome: Students will be able to construct the matrix representation of a linear transformation with respect to given bases.

- Lecture 4: The Big Isomorphism Theorem

  Learning Outcome: Students will be able to state and prove the Big Isomorphism Theorem.

Module 14 — The Pseudoinverse

Reading Assignment

LADS 18.1-18.5

Problem Set

Solve Exercises: 18.1, 18.10, 18.11

- Lecture 1: When the Inverse Fails

  Learning Outcome: Students will be able to identify the conditions under which a matrix fails to have an inverse.

- Lecture 2: Fundamental Subspaces and the Pseudo-Inverse

  Learning Outcome: Students will be able to relate the fundamental subspaces of a matrix to its pseudo-inverse.

- Lecture 3: Computing the Pseudo-Inverse via the SVD

  Learning Outcome: Students will be able to compute the pseudo-inverse of a matrix using its singular value decomposition.

- Lecture 4: Linear Systems

  Learning Outcome: Students will be able to use the pseudo-inverse to solve linear systems that lack a unique solution.

- Lecture 5: The Moore-Penrose Conditions

  Learning Outcome: Students will be able to verify the Moore-Penrose conditions that characterize the pseudo-inverse.

Module 15 — The Pseudo-Inverse of Linear Transformations

Reading Assignment

LADS 18.6

Problem Set

Solve Exercises: 18.7

- Lecture 1: Extending to Abstract Linear Transformations

  Learning Outcome: Students will be able to extend the definition of the pseudo-inverse to abstract linear transformations.

- Lecture 2: The Pseudoinverse of Integration

  Learning Outcome: Students will be able to compute the pseudo-inverse of the integration operator.